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Latest lattice results of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with some topological insights

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We summarise the latest results of our collaboration concerning $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in four dimensions on the lattice. We investigate the expected formation of supersymmetric multiplets of the lightest particles and the behaviour of the topological susceptibility approaching the supersymmetric limit of the theory.

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PoS(LATTICE2014)273

1. Introduction

Supersymmetry (SUSY) has proved to be a powerful concept which has been explored by physicists in different contexts for decades. It can be seen as an extension of the Poincaré symmetry of space-time, realised by the introduction of supercharges, *i.e.* the generators of supersymmetry transformations. Supercharges are operators which transform bosons into fermions, and vice versa.

In high energy physics SUSY is well known as a promising extension of the Standard Model, with strong support at both mathematical and physical level. SUSY can be realised in many different ways. The Large Hadron Collider (LHC) can probe extensively the low-energy realisation of some of them. The first run of the LHC has restricted significantly the parameter space of various SUSY models. Moreover, the non-observation of SUSY states has shifted the SUSY partner masses into the TeV region. As a consequence, even what was the first motivation of its introduction, *i.e.* the possibility of solving the hierarchy problem, has been questioned. Still, the last word about the realisation of this symmetry in nature, at least in this context, has not been said [1].

Moreover, SUSY is the key ingredient in many other areas of research. When *local* supersymmetry is imposed, a new field theory is obtained where supersymmetry and general relativity live together in what we call supergravity. SUSY has been incorporated in string theory, extending the previous bosonic string theory, including fermionic degrees of freedom and originating the so called superstring theory. In physical cosmology it is used to explain the presence of a small but nonzero cosmological constant. It has been added in quantum mechanics before as an attempt to study the consequences of SUSY in a simpler setting, but later as an interesting topic by itself [2]. There are applications in condensed matter physics in studying disordered and mesoscopic systems [3]. It is also used in optical physics to tackle various theoretical problems [4, 5]. It is clear then that the study of the properties of supersymmetric theories, in particular the non-perturbative ones, continues to be of extreme interest.

The simplest non-abelian supersymmetric gauge theory, which is studied in this work, is the $\mathcal{N} = 1$ supersymmetric Yang-Mills (SYM) theory with gauge group $SU(2)$. It describes the interaction between gluons and gluinos. The Lagrangian looks like the one of QCD with only one flavour, except that in this theory the fermion field transforms in the adjoint representation and it is a Majorana field. Usually a mass term is considered, which breaks SUSY softly. When the mass term is zero supersymmetry is predicted to be unbroken, even in the quantised theory [6].

Like in QCD, the theory in SYM is asymptotically free at high energies and becomes stronly coupled in the infrared limit. Due to confinement, the spectrum of particles is expected to consist of colourless bound states. If supersymmetry is unbroken the particles should belong to mass degenerate SUSY multiplets.

Many predictions concerning the properties of SYM theories are based on perturbation theory or semiclassical methods. However, some important properties are of a non-perturbative nature. The first predictions on the spectrum of the theory were possible exploiting the fact that the symmetries of the theory constrain the form of the low-energy effective actions [7, 8]. Verifying the formation of the predicted supermultiplets is a central task of our investigations.

Some important results have already been obtained by our collaboration in previous studies in the framework of a lattice-regularised version of SYM, see Refs. [9, 10, 11]. We have found that a rather small lattice spacing is necessary to investigate the restoration of SUSY. In this work we

PoS(LATTICE2014)273

have added the results of a further, even smaller, lattice spacing. Due to the small lattice needed to reduce the supersymmetry breaking, a closer look at the topological properties is required. In particular, some results regarding the topological susceptibility are presented.

2. Chiral symmetry, SUSY, and continuum limit

As discussed in Ref. [12, 13], SUSY gauge theories can be studied on the lattice. The main idea is that, rather than trying to have some version of SUSY on the lattice, which can be realized only in a non-local way, one should only require that it is recovered in the continuum limit. The conclusion of the two papers is that, in the continuum limit, the chiral limit defines the SUSY point and vice versa.

