PROCEEDINGS OF SCIENCE

Renormalization constants for $N_f = 2+1+1$ twisted mass QCD

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We summarize recent non-perturbative results obtained for the renormalization constants computed in the RI'-MOM scheme for $N_f = 2+1+1$ twisted mass QCD. Our implementation employs the Iwasaki gauge action and four dynamical degenerate twisted mass fermions. Renormalization constants for scalar, pseudo-scalar, vector and axial operators, as well as the quark propagator renormalization, are computed at three different values of the lattice spacing, two different volumes and several values of the twisted mass. Our method allows for a precise cross-check of the running, because of the particular proper treatment of the hypercubic artifacts. Preliminary results for twist-2 operators are also presented.

The 32nd International Symposium on Lattice Field Theory
23-28 June, 2014
Columbia University New York, NY

*Speaker.

PoS(LATTICE2014)284

1. Introduction

Quantum Chromodynamics (QCD) is strongly coupled in the scales relevant for Hadron Physics and thus requires a non-perturbative treatment. At the moment the only approach which avoids approximations and modelling assumptions is QCD discretized on a spacetime lattice. However, lattice QCD is a bare field theory where the results of matrix elements are computed at a fixed value of the UV cutoff $\Lambda = 1/a$. One must renormalize in order to obtain continuum physics and this can be done perturbatively or non-perturbatively. Lattice perturbation theory is infamous for its slow convergence properties so we will employ the non-perturbative (NP) method that fits naturally to the whole non-perturbative setup of the computation. Of course the lattice computation is hindered by $\mathcal{O}(a^2)$ discretization effects if e.g. one employs a Symanzik improved action or the Twisted mass (TM) formulation. The two main players in the game of NP renormalization are the RI'-MOM scheme [1] and the Schrödinger functional [2]. There has been a lot of work utilizing the RI'-MOM scheme [3] and a significant amount of it has been within the framework of TM QCD [4, 5, 6, 7, 8, 9]. In this study we employ the RI'-MOM scheme to compute the renormalization constants (RCs) of fermionic bilinears for $N_f = 4$ using TM fermions. We perform the renormalization of the densities, the currents as well as of the twist-2 operator $O_{44}(x) \propto \bar{u}(x)[\gamma_4 \overset{\leftrightarrow}{D}_4 - \frac{1}{3}\sum_k \gamma_k \overset{\leftrightarrow}{D}_k]u(x)$ which determines the average momentum fraction $\langle x \rangle_q = \int_0^1 dx\, x(q(x)+\bar{q}(x))$ in the hadrons [10]. Our methods allow for the extraction of the $\langle A^2 \rangle$, the dimension-2 gluon condensate that has rich phenomenological implications.

2. RI'-MOM and the computational setup

We use the RI'-MOM scheme [1] and we focus on local fermion bilinears

$$O_\Gamma = \bar{\psi}(x_1)\Gamma\psi(x_2), \tag{2.1}$$

where Γ can be any Dirac structure and contains covariant derivatives for the case of twist-2 operators. We insert O_Γ in the fermion 2-pt function, which determines the 3-pt function

$$G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle, \tag{2.2}$$

then we compute the amputated Green's function, or bare vertex, as

$$\Lambda_O(p) = S_u^{-1}(p) G_O(p) S_d^{-1}(p), \tag{2.3}$$

with $S(p)$ the quark propagator. The operators that we consider are multiplicatively renormalized. Since the RC is a scalar, one works with the projected quantity

$$\Gamma_O(p) = \frac{1}{12}\mathrm{Tr}[P_O \Lambda_O(p)], \tag{2.4}$$

where P_O is a projector specific for each operator.
The operator RC (Z_O) is fixed by the following renormalization condition

$$\Gamma_O(\mu, g_R, m_R = 0)\big|_R = Z_q^{-1}(a\mu, g_0) Z_O(a\mu, g_0) \Gamma_O(p, g_0, m)\big|_{\substack{p^2 = \mu^2 \\ m \to 0}} = 1, \tag{2.5}$$

after having computed the quark wave function RC which is given by

$$Z_q(\mu^2 = p^2) = -\frac{i}{12p^2}\mathrm{Tr}[S^{-1}_{bare}(p)\not{p}]. \tag{2.6}$$

