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Current status of ε_K calculated with lattice QCD inputs

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We present results for ε_K, the indirect CP violation parameter, calculated in the Standard Model using inputs from lattice QCD: the kaon bag parameter $\hat{B}_K$, and the CKM matrix element V_{cb} from the axial current form factor for the exclusive decay $\bar{B} \to D^* \ell \bar{\nu}$ at zero-recoil. In addition, we take the coordinates of the unitarity triangle apex $(\bar{\rho}, \bar{\eta})$ from the angle-only fit of the UTfit Collaboration and use V_{us} to fix λ. In order to estimate the systematic error, we also use Wolfenstein parameters from the CKMfitter and UTfit. We find a $3.3(2)\sigma$ difference between ε_K and experiment with exclusive V_{cb}. We report details of this preliminary result.

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1. Introduction

Indirect CP violation in the neutral kaon system is parametrized by ε_K

$$\varepsilon_K \equiv \frac{A[K_L \to \pi\pi(I=0)]}{A[K_S \to \pi\pi(I=0)]} \,. \tag{1.1}$$

Experimentally [1],

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon} \,, \qquad \phi_\varepsilon = 43.52 \pm 0.05^\circ \,. \tag{1.2}$$

We can also calculate ε_K in the Standard Model (SM). In the SM, the CP violation comes solely from a single phase in the CKM matrix elements [2, 3]. The SM allows the mixing of neutral kaons K^0 and $\overline{K}^0$ through loop processes, and describes contributions to the mass splitting ΔM_K and ε_K. Hence, we can test the SM through the CP violation by comparing the experimental and theoretical values of ε_K.

We can express ε_K in terms of input parameters from lattice QCD and experiments. Among them, the input parameters $\hat{B}_K$ and V_{cb} long dominated the statistical and systematic uncertainty in the SM evaluation of ε_K. During the past decade, lattice QCD has reduced the $\hat{B}_K$ error dramatically, to $\approx 1.3\%$. The average of the lattice results is available from Flavour Lattice Averaging Group (FLAG) [4]. We calculate ε_K using the lattice average for $\hat{B}_K$ from FLAG and compare the value of ε_K calculated with the updated result for $\hat{B}_K$ from the SWME Collaboration, which has a larger uncertainty of $\approx 5\%$ [5].

There exists a 3σ difference in V_{cb} between exclusive and inclusive channels [6]. Our analysis shows how this discrepancy propagates to ε_K. The axial current form factor for the semi-leptonic decay $\bar{B} \to D^* \ell \bar{\nu}$ at zero recoil, with the experimental branching fraction, can be used to determine V_{cb}. The Fermilab Lattice and MILC Collaborations (FNAL/MILC) have updated their lattice calculation of the form factor [7]. We compare ε_K obtained using the exclusive V_{cb} from the FNAL/MILC result with ε_K obtained using the inclusive V_{cb} in Ref. [6].

We use the Wolfenstein parametrization for the CKM matrix, truncating the series at $\mathcal{O}(\lambda^7) \approx 10^{-5}$. We examine three different choices of Wolfenstein parameters: (1) λ, $\bar{\rho}$, and $\bar{\eta}$ from the global unitarity triangle (UT) fit of CKMfitter, (2) λ, $\bar{\rho}$, and $\bar{\eta}$ from the global UT fit of UTfit, and (3) $\bar{\rho}$ and $\bar{\eta}$ from an angle-only UT fit from UTfit, with λ from V_{us} [1, 8]. In all cases we take V_{cb} instead of A. The angle-only fit (AOF) does not use ε_K, $\hat{B}_K$, and V_{cb} to determine the UT apex $\bar{\rho}$ and $\bar{\eta}$. Hence, it provides a way to test the validity of the SM with ε_K, using the lattice results of $\hat{B}_K$ and V_{cb}.

