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K_L - K_S mass difference computed with a 171 MeV pion mass

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In this work, I used a $32^3 \times 64 \times 32$, 2+1 flavor domain wall lattice with Iwasaki+DSDR gauge action. The pion mass is 171 MeV and the kaon mass is 492 MeV. We implement the Glashow-Iliopoulos-Maiani (GIM) cancellation using charm quark masses of 750 MeV and 592 MeV. This is an intermediate calculation, in that we are using both a coarse lattice spacing (1/a = 1.37GeV) so we expect significant discretization error coming from charm quark mass and we are also using unphysical kinematics for the pion. The main purpose of this calculation is to study the contribution from the two-pion intermediate state when the energy of a two-pion state is lower than that of the kaon, as well as the corresponding finite volume correction to the ΔM_K.

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1. Introduction

The $K_L - K_S$ mass difference ΔM_K, with an experimental value of $3.483(6)\times 10^{12}$ MeV is an important quantity in particle physics, for the reason that it leads to the prediction of charm quark mass scale, and it's small size provides an important test for the Standard Model. Perturbation theory calculation fails to make a convincing prediction of ΔM_K. As pointed out in [3], the size of NNLO is about 0.36 of the size of LO contribution, and the purely non-perturbative, long distance part, is estimated to be about 30% of the total contribution. Therefore, lattice QCD is the only reliable way to calculate ΔM_K in the Standard Model, with all systematic error controlled.

Previous attempts to calculate ΔM_K[5][1] are encouraging. These are done using an unphysically large 329 MeV pion mass. If we go to physical or near physical pion mass, two issues might arise: the treatment of the two-pion intermediate state which gives an exponential growing contribution to our integrated correlator, as well as the corresponding finite volume correction. This work is done primarily to address these issues. This is an intermediate calculation, in that we are using both a coarse lattice spacing(1/a = 1.37GeV) so we expect significant discretization error coming from the charm quark mass and that we are also using unphysical kinematics for the pion.

2. Evaluation of ΔM_K on the Lattice

The $K^0 - \bar{K^0}$ mixing is represented in Figure 1. We have the $\Delta S = 1$ weak Hamiltonian H_W:

$$H_W = \frac{G_F}{2}\sum_{q,q'=u,c} V_{qd}V^*_{q's}(C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \tag{2.1}$$

$$Q_1^{qq'} = (\bar{s}_i d_i)_{V-A}(\bar{q}_j q'_j)_{V-A} \tag{2.2}$$

$$Q_2^{qq'} = (\bar{s}_i d_j)_{V-A}(\bar{q}_j q'_i)_{V-A}\,. \tag{2.3}$$

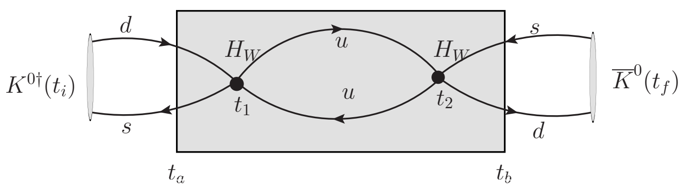

Figure 1: Example diagram for $K^0 - \bar{K^0}$ mixing. The two weak Hamiltonians are integrated over the time interval $[t_a, t_b]$.

By integrating the time-ordered product of the two $\Delta S = 1$ operator, we can obtain the integrated correlator for this $\Delta S = 2$ process. The integrated correlator is defined as:

$$\mathscr{A} = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b} \langle 0|T\left\{\overline{K}^0(t_f)H_W(t_2)H_W(t_1)\overline{K}^0(t_i)\right\}|0\rangle\,. \tag{2.4}$$

If we insert a complete set of intermediate states, we can find:

$$\mathscr{A} = N_K^2 e^{-M_K(t_f-t_i)}\left\{\sum_n \frac{\langle\bar{K}^0|H_w|n\rangle\langle n|H_w|K^0\rangle}{M_K - M_n}\left(-T + \frac{e^{(M_K-M_n)T}-1}{M_K-M_n}\right)\right\}. \tag{2.5}$$

The term proportional to T can be related to ΔM_K:

$$\Delta M_k = 2\sum_n \frac{\langle \bar{K}^0|H_w|n\rangle\langle n|H_w|K^0\rangle}{M_K - M_n} . \tag{2.6}$$