Unity on the RHS of the renormalization condition represents the tree level value of Γ_0. Note that since the 2-pt and 3-pt correlation functions are not gauge invariant, one needs to fix the gauge for the determination of the RCs. We have chosen the lattice version of the Landau gauge in this study. In order to obtain sensible results through this renormalization procedure one needs to be within the window of applicability of the RI'-MOM scheme which is defined in the case of a lattice regularization as follows

$$\Lambda_{QCD} \ll \mu \ll \frac{\pi}{a}, \tag{2.7}$$

where the first inequality ensures the possibility of matching with some perturbative scheme like the $\overline{\mathrm{MS}}$ and protects from infrared effects such as Goldstone pole contaminations while the second inequality ensures small cutoff effects. With the available resources the statistical error in the computation of RCs is rather miniscule while the systematic errors are originating mainly from cutoff effects which are considerably larger. Thus we have tried to isolate the different cutoff effects which contaminate our results and we treat them non-perturbatively utilizing group theoretical methods [11, 12]. There are two dominant type of cutoff effects. Those which are invariant under $\mathrm{O}(4)$ and those that are only invariant under $\mathrm{H}(4)$, the group of hypercubic rotations in four dimensions.

In the current computations of $N_f = 2+1+1$ flavors the ETMC is employing the Iwasaki action in the gauge sector and a twisted mass action for the heavy as well as the light quarks [13]. The full action reads

$$S = S^{YM}_{Iwa} + S^f_l + S^f_h, \tag{2.8}$$

while the fermionic part of the action [14]

$$\begin{aligned} S^f_l + S^f_h &= a^4 \sum_x \bar{\chi}_l \left(\gamma\cdot\nabla - \frac{a}{2}\nabla\cdot\nabla + m_{0l} + i\mu_l\gamma_5\tau_3\right)\chi_l \\ &+ a^4 \sum_x \bar{\chi}_h \left(\gamma\cdot\nabla - \frac{a}{2}\nabla\cdot\nabla + m_{0h} + i\mu_h\gamma_5\tau_1 + \mu_\delta\tau_3\right)\chi_h. \end{aligned} \tag{2.9}$$

The polar mass is defined as $M = \sqrt{m^2+\mu^2}$ and the twist angle as $\omega = \arctan(\mu/m)$ where $m = Z_A m_{\mathrm{PCAC}}$. The quark doublet in the twisted basis is related to the one in the physical basis by the transformation

$$\begin{aligned} \psi_l &= e^{\frac{i}{2}\omega_l\gamma_5\tau_3}\chi_l, \\ \bar{\psi}_l &= \bar{\chi}_l e^{\frac{i}{2}\omega_l\gamma_5\tau_3}, \\ \psi_h &= e^{-i\omega_1\gamma_5\tau_1/2}e^{i\omega_2\tau_2/2}\chi_h, \\ \bar{\psi}_h &= \bar{\chi}_h e^{-i\omega_2\tau_2/2}e^{-i\omega_1\gamma_5\tau_1/2}. \end{aligned} \tag{2.10}$$

which transforms the action to the conventional Dirac action

$$S_{ph} = a^4 \sum_{f=h,l} \bar{\psi}_f \left(D_{\mathrm{tW}} + M_f\right)\psi_f. \tag{2.11}$$

In order to achieve the benefits of the TM formulation, such as automatic $\mathcal{O}(a)$ improvement, one needs to work at maximal twist $\omega = \pi/2$ [13] which amounts to tuning m_{PCAC} to zero. For the