To estimate the effect of correlations in lattice input parameters, we note that the V_{cb} dominates the error in ε_K, and the FLAG $\hat{B}_K$ is dominated by the BMW result [9]. The correlation between the BMW $\hat{B}_K$ and the exclusive V_{cb} from the FNAL/MILC form factor is negligible. Hence, we assume that the correlation between the lattice input parameters $\hat{B}_K$, V_{cb} and ξ_0 are negligible. To determine the value of ε_K, we take uncorrelated inputs for all the parameters, and use the Monte Carlo method to determine the error. We also compare the results with standard error propagation to cross-check them. In the error budget, we quote results obtained using the error propagation method.

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2. Indirect CP Violation in the Kaon System: ε_K

We use the master formula in Eq. (2.2) to evaluate the SM value of ε_K.

$$\varepsilon_K^{SM} = \frac{\tilde{\varepsilon} + i\xi_0}{1 + i\tilde{\varepsilon}\xi_0} = \tilde{\varepsilon}_0 + i\xi_0 + \mathscr{O}(\tilde{\varepsilon}_0^3) \tag{2.1}$$

$$= e^{i\theta}\sqrt{2}\sin\theta\left(C_\varepsilon \hat{B}_K X + \xi_0\right) + \xi_{\text{LD}} + \mathscr{O}(\tilde{\varepsilon}_0^3) \tag{2.2}$$

where $\tilde{\varepsilon} = \tilde{\varepsilon}_0(1+\tilde{\varepsilon}^2)$ [10, 11], ξ_0 is defined in Eq. (3.3), and ξ_{LD} is the long distance effect of $\approx 2\%$, which we neglect in this paper. We also neglect the truncation error of $\mathscr{O}(\tilde{\varepsilon}_0^3) \cong 10^{-9}$. The mixing parameter $\tilde{\varepsilon}$ is defined by the following.

$$|K_S\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}(|K_1\rangle + \tilde{\varepsilon}|K_2\rangle), \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}(|K_2\rangle + \tilde{\varepsilon}|K_1\rangle), \tag{2.3}$$

where $|K_1\rangle$ and $|K_2\rangle$ are CP even and odd states, respectively. In our phase convention of $CP|K^0\rangle = -|\overline{K}^0\rangle$, they are

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right), \qquad |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right). \tag{2.4}$$

The factor X is

$$X = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_2 S_0(x_t) + \eta_3 S_0(x_c,x_t) - \eta_1 S_0(x_c)\right] \tag{2.5}$$

where $x_i = m_i^2/M_W^2$ with $(i = c,t)$, and S_0's are the Inami-Lim functions. X takes into account the short-distance contribution of the box-diagram [12].

We use the experimental value for ΔM_K because the theoretical value does not have enough precision yet [13]. Other input parameters which appear in Eq. (2.5), the factor C_ε,

$$C_\varepsilon = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}, \tag{2.6}$$

and $\hat{B}_K$ will be explained in the next section.

3. Input Parameters

The input values that we use for V_{cb} are summarized in Table 1a. The inclusive determination considers the following inclusive decays: $B \to X_c l\nu$, and $B \to X_s\gamma$. Moments of lepton energy, hadron masses, and photon energy are measured from the relevant decay. Those moments are fit to the theoretical expressions which are obtained by applying the operator product expansion (OPE) to the decay amplitude with respect to the strong coupling α_s, and inverse heavy quark mass Λ/m_b. There are two schemes for the choice of b quark mass m_b in the heavy quark expansion: kinetic scheme and 1S scheme. We use the value obtained using the kinetic scheme, which has somewhat larger errors [1].

The exclusive determination considers the semi-leptonic decay of $\bar{B}$ to D or D^*. Here, we use the most up-to-date value from FNAL/MILC lattice calculation of the form factor of the semi-leptonic decay $\bar{B} \to D^*\ell\bar{\nu}$ at zero-recoil [7].

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$\lvert V_{cb}\rvert$	42.42(86)	[6] Incl.
	39.04(49)(53)(19)	[7] Excl.