In order to extract the linear term with T from the integrated correlator $\mathscr{A}$, we have to deal with the second term which involves an exponential contribution. Our choice for T is enough to make most of the intermediate state with energy higher than the kaon highly suppressed (except for the η). For intermediate states that have lower energy than the kaon, we must subtract their exponentially growing contribution to our integrated correlator. We have two ways of doing this: we can either directly calculate the matrix element and determine their exponential contribution:

$$N_K^2 e^{-M_K(t_f-t_i)} \frac{\langle \bar{K}^0|H_w|n\rangle\langle n|H_w|K^0\rangle}{(M_K - M_n)^2} e^{(M_K-M_n)T} , \tag{2.7}$$

and subtracted it. Or, we can add a scalar operator $\bar{s}d$, or a pseudoscalar operator $\bar{s}\gamma_5 d$, to our weak Hamiltonian, chosen to make their contribution disappear. We have this freedom because these two operators can be written as a total divergence of a vector or axial current, and therefore they will not affect the physical linear term in our integrated correlator.

In this calculation, the states lighter than the kaon are single pion state and two-pion state with either isospin 0 or 2. The η meson is slightly heavier than the kaon, but it's energy difference is not enough to make it highly suppressed, so we also have to subtract the η state. Because we have disconnected diagrams, we must also subtract vacuum state, which might be a major source of statistical noise.

3. Details of simulation

We work on a $2+1$ flavor, $32^3 \times 64 \times 32$ DWF lattice, with the Iwasaki + DSDR gauge action, and an inverse lattice spacing $1/a = 1.37$ GeV. The pion mass is 171 MeV and the kaon is 492 MeV. We implement GIM cancellation by including a quenched charm quark. We use two choices of charm quark mass, 0.38 and 0.3 in lattice units, which correspond to 750 MeV and 592 MeV. One might think that 0.38 is too high because it can produce an unphysical state that propagates on the 5th dimension. However, because we are only interested in the physics on the domain wall of 5th dimension, which couples weakly to this unphysical state, having a charm mass of 0.38 will not give rise to much systematic error. In order to accelerate the inversion, we used low-mode deflation with 580 eigenvectors obtained using the Lanczos algorithm. Also, we use Mobius fermions with $b+c = 2.667$, $L_s = 12$, which leaves our residual mass unchanged from its unitary value. We are using 405 configurations, about twice the number presented in the talk.

We calculate all the four point diagrams corresponding to the $K^0 - \bar{K}^0$ mixing process, as shown in Figure 2. In order to subtract the two-pion intermediate state, we must also calculate the kaon to two-pion matrix element $\langle\pi\pi|H_W|K\rangle$, as shown in Figure 3. We use Coulomb gauge fixed wall source for the kaon, and a point source propagator at each time slice for the internal quark lines coming from one of the weak vertex in type 1/2 four point diagrams. For the self-loop in type 3/4 four point diagrams and type 3/4 kaon to two-pion diagrams, we use a random space-time volume source with 80 hits. This can significantly reduce the number of inversions required compared to the random wall source with 5 hits that we used in [1], while leaving the statistical error the same size. To suppress vacuum noise in the computation of $\langle\pi\pi|H_W|K\rangle$, we separate the two-pion in the sink by 4 units in time direction.

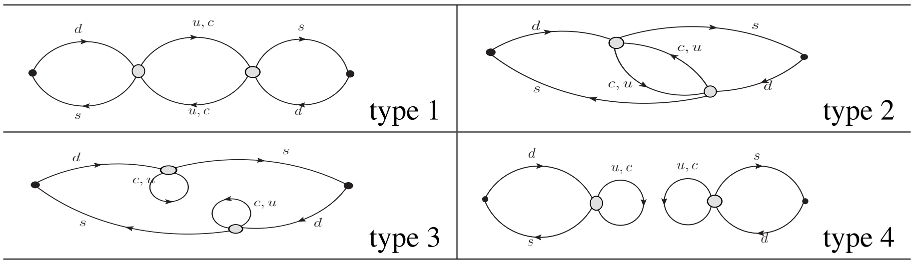

Figure 2: Four types of four point diagrams in the calculation of integrated correlator