A fine tuning of the gluino mass m_g is sufficient to approach supersymmetry in the continuum theory. This tuning is efficiently done by means of the mass of an *unphysical* particle: the adjoint pion a-π. Practically, the adjoint pion is defined by the connected contribution of the correlator of the a–η' particle. It has been suggested [7] that, in the OZI approximation, the adjoint pion mass should vanish for a massless gluino. This has been then proved in a more formal way, using a partially quenched setup [14], arriving at the important conclusion that $m^2_{\text{a}-\pi} \propto m_g$.

The strategy we follow to reach these limits consists of two steps: in the first, we fix the lattice spacing, *i.e.* we run our simulations at fixed β (the inverse of the coupling constant) and using several (3 or 4) values of the mass parameter κ, we extrapolate our results to the chiral limit. In the second step, we extrapolate to the continuum limit, repeating the first step for 3 or 4 values of β.

3. Fixing the scale: r_0 and w_0

The results the collaboration presented in Ref. [11] were characterised by the constant $\beta = 1.60$. A rather large gap, between fermionic and bosonic masses inside the same supermultiplet, was obsterved. In the following we decreased the value of the lattice spacing by $\sim 40\%$, increasing the value of β to 1.75. We presented the results in Refs. [9, 15] and for the first time we had some indications of a restoration of SUSY in the theory we are studying. The results we present in this paper have been obtained on a $32^3 \times 64$ lattice and they are characterised by $\beta = 1.90$, which means a further reduction of the value of the lattice spacing by $\sim 30\%$. Comparing the scale of this theory with that of QCD, we determined the value of the lattice spacing to be $a = 0.03610(65)$ fm. We are now very close to the continuum limit and the fundamental picture starts to emerge. Because results with different lattice spacings have been obtained it is now crucial to determine very accurately a scale so that all our results can be compared and extrapolations to the continuum limit can be carried out. So far the scale has been determined using the Sommer parameter r_0 [16]. It is determined from the static quark potential, improving the signal using APE smearing. The method requires two consecutive fitting procedures. Overall the method can be characterised by a few systematic errors which sometimes are not easily taken into account.

To see how good our determinations are, we compared the values of r_0 with the expected scaling. The β-function for SYM has been determined analytically [17]:

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N_c}{1-\frac{g^2N_c}{8\pi^2}} , \tag{3.1}$$

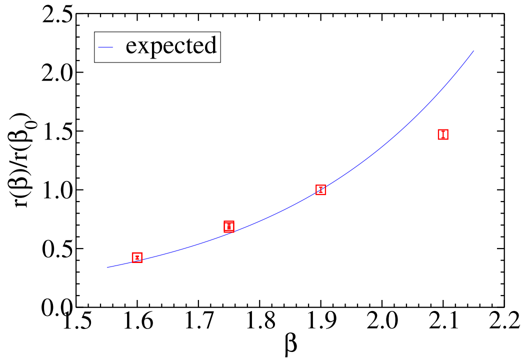

Figure 1: Comparison of the Sommer parameter data, normalised to $\beta = 1.90$, with the expected value determined from the analytical β-function.

where N_c is the number of colours and g the coupling constant. The first two terms in the coupling constant expansion of the β-function are universal, namely scheme independent.

In Fig. 1 we plot the ratio $a(\beta_0)/a(\beta)$, with $\beta_0 = 1.90$, determined using a second order expansion in g to calculate the integral of the β-function, together with our numerical determination of the ratio of the Sommer parameters: $r(\beta)/r(\beta_0) \equiv a(\beta_0)/a(\beta)$. The result is pretty good: the first three points, taking in account the errors, are in reasonable agreement with the theoretical expectation. The fourth point, which is our preliminary result for $\beta = 2.10$, has still some strong systematic error which will be discussed later in Sec. 5.

