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Partially quenched chiral perturbation theory for $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

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Adding a gluino mass term to $\mathcal{N} = 1$ supersymmetric Yang-Mills theory breaks supersymmetry softly. In order to approach the supersymmetric continuum limit in numerical simulations with the Wilson action, the bare gluino mass has to be tuned to the limit of vanishing renormalised gluino mass. This can be done efficiently by means of the mass of the adjoint pion, which is, however, an unphysical particle. We discuss how the adjoint pion can be defined in the framework of partially quenched chiral perturbation theory. A relation between its mass and the mass of the gluino, analogous to the Gell-Mann-Oakes-Renner relation of QCD, can be derived.

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1. The Model

1.1 $\mathcal{N}=1$ SUSY Yang-Mills Theory

The $\mathcal{N}=1$ supersymmetric Yang-Mills theory (SYM) is the simplest supersymmetric field theory containing non-Abelian gauge fields. Its structural complexity is, however, comparable to that of QCD. Its field content is given by a vector supermultiplet, consisting of a gauge field $A_\mu^a(x)$, $a=1,\ldots,N_c^2-1$, describing gluons belonging to the gauge group SU(N_c), a fermionic spinor field $\lambda^a(x)$, obeying the Majorana condition $\bar{\lambda}=\lambda^T C$, and an auxiliary field. The Majorana field describes gluinos, the superpartners of gluons, and it transforms under the adjoint representation of the gauge group: $\mathscr{D}_\mu\lambda^a=\partial_\mu\lambda^a+g\,f_{abc}A_\mu^b\lambda^c$. The on-shell Lagrangean of SYM in Euclidean space-time reads

$$\mathscr{L}=\frac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a+\frac{1}{2}\bar{\lambda}^a\gamma_\mu(\mathscr{D}_\mu\lambda)^a. \tag{1.1}$$

It is invariant under the SUSY transformations

$$\delta A_\mu^a=-2\,\mathrm{i}\,\bar{\lambda}^a\gamma_\mu\varepsilon,\quad \delta\lambda^a=-\sigma_{\mu\nu}F_{\mu\nu}^a\varepsilon. \tag{1.2}$$

Being part of the supersymmetrically extended Standard Model, SYM represents an interesting field theory. It has some similarities to QCD, the important differences being that gluinos are Majorana particles, and that they are in the adjoint representation, in contrast to quarks.

The Lagrangean can be extended to include a gluino mass term $m_{\tilde{g}}\bar{\lambda}^a\lambda^a$. The gluino mass breaks SUSY softly. The action is only invariant under supersymmetry transformations in the limit $m_{\tilde{g}}=0$.

1.2 Motivation

SYM is an interesting laboratory for the study of the properties of supersymmetric models. As in the case of QCD, SYM is characterised by a number of non-perturbative aspects, which can be investigated in a lattice-discretised version:

- SYM has a discrete chiral symmetry Z_{2N_c}, which is predicted to be broken spontaneously down to Z_2. The breaking is associated with a gluino condensate $<\lambda\lambda>\neq 0$.
- SYM is expected to show confinement, and the particle states should be bound states, forming supermultiplets.
- Static quarks, belonging to the fundamental representation of the gauge group, are expected to be confined.
- Spontaneous breaking of SUSY is predicted not to occur for SYM.
- SUSY is broken by the lattice regularisation. A question which is still open is if there is a continuum limit in which SUSY is restored?
- Predictions about the low-lying particle spectrum from effective Lagrangeans [1, 2] should be checked on the lattice.

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1.3 SUSY on the Lattice

Lattice discretisation generically breaks SUSY [3]. In the case of SYM a fine-tuning of the bare gluino mass parameter in the continuum limit is enough to approach both the (spontaneously broken) chiral symmetry and supersymmetry of the continuum theory [4, 5]. Based on this, Curci and Veneziano [4] proposed to use the Wilson action and to search for a supersymmetric continuum limit by an appropriate tuning of parameters. The Wilson action for SYM is given by

$$S = -\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\,U_p + \frac{1}{2}\sum_x \left\{ \overline{\lambda}_x^a \lambda_x^a - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}_{x+\hat{\mu}}^a V_{ab,x\mu}(1+\gamma_\mu)\lambda_x^b + \overline{\lambda}_x^a V_{ab,x\mu}^t (1-\gamma_\mu)\lambda_{x+\hat{\mu}}^b \right] \right\}, \quad (1.3)$$

where $V_{ab,x\mu}$ are the link variables in the adjoint representation. The parameters in the lattice action are the inverse gauge coupling $\beta = 2N_c/g^2$ and the hopping parameter $\kappa = 1/(2m_0+8)$, related to the bare gluino mass m_0.

