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B meson decay constants and $\Delta B = 2$ matrix elements with static heavy and domain-wall light quarks

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Neutral B meson mixing matrix elements and B meson decay constants are calculated. Static approximation is used for b quark and domain-wall fermion is employed for light quarks. The calculations are carried out on $N_f = 2+1$ dynamical ensembles with lattice spacings of 0.086 fm and 0.11 fm, and a fixed physical spatial volume of about $(2.7\text{ fm})^3$, generated by RBC-UKQCD Collaborations. We employ two kinds of link-smearing for the static action and their results are combined in taking a continuum limit. For the matching between the lattice and the continuum theory, one-loop perturbative calculations are employed with $O(a)$ improvement to reduce discretization errors. We also show statistical improvements by the all-mode-averaging technique.

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1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements are a key to the elementary particle physics. Constraints on V_{ts} and V_{td} can be obtained through $B^0 - \overline{B^0}$ mixing phenomena. In the Standard Model (SM) framework, the mass difference between the two neutral B meson mass eigenstates is related to the CKM matrix elements by

$$\Delta m_q = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} \left| V_{tq}^* V_{tb} \right|^2 S_0(x_t)\, \eta_B \hat{\mathscr{M}}_{B_q}, \tag{1.1}$$

where $q = \{d, s\}$. In the formula (1.1), both the Inami-Lim function $S_0(x_t)$ ($x_t = m_t^2/m_W^2$) and the QCD coefficient η_B can be perturbatively calculated. $\hat{\mathscr{M}}_{B_q}$ is a renormalization group invariant (RGI) $\Delta B = 2$ four-fermion operator matrix element in the effective Hamiltonian of the W-boson exchanging box diagram at low-energy scale. The hadronic matrix element $\hat{\mathscr{M}}_{B_q}$ is highly nonperturbative. Current reliable way to access it is, hence, only numerical simulations using the lattice QCD. In the $B^0 - \overline{B^0}$ mixing, a ratio quantity is important, which gives strong restriction on CKM matrix elements:

$$\left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta m_d}{\Delta m_s} \frac{m_{B_s}}{m_{B_d}}}, \qquad \xi = \frac{m_{B_d}}{m_{B_s}} \sqrt{\frac{\mathscr{M}_{B_s}}{\mathscr{M}_{B_d}}}, \tag{1.2}$$

where ξ is called SU(3) breaking ratio [1]. The ratio constrains the apex of the CKM unitary triangle and new quark-flavor-changing interactions from Beyond Standard Model (BSM) would affect this. In the ratio many uncertainties and data fluctuations are canceled and precise determination of ξ would lead to a constraint on the CKM unitary triangle and some hints for the BSM by observing inconsistency of the unitary triangle in the SM. (See Review of lattice results by Flavor Lattice Averaging Group (FLAG) [2] for the summary.)

In the lattice QCD simulation with b quark, we need to manage the large energy scale difference between light quarks (u and d) and b quark. Heavy Quark Effective Theory (HQET) provides a realistic solution to this problem, in which the heavy quark dynamics is integrated out and we may treat only light quark degrees of freedom. The theory is described by systematic expansion in terms of inverse of heavy quark mass, $1/m_Q$. We employ the static approximation as b quark treatment in this study [3, 4]. This approximation itself has $O(\Lambda_{\rm QCD}/m_b) \sim 10\%$ uncertainty. The results at the static limit is, however, valuable as an anchor point when combined with simulations in lower quark mass region. We here consider a heavy quark expansion of some heavy-light quantity $\Phi_{\rm hl}$, which has a finite asymptotic limit as $m_Q \to \infty$,

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(0) \exp\left[\sum_{p=1}^{\infty} \gamma_p \left(\frac{\Lambda_{\rm QCD}}{m_Q} \right)^p \right], \tag{1.3}$$

where m_Q is a heavy quark mass. Equivalently, the expansion is written as

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(1/m_{Q_A}) \exp\left[\sum_{p=1}^{\infty} \gamma_p \left\{ \left(\frac{\Lambda_{\rm QCD}}{m_Q} \right)^p - \left(\frac{\Lambda_{\rm QCD}}{m_{Q_A}} \right)^p \right\} \right], \tag{1.4}$$

using some "anchor" point m_{Q_A}. (The static limit $m_Q \to \infty$ in Eq. (1.3) is regarded as an anchor point.) If we obtain the expansion coefficients γ_p and the overall factor $\Phi_{\rm hl}(1/m_{Q_A})$, the physical b quark point can be reached. There are several ways to the determination:

(i) The anchor point sits in the static limit, $m_Q \to \infty$. To treat the heavy quark expansion from the static limit, $O(1/m_Q)$ operators in the expansion are included. The HQET must be matched with the original QCD full theory, in which the matching beyond the static approximation cannot be carried out perturbatively [5, 6].

(ii) The anchor point sits in lower mass region, typically c quark region. The usual relativistic lattice formulations can be employed in that region, while relatively finer lattices are required.

(iii) The anchor point is the static limit, while γ_ps are explored by using usual relativistic formulations in lower quark mass region.