$N_f = 4$ configurations this tuning was a highly non trivial task at the time these configurations where produced. So an alternative strategy was followed, to average results obtained at two opposite values of m_{PCAC}. The combination of the results with positive and negative m_{PCAC} that goes usually under the name of θ average is the first step of our calculations in order to get rid of the $\mathcal{O}(a)$ cutoff effects. The $N_f = 4$ configurations were generated with the main purpose to renormalize accurately the physical $N_f = 2+1+1$ configurations. The reason is that since we employ a mass independent renormalization scheme (where RCs are defined in the chiral limit), the $N_f = 4$ ensembles with four light degenerate flavors allow for a reliable chiral extrapolation. In this analysis the configurations used comprise two different volumes, three values of the lattice spacing, as well as several values of the twisted mass. The values of the lattice spacing are respectively $a = 0.062$ fm for $\beta = 2.10$, $a = 0.078$ fm for $\beta = 1.95$ and $a = 0.086$ fm for $\beta = 1.90$ [7]. The exact parameters of the runs are summarized in Table 1. The next step in the analysis is to take the chiral limit in the valence

Table 1: *The $N_f = 4$ ETMC ensembles utilized in this study note that $a\mu_{sea}$ is given in bold*

ensemble	κ	am_{PCAC}	$a\mu$	confs #
$\beta = 2.10$ - $32^3.64$				
3p	0.156017	+0.00559(14)	0.0025, **0.0046**, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297	250
3m	0.156209	-0.00585(08)	0.0025, **0.0046**, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297	250
4p	0.155983	+0.00685(12)	0.0039, **0.0064**, 0.0112, 0.0184, 0.0240, 0.0295	210
4m	0.156250	-0.00682(13)	0.0039, **0.0064**, 0.0112, 0.0184, 0.0240, 0.0295	210
5p	0.155949	+0.00823(08)	0.0048, **0.0078**, 0.0119, 0.0190, 0.0242, 0.0293	220
5m	0.156291	-0.00821(11)	0.0048, **0.0078**, 0.0119, 0.0190, 0.0242, 0.0293	220
$\beta = 1.95$ - $24^3.48$				
2p	0.160826	+0.01906(24)	**0.0085**, 0.0150, 0.0203, 0.0252, 0.0298	290
2m	0.161229	-0.02091(16)	**0.0085**, 0.0150, 0.0203, 0.0252, 0.0298	290
3p	0.160826	+0.01632(21)	0.0060, 0.0085, 0.0120, 0.0150, **0.0180**, 0.0203, 0.0252, 0.0298	310
3m	0.161229	-0.01602(20)	0.0060, 0.0085, 0.0120, 0.0150, **0.0180**, 0.0203, 0.0252, 0.0298	310
8p	0.160524	+0.03634(14)	**0.0020**, 0.0085, 0.0150, 0.0203, 0.0252, 0.0298	310
8m	0.161585	-0.03627(11)	**0.0020**, 0.0085, 0.0150, 0.0203, 0.0252, 0.0298	310
$\beta = 1.90$ - $24^3.48$				
1p	0.162876	+0.0275(04)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	450
1m	0.163206	-0.0273(02)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	450
4p	0.162689	+0.0398(01)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	370
4m	0.163476	-0.0390(01)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	370

sector where we also take care of the Goldstone pole in the case of the pseudoscalar RC. We use the following ansatz for the amputated pseudoscalar vertex

$$\Gamma_P = a_P + b_P m_\pi^2 + \frac{c_P}{m_\pi^2}, \tag{2.12}$$

and we "subtract" the pole contributions according to

$$\Gamma_P^{sub} = \Gamma_P - \frac{c_P}{m_\pi^2}. \tag{2.13}$$

3. Correcting for discretization effects

With the improvement provided by the TM formulation we still have $\mathcal{O}(a^2)$ lattice artifacts contaminating our results. We will correct for the H(4), hypercubic artifacts by employing the

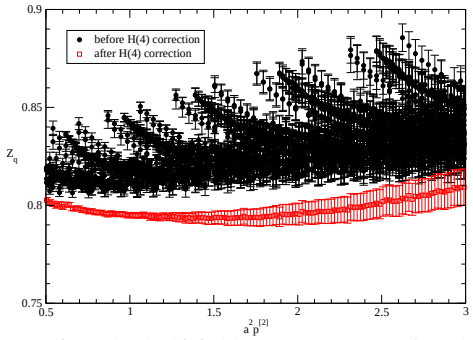

Figure 1: The half fishbone structure arises because of the hypercubic artifacts. The same value of p^2 corresponds to different hypercubic orbits (black data). The "$H(4)$ - extrapolation" (red data) treats these cut-off effects non perturbatively. Results are shown for $\beta = 2.10$, volume $32^3 \times 64$ and $\mu = 0.0046$.