(a)

$\hat{B}_K$	0.7661(99)	[4] FLAG
	0.7379(47)(365)	[5] SWME

(b)

Table 1: The magnitudes of inclusive and exclusive V_{cb} are given in units of 10^{-3}. The inclusive V_{cb} value is determined in the kinetic scheme for the heavy quark expansion.

Several lattice calculations of $\hat{B}_K$ are available. FLAG summarizes lattice results with $N_f = 2+1$ and provides the lattice average. Here, we use the $N_f = 2+1$ FLAG average [14, 15, 16, 9, 4] and SWME calculation as inputs, Table 1b. FLAG uses the previous $\hat{B}_K$ result of SWME collaboration [14], and it is not much different from the updated value [5] that we use in this analysis.

The CKMfitter and UTfit groups provide the Wolfenstein parameters $\lambda, \bar{\rho}, \bar{\eta}$ and A from the global UT fit. Here, we use $\lambda, \bar{\rho}, \bar{\eta}$ from CKMfitter and UTfit, and we use V_{cb} instead of A when we calculate ε_K as in Eq. (2.5).

$$|V_{cb}| = A\lambda^2 + \mathscr{O}(\lambda^7)\,, \tag{3.1}$$

where $\mathscr{O}(\lambda^7) \approx 2\times 10^{-5}$ is negligible. The parameters λ, $\bar{\rho}$, and $\bar{\eta}$ are collected in the Table 2a.

The parameters $\varepsilon_K, \hat{B}_K$, and V_{cb} are inputs to the global UT fit. Hence, the Wolfenstein parameters extracted from the global UT fit of the CKMfitter and UTfit groups contain unwanted dependence on the ε_K calculated from the master formula, Eq. (2.2). To self-consistently determine ε_K, we take another input set from the angle-only fit (AOF). The AOF does not use $\varepsilon_K, \hat{B}_K$, and V_{cb} as inputs to determine the UT apex of $\bar{\rho}$ and $\bar{\eta}$ [8]. The AOF gives the UT apex $(\bar{\rho}, \bar{\eta})$ but not λ. We can take λ independently from the CKM matrix element V_{us}, because this is parametrized by

$$|V_{us}| = \lambda + \mathscr{O}(\lambda^7)\,. \tag{3.2}$$

Here we use the average of results extracted from the $K_{\ell 3}$ and $K_{\mu 2}$ decays [1].

The RBC-UKQCD collaboration provides lattice results of Im A_2 and ξ_0 [17]. They obtain ξ_0 using the relation

$$\mathrm{Re}\left(\frac{\varepsilon_K'}{\varepsilon_K}\right) = \frac{\cos(\phi_{\varepsilon'} - \phi_\varepsilon)}{\sqrt{2}|\varepsilon_K|}\frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \xi_0\right), \qquad \xi_0 \equiv \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\,. \tag{3.3}$$

In using this relation, input parameters except ξ_0 and Im A_2 are taken from the experimental values, as suggested in Ref. [17]. In particular, they use the experimental value of ε_K as an input parameter to determine ξ_0. However, the error is dominated by the experimental error of $\mathrm{Re}(\varepsilon_K'/\varepsilon_K) \approx 14\%$. In the numerator, $\cos(\phi_{\varepsilon'} - \phi_\varepsilon)$ is approximated by 1, because the two phases are very close to each other. The result of ξ_0 is given in Table 2b.

The remaining input parameters are the Fermi constant G_F, W boson mass M_W, quark masses m_q, kaon mass m_K^0, mass difference ΔM_K, kaon decay constant F_K, and QCD short distance correction factors η_i; these are summarized in Table 2b. The factors η_1 and η_2 are next-to-leading order (NLO) results.[1] Recently, the next-to-next-to-leading order (NNLO) calculation became available for η_3 [20], and we take this value as an input.

[1]The NNLO result of η_1 $(=\eta_{cc})$ is available in Ref. [18]. However, there is a claim that the error is overestimated [19]. This issue is under further investigation. We plan to address this issue in Ref. [11].