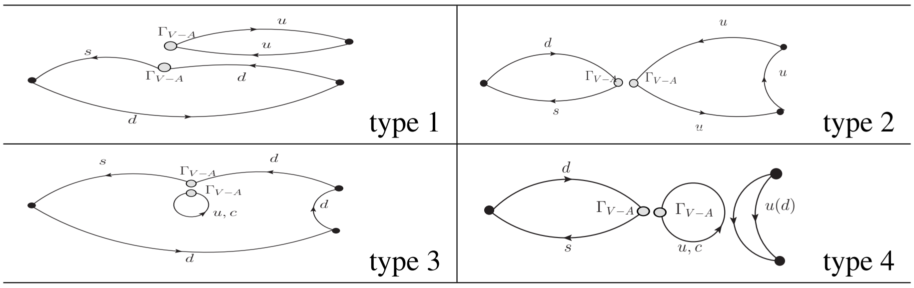

Figure 3: Four types of diagrams in the calculation of $\langle\pi\pi|H_W|K\rangle$

4. Results and finite volume correction

Before we do the linear fitting to obtain ΔM_K, we have to first subtract all the intermediate states with energy lower than or close to the kaon mass. Firstly, because the vacuum intermediate state has the largest matrix element $\langle 0|H_W|K\rangle$, if we directly subtract the vacuum state, then the statistical error is too large in our final integrated correlator, making fitting impossible. Therefore, we must use our freedom of adding a pseudoscalar operator $\bar{s}\gamma_5 d$ to eliminate the vacuum contribution. Because $\langle\eta|H_W|K\rangle$ has the largest statistical error coming from the disconnected diagrams, we use a scalar operator $\bar{s}d$ to eliminate the η contribution. We tune the coefficient c_s and c_p in our modified Hamiltonian $H_W' = H_W + c_p\bar{s}\gamma_5 d + c_s\bar{s}d$, so that

$$\langle 0|H_W + c_p\bar{s}\gamma_5 d|K\rangle = 0, \tag{4.1}$$

$$\langle\eta|H_W + c_s\bar{s}d|K\rangle = 0. \tag{4.2}$$

The subtraction coefficients c_s, c_p are shown in Table 1. With the modified Hamiltonian, we calculate the kaon to two-pion matrix element $\langle\pi\pi|H_W'|K\rangle$, with both isospin 0 and 2, the results are given in Table 2.

After we have subtracted all the exponentially growing intermediate state contributions, we can fit our integrated correlator as a linear function of T. This is shown in Figure 4, as well as the corresponding effective slope. We show the individual contributions to ΔM_K, in Table 3. All of

m_c	c_{1s}	c_{2s}	c_{1p}	c_{2p}
750 MeV	$6.4(14)\times 10^{-4}$	$-8.2(10)\times 10^{-4}$	$-4.356(14)\times 10^{-4}$	$7.567(15)\times 10^{-4}$
592 MeV	$5.7(15)\times 10^{-4}$	$-7.2(12)\times 10^{-4}$	$-4.052(18)\times 10^{-4}$	$6.626(18)\times 10^{-4}$

Table 1: The subtraction coefficient c_s, c_p, for Q_1 and Q_2 separately. $m_c = 0.38, 0.30$ respectively on lattice

m_c	$\langle\pi\pi_{I=2}\vert Q_1\vert K\rangle$	$\langle\pi\pi_{I=2}\vert Q_2\vert K\rangle$	$\langle\pi\pi_{I=0}\vert Q_1\vert K\rangle$	$\langle\pi\pi_{I=0}\vert Q_2\vert K\rangle$
750 MeV	$1.291(8)\times 10^{-4}$	$1.291(8)\times 10^{-4}$	$-5.2(35)\times 10^{-4}$	$7.2(31)\times 10^{-4}$
592 MeV	$1.284(10)\times 10^{-4}$	$1.284(10)\times 10^{-4}$	$-4.4(34)\times 10^{-4}$	$8.7(28)\times 10^{-4}$

Table 2: Kaon to two-pion matrix elements. The fact that the two $I = 2$ matrix element are exactly the same is not surprising because they result from the same diagrams.

these numbers have been multiplied by the Wilson coefficient at the energy scale 3 GeV, which can be found in Table 4.