Numerical simulations of this model, with gauge group SU(2), have been performed by the Münster-DESY-Frankfurt group in recent years, see the contributions to this conference by P. Giudice and S. Piemonte, and Refs. [6, 7].

2. The Goals

2.1 Phase transition for SU(2)

As a function of the hopping parameter κ the gluino condensate makes a jump at a certain value κ_c. In the phase diagram the line $\kappa = \kappa_c(\beta)$ represents a first order phase transition.

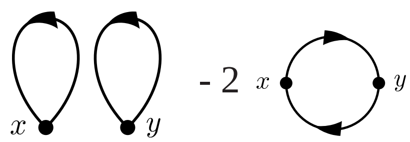
The recovery of both supersymmetry and chiral symmetry in the continuum limit requires to tune the hopping parameter to the point $\kappa_c(\beta)$, where the renormalised gluino mass vanishes [4, 5].

2.2 The adjoint pion

The gluino mass term is not protected against additive renormalisation in the Curci-Veneziano formulation. Therefore the point of vanishing gluino mass is not given a priori, but has to be determined with suitable observables. A numerically relatively cheap and therefore attractive way to tune to κ_c is to search for the point where the adjoint pion mass vanishes: $m_{\mathrm{a}-\pi} \to 0$.

However, SYM does not have a continuous chiral symmetry and thus the spontaneous chiral symmetry breaking is not accompanied by (pseudo-)Goldstone bosons like pions, whose masses would vanish in the chiral limit. So, what is the adjoint pion a–π ?

A pseudoscalar mesonic bound state, called a–η', is represented by the interpolating field $\overline{\lambda}\gamma_5\lambda$. Its correlator has connected and disconnected pieces:



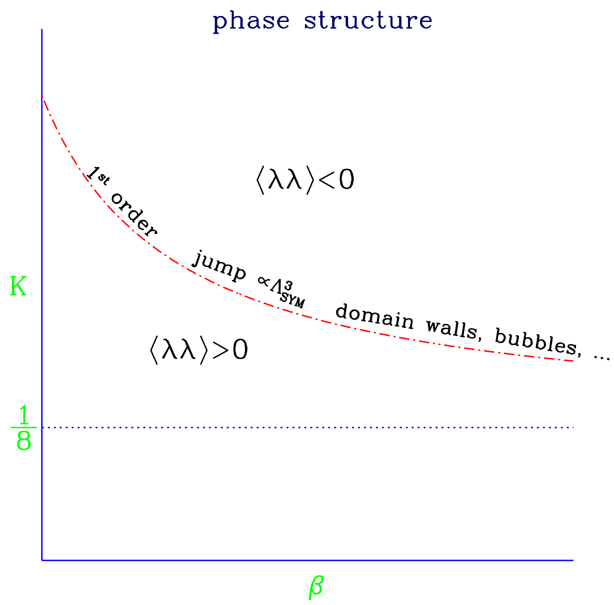


Figure 1: The phase diagram of SYM with gauge group SU(2)

The correlator of the a–π is now given by the connected part of the a–η' correlator, and the adjoint pion mass can be obtained unambiguously from it. The a–π correlator has the form of the correlator of a meson formed out of different gluino species. But since SYM only contains one gluino, the a–π is not a physical particle in the Hilbert space of the theory.

The assumption underlying the tuning of κ is that the adjoint pion mass vanishes with the renormalised gluino mass as

$$m^2_{\text{a}-\pi} \propto m_{\tilde{g}}\,, \tag{2.1}$$

analogously to the Gell-Mann-Oakes-Renner (GOR) relation of QCD [8],

$$m^2_{\pi} \propto m_q\,. \tag{2.2}$$

An argument for this relation, based on the OZI-approximation of SYM, has been given in [1].