An important improvement in the analysis of our data has been the determination of the parameter w_0 [18], determined by Wilson flow [19], to fix the scale. This parameter does not suffer from the systematic uncertainties which are present in the Sommer parameter, providing a more reliable comparison of our results. A plot similar to Fig. 1 has been obtained confirming the previous comments. More details will be presented in an upcoming paper.

4. Light particle spectrum

The low-lying spectrum of particles has been predicted by means of effective Lagrangians. It consists of colour neutral bound states of gluons and gluinos, forming supermultiplets: glueballs gg, gluinoballs (mesons) $\tilde{g}\tilde{g}$ and gluino-glueballs $\tilde{g}g$.

In Ref. [7] interpolating operators for pure gluonic states have not been included, and only one supermultiplet was described. It consists of a scalar (0^+ gluinoball: a–$f_0 \sim \bar{\lambda}\lambda$), a pseudoscalar ($0^-$ gluinoball: a–$\eta' \sim \bar{\lambda}\gamma_5\lambda$), and a Majorana fermion (spin 1/2 gluino-glueball: $\chi \sim \sigma^{\mu\nu}\mathrm{Tr}\left[F_{\mu\nu}\lambda\right]$).

The effective Lagrangian of Ref. [7] was generalised in Ref. [8]. In addition to the first chiral supermultiplet a new one appears: a 0^- glueball, a 0^+ glueball, and again a gluino-glueball.

As stressed by the authors, neither of these supermultiplets contain *pure* gluino-gluino, gluino-gluon or gluon-gluon bound states. As a matter of fact, the physical excitations are mixed states of them: actually in the limit when there is no mixing the two supermultiplets are degenerate. This fact can have important consequences for the interpretation of the numerical results: *e.g.* analysing a pure gluonic operator does not imply that we are determining the spectrum of a gluon-gluon bound state. In Fig. 2 four bound states are plotted, and their chiral limit, linearly extrapolated, is shown. The mass of the gluino-glue and the a–η' are the ones extrapolated with a better precision (relative error $\sim 10\%$ and $\sim 15\%$ respectively), than there is the a–f_0 and the scalar glueball (both with a relative error of $\sim 30\%$).

From this figure it is now clear that the three bound states belonging to the first supermultiplet are degenerate within the error bars. Actually also the mass we determine studying the pure 0^+ glueball operator is compatible with the other three masses.

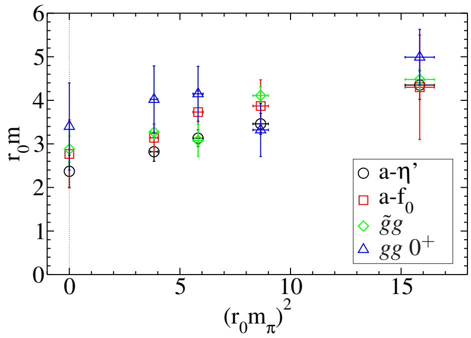

Figure 2: Spectrum of the theory at $\beta = 1.90$ for four values of the a–π mass and in the extrapolated chiral limit.

In contrast, from our preliminary results the mass of the pseudoscalar glueball 0^- is almost two times the mass of the first supermultiplet. A possible explanation could be that due to mixing the 0^+ glueball operator has a significant overlap with the scalar state of the lower supermultiplet. As a consequence, studying the corresponding correlator at large euclidean-time distance, we find again the mass of the low-lying supermultiplet particle with the same quantum number, *i.e.* the a–f_0.

Note that, contrary to what was assumed in Ref. [8], this would imply that the lighter supermultiplet is gluinoball-like and the higher is glueball-like.

5. Topological susceptibility

$\mathcal{N}=1$ SYM is characterised by the presence of topological sectors. One of the greatest problem with this kind of theories, when local update algorithms are used as in our case, is that the simulation may get stuck inside the same topological sector. The transition between different topological sectors is suppressed going closer to the continuum limit. It is therefore necessary to verify the shape and the position of the distribution of the topological charge in every set of configurations generated.