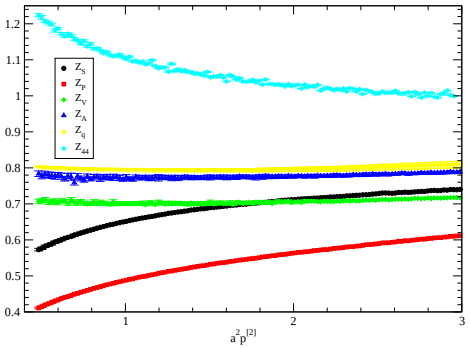

Figure 2: RCs as a function of a^2p^2 after the removal of the $H(4)$ artifacts for the ensemble with $\beta = 2.10$, volume $32^3 \times 64$ and $\mu = 0.0046$.

"$H(4)$"-extrapolation method of [11] which treats them non-perturbatively. From basic group theory [15] one can show that any H(4) invariant polynomial can be expanded in the basis of the following H(4) invariants

$$p^{[2]} = \sum_{\mu=1}^{4} p_\mu^2, \quad p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4, \quad p^{[6]} = \sum_{\mu=1}^{4} p_\mu^6, \quad p^{[8]} = \sum_{\mu=1}^{4} p_\mu^8 \ldots \tag{3.1}$$

We start by expanding the RC already averaged over the cubic orbits around $p^{[4]} = 0$ as,

$$Z_{latt}(a^2p^2, a^4p^{[4]}, a^6p^{[6]}, ap_4, a^2\Lambda_{QCD}^2) = Z_{hypcorrected}(a^2p^2, ap_4, a^2\Lambda_{QCD}^2) + R(a^2p^2, a^2\Lambda_{QCD}^2)\frac{a^2p^{[4]}}{p^2} + \ldots \tag{3.2}$$

$$\text{where } R(a^2p^2, a^2\Lambda_{QCD}^2) = \frac{dZ_{latt}(a^2p^2, 0, 0, 0, a^2\Lambda_{QCD}^2)}{d\varepsilon}|_{\varepsilon = p^{[4]}/p^2 \ll 1} = c_{a2p4} + c_{a4p4}a^2p^2. \tag{3.3}$$

The effect of the $H(4)$ corrections can be seen in Figs. 1 and 2.

Then in order to take into account the lattice artifacts which are invariant under $O(4)$ we explicitly add $O(4)$ lattice artifacts in the running of the RCs. In the case of the quark wave function renormalization we utilize the OPE inspired formula for the perturbative running of Z_q [6]

$$\begin{aligned} Z_q^{hyp-corr}(a^2p^2) \;=\; & Z_q^{pert\,RI'}(\mu^2)\, c_{0Z_q}^{RI'}\Big(\frac{p^2}{\mu^2}, \alpha(\mu)\Big) \\ & \times \left(1 + \frac{\langle A^2\rangle_{\mu^2}}{32p^2}\, \frac{c_{2Z_q}^{\overline{\rm MS}}\big(\frac{p^2}{\mu^2}, \alpha(\mu)\big)}{c_{0Z_q}^{RI'}\big(\frac{p^2}{\mu^2}, \alpha(\mu)\big)}\, \frac{c_{2Z_q}^{RI'}\big(\frac{p^2}{\mu^2}, \alpha(\mu)\big)}{c_{2Z_q}^{\overline{\rm MS}}\big(\frac{p^2}{\mu^2}, \alpha(\mu)\big)}\right) \\ & + \; c_{a2p2}\, a^2\, p^2 + c_{a4p4}\, (a^2p^2)^2. \end{aligned} \tag{3.4}$$

The coefficients $c_{0Z_q}^{RI'}$, $c_{0Z_q}^{RI'}$ and $c_{2Z_q}^{\overline{\rm MS}}$ are known perturbatively [16, 17]. While $Z_q^{pert\,RI'}(\mu^2)$, $\langle A^2\rangle_{\mu^2}$, c_{a2p2} and c_{a4p4} will be determined through fitting, see Fig. 3. Note the presence of the gluon