On the other hand, the renormalised gluino mass $m_{\tilde{g}}$ can be determined by means of the lattice supersymmetric Ward identities as discussed in [9]. Numerical investigations of both $m_{\tilde{g}}$ from supersymmetric Ward identities and $m_{\text{a}-\pi}$ have been performed in [10]. The results are in agreement with

$$am_{\tilde{g}}Z_S = \frac{1}{2}\left(\frac{1}{\kappa}-\frac{1}{\kappa_c}\right), \qquad (am_{\text{a}-\pi})^2 \simeq A\left(\frac{1}{\kappa}-\frac{1}{\kappa_c}\right), \tag{2.3}$$

and support the validity of the above assumption, see Fig. 2. The a–π, however, yields a more precise signal for the tuning than the supersymmetric Ward identities.

2.3 Goals

The goals of this work are:

- define the adjoint pion a–π properly,
- establish the relation $m^2_{\text{a}-\pi} \propto m_{\tilde{g}}$.

2.4 Approach

In QCD the GOR relation can be derived in the framework of chiral perturbation theory. Thus the idea is to use this as a starting point for SYM, too. In contrast to QCD, however, SYM does not have a continuous chiral symmetry. Therefore the approach consists in adding additional flavours of gluinos, $\lambda_i(x)$, $i=2,\ldots,N$, which are quenched, in order to keep SUSY intact. This is a particular case of Partially Quenched Chiral Perturbation Theory (PQChPT), in the spirit of the case of one-flavour QCD [11]. With the help of the additional gluinos, adjoint pions can be formed as $\overline{\lambda}_i\gamma_5(\tau_\alpha)_{ij}\lambda_j$ with $i,j=1,2$.

3. The Calculation

3.1 Adding gluinos

Let us start by extending SYM with $N-1$ additional flavours of gluinos $\lambda_i(x)$, $i=2,\ldots,N$. In contrast to QCD, where the chiral symmetry group of N quarks is given by $\mathrm{SU}(N)_L\otimes\mathrm{SU}(N)_R$, due to the Majorana condition the chiral symmetry group of extended SYM turns out to be given by a subgroup isomorphic to SU(N). If the gluinos are represented as Weyl fermions, this SU(N) is the group of transformations of N Weyl fermions.

Spontaneous breakdown of chiral symmetry, accompanied by non-vanishing gluino condensates, breaks the group $G=\mathrm{SU}(N)$ down to $H=\mathrm{SO}(N)$. To be specific, we consider the case $N=2$ in the following. The Goldstone manifold is then the coset space $G/H=\mathrm{SU}(2)/\mathrm{SO}(2)\sim S^2$. It can be parameterised by $u=\exp(\mathrm{i}\alpha_1T_1+\mathrm{i}\alpha_3T_3)$, where T_i are the generators of SU(2). It is now convenient to define the nonlinear Goldstone boson field by $U(x)=u(x)^2=u(x)u(x)^T\doteq\exp\left(\mathrm{i}\,\phi(x)/F\right)$,

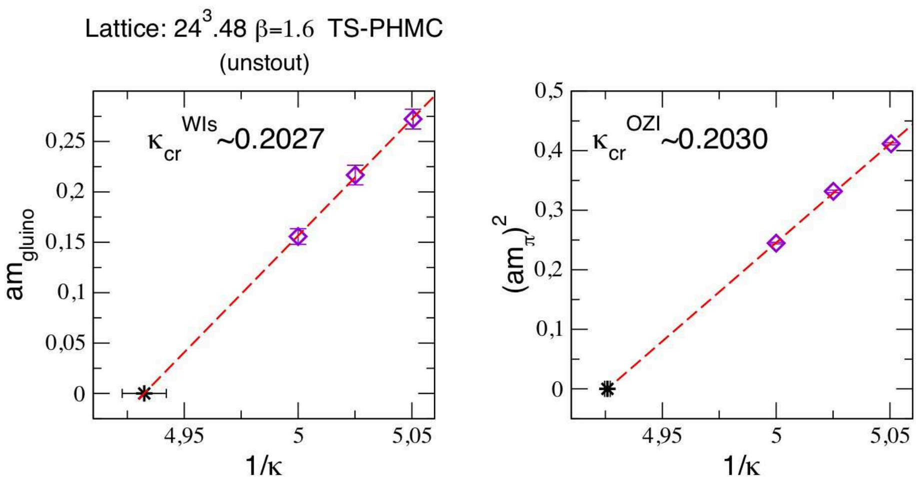


Figure 2: The renormalised gluino mass from SUSY Ward identities (left figure) and the adjoint pion mass squared (right figure) as functions of κ