In the case of QCD the topological susceptibility is a commonly studied observable, a quantity which reflects the dependence of the vacuum energy on the vacuum angle. This quantity has not yet been studied intensively in SUSY models. An exception is Ref. [20], where the topological susceptibility is discussed in the context of the orbifold equivalence. The relevant result for us is that the topological susceptibility is expected to go to zero proportionally to the quark mass, *i.e.* proportionally to the square of the adjoint pion mass, according to the discussion in Sec. 2. This behaviour has been verified in our analysis.

The topological charge Q, discussed in this paper, is defined as follows [21]:

$$Q = \text{round}(\alpha Q_L)\ , \tag{5.1}$$

where $\text{round}(x)$ denotes the closest integer to x and Q_L is the lattice definition in terms of smeared plaquettes. The scaling factor α, which has been proved to improve the quality of the charge distributions, is determined minimising the quantity:

$$\langle(\alpha Q_L - \text{round}(\alpha Q_L))^2\rangle\ . \tag{5.2}$$

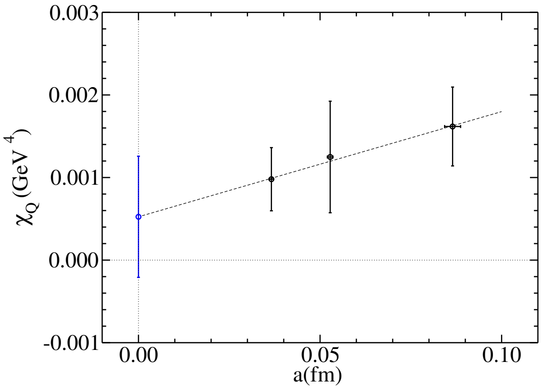

Figure 3: Topological susceptibility extrapolated to the continuum limit. Each point represents the chiral extrapolated value obtained fitting at least 3 values of κ. The conversion in dimensionful unit has been done using the value of the Sommer parameter used in QCD: $r_0 = 0.5$ fm.

The susceptibility of the topological charge is then defined by:

$$\chi_Q = \frac{1}{V}\left(\langle Q^2\rangle - \langle Q\rangle^2\right) . \tag{5.3}$$

The lattice definition of the topological charge is affected by UV fluctuations, and as a consequence it does not take integer values. This means that it is necessary to introduce a multiplicative renormalisation and even an additive one for the topological susceptibility. The way we deal with such renormalisations is based on smoothing methods: initially we compared APE, HYP and stout smearing, and for the final measurements we focussed on APE smearing.

In Fig. 3 the topological susceptibility is plotted against the lattice spacing as determined for our three values of β, and its value extrapolated linearly to the continuum limit. A non-vanishing slope is not unexpected, because the fermion part of the action is improved only by applying a few levels of stout smearing to the link variables, and even using the tree-level Symanzik improved gauge action a linear dependence of observables on the lattice spacing is not prevented. It should be noted that the extrapolated value is compatible with zero as expected.

The fact that in Fig. 1 the point at $\beta = 2.1$ is far below the expected value is related to the freezing of topology at that value of the lattice spacing, $a \sim 0.019$ fm in QCD units, namely the scale parameter is dependent on the topological charge. We expect, however, that this difficulty can be overcome by optimising the parameters (*e.g.* trajectory length) of our updating algorithm [22].

6. Conclusions and outlooks

We have presented our latest results on $\mathcal{N}=1$ supersymmetric Yang-Mills theory with gauge group $SU(2)$. We are now able to see the degeneracy of the first supermultiplet, in accordance with the existence of a SUSY limit of the theory.

Some issues related to the mixing of the states, and therefore with the content of the second supermultiplet, have to be clarified. We have started a systematic analysis of some topological properties of the theory, shedding light on this less known aspect of SUSY models. Recently we have also started to explore the finite temperature properties of this theory [23].

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