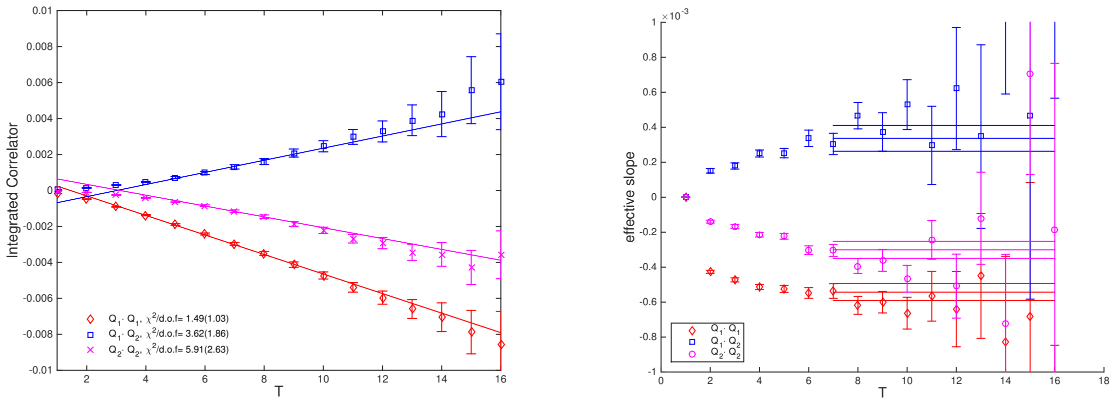

Figure 4: Integrated correlator using $m_c = 0.38$, starting fitting range $T_{min} = 7$, and the corresponding effective slope plot.

We calculated the Wilson coefficient at 3 GeV to reduce error in the perturbative calculation, but it might be too high for our coarse lattice with $1/a = 1.37$ GeV. Therefore, we first match our lattice operator non-perturbatively to a regularization independent, Rome-Southampton scheme [8, 2] with non-exceptional momentum under 1.5 GeV. We used both the RI/SMOM($\gamma_\mu, \not{q}$) and RI/SMOM(γ_μ, γ_μ) schemes [7]. Then we perform step scaling, i.e. by matching to higher energy scale using a finer intermediate lattice, and finally we match to 3 GeV in the RI/SMOM scheme. The matching factors from RI/SMOM to $\overline{MS}$ is given by $1 + \Delta r$ [1] , and the Δr in four-flavor theory can also be found in Table 4. The $\overline{MS}$ Wilson coefficient at 3 GeV can be calculated using equations in [4].

We have about a 5% discrepancy between out lattice Wilson coefficient from the two different intermediate schemes, and this will introduce about a 10% of systematic error in our final results for ΔM_K. We can potentially overcome this by working on a finer lattice which minimize lattice

[1] C.Lehner and C. Sturm, private communications

PoS(LATTICE2014)368

m_c	T_{min}	Q_1Q_1	Q_1Q_2	Q_1Q_2	ΔM_K
0.30	6	0.59(5)	1.32(17)	2.58(36)	4.50(49)
0.30	7	0.56(6)	1.20(23)	2.72(56)	4.50(75)
0.30	8	0.67(9)	1.77(31)	3.39(59)	5.83(85)
0.38	6	0.73(4)	1.37(20)	3.07(32)	5.17(47)
0.38	7	0.70(6)	1.22(27)	3.04(50)	4.96(73)
0.38	8	0.79(8)	1.70(34)	3.72(65)	6.22(94)

Table 3: ΔM_K and the individual contribution from different operator combinations, in the units of 10^{-12} MeV. We have varied the starting point of our fitting range. All numbers are multiplied by the appropriate Wilson coefficient computed in the (γ_μ, γ_μ) scheme.