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λ	0.22535(65)	[1] CKMfitter
	0.22535(65)	[1] UTfit
	0.2252(9)	[1] $\lvert V_{us}\rvert$ (AOF)
$\bar{\rho}$	$0.131^{+0.026}_{-0.013}$	[1] CKMfitter
	0.136(18)	[1] UTfit
	0.130(27)	[8] UTfit (AOF)
$\bar{\eta}$	$0.345^{+0.013}_{-0.014}$	[1] CKMfitter
	0.348(14)	[1] UTfit
	0.338(16)	[8] UTfit (AOF)

(a) Wolfenstein Parameters

G_F	$1.1663787(6)\times10^{-5}$ GeV^{-2}	[1]
M_W	80.385(15) GeV	[1]
$m_c(m_c)$	1.275(25) GeV	[1]
$m_t(m_t)$	163.3(2.7) GeV	[21]
η_1	1.43(23)	[22]
η_2	0.5765(65)	[22]
η_3	0.496(47)	[20]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	497.614(24) MeV	[1]
ΔM_K	$3.484(6)\times10^{-12}$ MeV	[1]
F_K	156.1(8) MeV	[1]
ξ_0	$-1.63(19)(20)\times10^{-4}$	[17]

(b)

Table 2: Wolfenstein parameters, ξ_0, and other inputs.

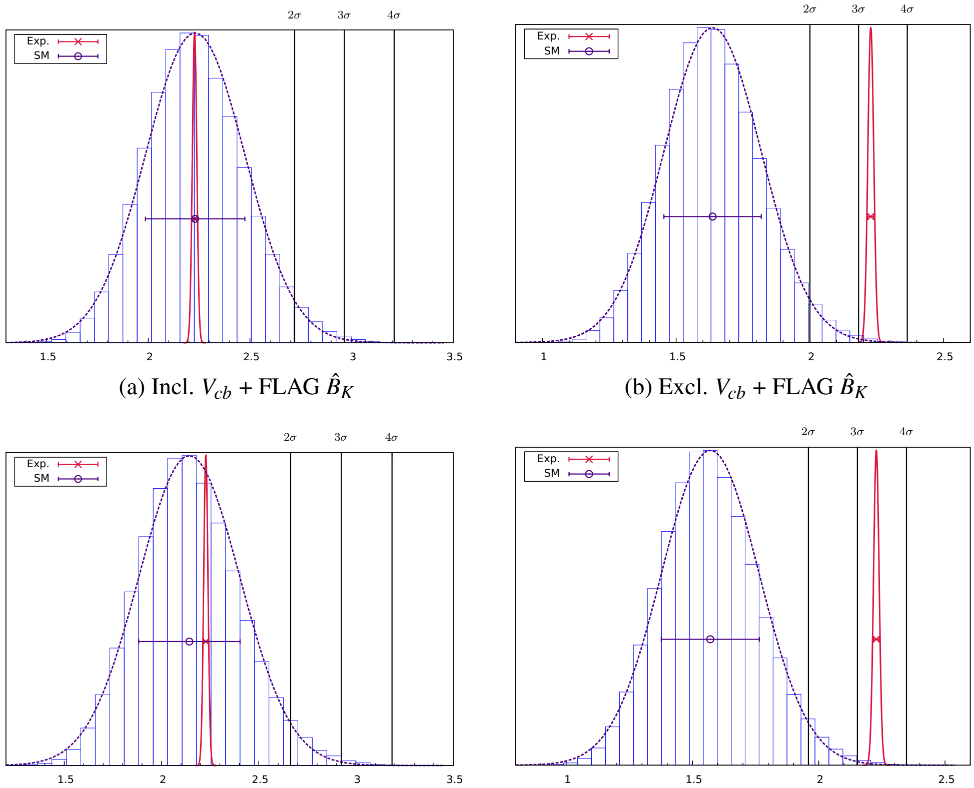
4. Results

We use the Monte Carlo method to calculate the value of ε_K in the SM. Assuming the input parameters are uncorrelated with each other and follow the Gaussian distribution with mean and standard deviation given in Tables 1 and 2, we generate 10^5 random sample vectors. The dimension of a sample vector is $n = 17$, the total number of input parameters which appear in the ε_K master formula of Eq. (2.2).