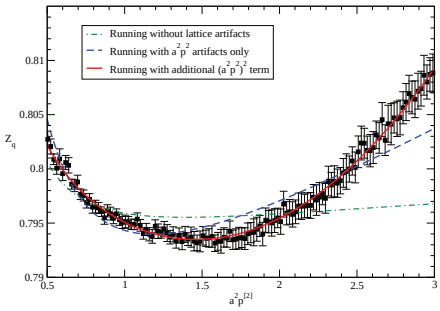

Figure 3: The running of Z_q for the ensemble 3mp.

condensate, $\langle A^2 \rangle_{\mu^2}$, which plays a crucial role in the RC of gauge variant quantities such as the quark field and since also the whole computation takes place in a gauge fixed setting. One can immediately see that the running formula contains $O(4)$ invariant lattice artifact terms $\propto a^2p^2$ and $\propto (a^2p^2)^2$. We use similar running formulae for all the RCs and we refer the reader to [18] for all the details. Note that for the other composite operators we might have terms $\propto 1/(a^2p^2)$ which also mainly accounts for the gluon condensate for Z_P, Z_S and Z_{44}. The perturbative running allows us to run the RCs up to large scales, such as, 10GeV and at this point we use perturbative formulae [19] to convert our results to the $\overline{\text{MS}}$ scheme which will allow us to make contact with phenomenological calculations and experiments. In the following table we tabulate all the results for the RCs in the $\overline{\text{MS}}$ scheme at 2GeV.

β	Z_q	Z_S	Z_P	Z_V	Z_A	Z_P/Z_S	Z_{44}
1.90	0.762(3)(5)(2)	0.722(3)(5)(9)	0.431(3)(3)(6)	0.623(2)(1)(5)	0.717(1)(2)(4)	0.597(4)(4) (3)	0.973(9)(7)(30)
1.95	0.770(2)(6)(6)	0.722(4)(5)(3)	0.461(2)(4)(5)	0.639(2)(1)(4)	0.726(2)(2)(4)	0.638(4)(4)(3)	0.977(12)(11)(30)
2.10	0.787(2)(6)(6)	0.725(2)(5)(3)	0.522(1)(4)(1)	0.687(1) (1)(2)	0.755(1)(2)(4)	0.720(4)(2)(5)	1.019(8)(6)(30)

Table 2: Final results for $N_f = 4$ RCs in the $\overline{\text{MS}}$ scheme at 2 GeV. We have quoted the statistical error in the first parenthesis, while in the second parenthesis the systematic error due to cutoff effects and in the third parenthesis we quote the systematic error originating from the chiral extrapolation.

4. Conclusions and Outlook

We have presented our results for the RCs of the quark propagator, densities and currents and for the O_{44} twist-2 operator for $N_f = 4$ twisted mass fermions. We have implemented a systematic and rigorous procedure for the correction of the hypercubic lattice artifacts. We have treated the main source of uncertainty, the cutoff effects in a non perturbative way for the $O(4)$ breaking artifacts as well with an OPE inspired perturbative formula for the $O(4)$ invariant artefacts. For a comparison with experiments and phenomenological calculations our results, obtained in the RI'-MOM scheme, have been converted to the $\overline{\text{MS}}$ scheme at 2GeV. We plan to apply our method to other twist-2 operators containing more than one covariant derivatives as well as to the new configurations that will be produced shortly by the ETMC at almost the physical point.

Acknowledgments.

We would like to thank P. Boucaud, N. Carrasco, P. Dimopoulos, R. Frezzotti, V. Lubicz, S. Simula and T. Vladikas for fruitful discussions. This work was granted access to the HPC resources of

CINES and IDRIS under the allocations 2013-052271 and 2014-052271 made by GENCI. Propagator computations have also extensively used CINECA GPUs in the framework of the DECI-9 project DEC09-NPR-LQCD. Most of the analysis took place in Lyon-CCIN2P3. We express our gratitude to the staff of all these Computing facilies for their constant help. This work was supported by the CNRS and the Humboldt Foundation (S.Z.).

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