because the transformation law of this group valued field,

$$U(x) \to U'(x) = VU(x)V^T\,, \quad V \in \mathrm{SU}(2), \tag{3.1}$$

is similar to that of the usual chiral perturbation theory.

Analogously to the approach used in QCD, the leading order effective Lagrangean can be determined to be

$$\mathscr{L}_2 = \frac{F^2}{4}\mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F^2}{4}\mathrm{tr}(\chi U^\dagger + U\chi^\dagger), \tag{3.2}$$

where

$$\chi = 2B_0 m_{\tilde{g}} \mathbf{1}$$

is the symmetry breaking mass term, and F and B_0 are low-energy constants.

Note that the theory with 2 gluinos might be conformal or near-conformal [12], implying a different breaking pattern. However, its discussion here just serves as a preparation of the following PQChPT analysis, which is not affected by this possibility.

3.2 PQChPT

In order that the dynamical content of the model is identical to that of SYM, and the correlation functions of the original fields are unchanged, the additional gluinos have to be quenched, which means that they are not taken into account in the fermionic functional integral. This is a case of PQChPT [13, 14]. Partial quenching can be described theoretically by the introduction of bosonic ghost fermions [15], in our case ghost gluinos. The contribution of the ghost gluinos exactly cancels the contribution of the additional gluinos, and only the contribution of the original single gluino remains. In the case of $N=2$ there is a single ghost gluino $\rho(x)$, compensating the contribution of the additional gluino. The resulting chiral symmetry group is the graded group SU(2|1), and the Goldstone boson field is a graded matrix field

$$\phi = \begin{pmatrix} \phi_{ss} & \phi_{sv} & \phi_{sg} \\ \phi_{vs} & \phi_{vv} & \phi_{vg} \\ \phi_{gs} & \phi_{gv} & \phi_{gg} \end{pmatrix}, \tag{3.3}$$

where s, v and g stand for sea, valence and ghost. Now the adjoint pion can in this formulation be defined to be the meson represented by ϕ_{sv}.

The leading order effective Lagrangean for the partially quenched theory is given by

$$\mathscr{L}_2^{PQ} = \frac{F^2}{4}\,\mathrm{str}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F^2}{4}\,\mathrm{str}(\chi U^\dagger + U\chi^\dagger), \tag{3.4}$$

where str denotes the supertrace. The next-to-leading order terms can be constructed analogously to the NLO terms for QCD [16], and are not reproduced here. They contain further low-energy constants L_i, the so-called Gasser-Leutwyler coefficients.

The masses of the pseudo-Goldstone bosons can be calculated in PQChPT by means of the effective Lagrangean. We have calculated them in NLO. Whereas the tree-level contributions are similar to the ones in QCD, the loop contributions differ due to the different group structure. For the mass of the adjoint pion $m_{\text{a-}\pi}$ we find

$$m^2_{\text{a-}\pi} = 2B_0 m_{\tilde{g}} + \frac{(2B_0 m_{\tilde{g}})^2}{F^2}(30L_8 - 2L_4 - 7L_5 + 8L_6), \tag{3.5}$$

with the low-energy coefficients L_i mentioned above. For small $m_{\tilde{g}}$ we recognise the desired GOR-relation

$$m^2_{\text{a}-\pi} = 2B_0 m_{\tilde{g}}. \tag{3.6}$$

3.3 Results

To summarise, the results of this investigation are:

- The adjoint pion a–π is defined in PQChPT,
- $m^2_{\text{a}-\pi} = 2B_0\, m_{\tilde{g}}$ in leading order PQChPT.

Details can be found in [17].

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