We compare the SM values of ε_K for our various input choices with the experimental value in Eq. (1.2). The Monte Carlo results with the AOF parameter inputs are given in Fig. 1. These results are consistent with those obtained using the input parameters of the CKMfitter and UTfit groups with their implicit dependence on ε_K, $\hat{B}_K$, and V_{cb}. Hence, regardless of the choice of Wolfenstein parameters, the SM is in good agreement with the experiment, if we use the inclusive V_{cb}. However, a substantial tension of $3.3(2)\sigma$ between the SM and the experiment exists with the exclusive V_{cb}, AOF inputs, and the FLAG $\hat{B}_K$. With input parameters from the global fits (CKMfitter and UTfit), this tension is relaxed but still exceeds 3.1σ. The SM appears to deviate from experiment by 3.1σ to 3.4σ; the former comes from taking the CKMfitter and FLAG $\hat{B}_K$ and the latter from taking the AOF and the SWME $\hat{B}_K$. The results are shown in Table 3a. The error budget for the AOF with FLAG $\hat{B}_K$ is given in Table 3b. The uncertainty in the value of V_{cb} dominates the error of the SM value.

5. Conclusion

With FLAG average for $\hat{B}_K$ and V_{cb} from the lattice (FNAL/MILC) form factor for $\bar{B} \to D^* \ell \bar{\nu}$, we find the SM value of ε_K differs from the experimental value by $3.3(2)\sigma$. However, with the inclusive V_{cb}, we do not observe any tension. The dominant error in ε_K comes from V_{cb}. New lattice QCD calculations and updated UT analyses are essential. To contribute to this effort, we



(a) Incl. V_{cb} + FLAG $\hat{B}_K$ (b) Excl. V_{cb} + FLAG $\hat{B}_K$

(c) Incl. V_{cb} + SWME $\hat{B}_K$ (d) Excl. V_{cb} + SWME $\hat{B}_K$

Figure 1: ε_K with the AOF set of Wolfenstein parameters. Each label shows the combination of V_{cb} and $\hat{B}_K$ inputs. The red narrow distribution represents experimental values. The dotted blue wide distribution represents the results of Monte Carlo method. With exclusive V_{cb} we observe a tension exceeding 3.1σ, which disappears with inclusive V_{cb}.

	FLAG $\hat{B}_K$	SWME $\hat{B}_K$
CKMfitter	1.674(180)	1.607(193)
$\lambda, \bar{\rho}, \bar{\eta}$	3.1σ	3.2σ
UTfit	1.683(178)	1.615(192)
$\lambda, \bar{\rho}, \bar{\eta}$	3.1σ	3.2σ
AOF	1.636(182)	1.570(195)
$\lambda, \bar{\rho}, \bar{\eta}$	3.3σ	3.4σ

(a) ε_K

source	error (%)	memo
V_{cb}	41.3	FNAL/MILC
$\bar{\eta}$	21.7	AOF
η_3	16.8	$c-t$ Box
η_1	5.1	$c-c$ Box
$\bar{\rho}$	4.6	AOF
m_t	3.4	
ξ_0	2.2	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
⋮	⋮	

(b) Error budget

Table 3: (a) ε_K with exclusive V_{cb}, and (b) relative error budget for the AOF set with FLAG $\hat{B}_K$.

plan to calculate the form factors for $\bar{B} \to D^* \ell \bar{\nu}$ using the Oktay-Kronfeld (OK) action, which is designed to reduce heavy quark discretization errors [23, 24, 25].

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