$C_1^{\overline{MS}}$	$C_2^{\overline{MS}}$	$\Delta r_{11} = \Delta r_{22}$	$\Delta r_{12} = \Delta r_{21}$	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$	C_1^{lat}	C_2^{lat}
-0.2394	1.1068	-0.0566	0.0065	0.5589	-0.0918	-0.2179	0.6027
-0.2394	1.1068	-0.0022	0.0065	0.5008	-0.0819	-0.2064	0.5713

Table 4: The $\overline{MS}$ Wilson Coefficients, $RI/SMOM \to \overline{MS}$ matching matrix Δr, $lat \to RI$ matching matrix Z obtained using Z_q calculated in different schemes, and finally the lattice Wilson Coefficient at scale 3.0GeV. 1st row: $(\gamma_\mu, \not{q})$ scheme, 2nd row: (γ_μ, γ_μ). We didn't include statistical error because all less than 1%.

artifact and using step scaling to match at a higher scale to minimize the errors in perturbation theory.

In this calculation, we have considered the two-pion intermediate state contribution to ΔM_K, which actually depend on the volume of the lattice. The finite volume and infinite volume result for ΔM_K are related by [6]:

$$2\sum_n \frac{f(E_n)}{m_k - E_n} = 2\mathscr{P}\int \mathrm{dE}\,\rho_V(E)\frac{f(E)}{m_k - E} + 2\left(f(m_K)\cot(h)\frac{\mathrm{dh}}{\mathrm{dE}}\right)_{\mathrm{m_K}} \tag{4.3}$$

$$f(m_K) = {}_V\langle \bar{K^0}|H_W|\pi\pi_{E=m_K}\rangle_V\, {}_V\langle \pi\pi_{E=m_K}|H_W|K_0\rangle_V \tag{4.4}$$

The two-pion contribution to the ΔM_K with $I = 2$ are highly suppressed compared to the $I = 0$ contribution, due to the $\Delta I = 1/2$ rule. We can apply 2.6 to calculate their individual contribution to ΔM_K, with results in Table 5. Therefore, we only consider the $I = 0$ intermediate state here, with the terms relevant for the finite volume correction in Table 6.

$E_{\pi\pi I=0}$	$E_{\pi\pi I=2}$	$\Delta M_K(\pi\pi_{I=0})$	$\Delta M_K(\pi\pi_{I=2})$
336.5(15)	346.5(9)	$-0.074(63)$	$-6.70(7)\times 10^{-4}$

Table 5: Two-pion energy (in MeV) and their contribution to ΔM_K (in 10^{-12} MeV). m_c = 750 MeV

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We can see that two-pion intermediate state only contribute a small fraction of the total ΔM_K, and the corresponding finite volume correction is even smaller (less than 1%). Therefore, being unable to measure precisely the kaon to two-pion matrix element does not give rise to much statistical error in ΔM_K.

$\Delta M_K(\pi\pi_{I=0})$	$h=\delta+\phi$	$\cot h$	dh/dE	$\cot h \times dh/dE$	$\Delta M_K(FV)$
0.084(70)	1.71(6)	-0.144(64)	15.3(3)	-2.2(10)	0.019(19)

Table 6: $\pi\pi_{I=0}$ contribution to ΔM_K, relevant terms for finite volume correction, and the finite volume correction term to mass difference $\Delta M_K(FV)$, in unit of $10^{-12}MeV$.

5. Conclusion and outlook

We have shown that having a two-pion intermediate state to subtract does not make the calculation much harder, and the finite volume corrections are controllable. The results are listed in Table 3. A smaller charm mass gives rise to smaller ΔM_K, because in all the 4-point diagrams, the charm quark enters as $u-c$. The smaller charm quark mass is more closer to up quark mass making the GIM cancellation more significant. Although ΔM_K for different T_{min} agree within errors, a noticeable difference appears if we go to $T_{min}=8$. We expect this to be improved with better statistics. The largest part of systematic error comes from the lattice discretization error, and an rough estimate gives about $(m_c a)^2 \approx 30\%$. We also have a systematic error in our Wilson Coefficient which is about 10%.

In future calculation, we can have more reliable results by going to physical kinematics, with $2+1+1$ flavor finer lattice and unquenched charm quark. The RBC collaboration is now working to generate a $80^2 \times 96 \times 192$ lattice with $1/a = 3$ GeV [9]. This work was supported in part by US DOE grant DE-SC0011941, and we thank the RIKEN BNL Research Center for the use of the IBM BG/Q supercomputers on which this calculation was performed.

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