The procedure (i) has been done with the step scaling strategy with Shcrödinger functional scheme for the nonperturbative matching with QCD full theory [6]. While the procedure (ii) requires relatively finer lattices with regular size of volume, the lattices to treat c quark is currently becoming available and the approach (ii) is getting its feasibility. A recent sophisticated implementation in this direction is “ratio method” [7], which is a viable option. The combination of the ratio method and the static limit as an anchor point would also be beneficial, which belongs to the category (iii). In this sense, the static limit is not only our theoretical interest, but also a valuable anchor point to explore physics at physical b quark point. The fact that “the static limit is close to the physical b quark mass in terms of $1/m_Q$” makes the static limit an important anchor point.

2. Calculation detail

2.1 Lattice action and gluon ensemble

We employ the standard static quark action with gluon link smearing for b quark, where we use HYP1 and HYP2 smearing to reduce the power divergence. For the light quark (u, d and s) sector, we use domain-wall fermion (DWF), which holds controllable approximate chiral symmetry at large enough fifth-dimension size. The chiral symmetry is important to prevent unphysical operator mixing. For the gluon part, we use Iwasaki action.

In our simulation, $2+1$ flavor dynamical DWF gluon ensemble generated by RBC-UKQCD Collaborations [8] is used. The ensemble parameters are shown in Tab. 1. Two lattice spacings $a \sim 0.11$ [fm] and 0.086 [fm] are used to take a continuum limit, for which we label the coarser and finer lattices as “24c” and “32c”, respectively. The physical box size is set to be modest, which is around 2.75 [fm]. The size of the fifth dimension is $L_s = 16$ making the chiral symmetry breaking small enough. Degenerate u and d quark mass parameters cover the pion mass range of 290-420 [MeV]. The smallest value of $m_\pi L$ is about 4, from which we assume finite volume effect would be small at simulation points in this work.

2.2 Perturbative matching with $O(a)$ improvement

In the HQET approach, we need two matchings: one is a matching between QCD and HQET in the continuum, another is a matching between continuum and lattice in the HQET. For the HQET matching beyond the leading order in $1/m_Q$ expansion, a nonperturbative procedure is required because of the linear divergence, otherwise we cannot take continuum limit. The perturbative matching is, however, justified when we stay in the static limit.

Table 1: $2+1$ flavor dynamical DWF ensembles generated by RBC-UKQCD Collaborations [8] used in this work. am_l and am_h represent the sea ud and s quark mass parameter, respectively.

label	β	$L^3 \times T \times L_s$	a^{-1} [GeV]	a [fm]	am_l/am_h	m_π [MeV]	$m_\pi L$
24c1	2.13	$24^3 \times 64 \times 16$	1.729(25)	0.114	0.005/0.04	327	4.54
24c2					0.01/0.04	418	4.79
32c1	2.25	$32^3 \times 64 \times 16$	2.280(28)	0.0864	0.004/0.03	289	4.05
32c2					0.006/0.03	344	4.83
32c3					0.008/0.03	393	5.52

The lattice fermions, we employ in this calculation, themselves have no $O(a)$ lattice artifact: the static action has no $O(a)$ when on-shell condition imposed and the DWF has also no $O(a)$ due to the existence of the chiral symmetry. The $O(a)$ lattice error, however, arises in the heavy-light system because the chiral symmetry only on the DWF does not prohibit $O(a)$ operators. We make $O(a)$ improvement for the heavy-light operators using one-loop perturbation to remove $O(g^2a)$ uncertainty [9].

2.3 Chiral and continuum extrapolations

In chiral fits for the chiral and continuum extrapolations, we basically use SU(2) heavy meson chiral perturbation theory (SU(2)HMχPT), and HYP1 and HYP2 data are combined including their correlation. To take into account uncertainty from chiral fit function ansatz, we take following criteria for the chiral and continuum extrapolations: (A) For B_d quantities and SU(3) breaking ratios, an average of results from SU(2)HMχPT and linear fit is taken as a central value, we then take half of the full difference between the SU(2)HMχPT and the linear results as an uncertainty from chiral fit function ansatz. (B) For B_s quantities, SU(2)HMχPT fit (linear fit) results are taken as central values, and we then take difference between the full data and cut data, where heavier quark mass data are removed, as an uncertainty of the chiral fit ambiguity.

3. Results

We summarize our results in the static limit of b quark [4]:

$$f_B = 218.8(6.5)_{\rm stat}(16.1)_{\rm sys}\ \text{MeV}, \quad f_{B_s} = 263.5(4.8)_{\rm stat}(18.7)_{\rm sys}\ \text{MeV}, \tag{3.1}$$

$$f_{B_s}/f_B = 1.193(20)_{\rm stat}(35)_{\rm sys}, \tag{3.2}$$

$$f_B\sqrt{\hat{B}_B} = 240(15)_{\rm stat}(17)_{\rm sys}\ \text{MeV}, \quad f_{B_s}\sqrt{\hat{B}_{B_s}} = 290(09)_{\rm stat}(20)_{\rm sys}\ \text{MeV}, \tag{3.3}$$

$$\xi = 1.208(41)_{\rm stat}(44)_{\rm sys}, \tag{3.4}$$

$$\hat{B}_B = 1.17(11)_{\rm stat}(19)_{\rm sys}, \quad \hat{B}_{B_s} = 1.22(06)_{\rm stat}(11)_{\rm sys}, \tag{3.5}$$

$$B_{B_s}/B_B = 1.028(60)_{\rm stat}(43)_{\rm sys}, \tag{3.6}$$

where $\hat{B}_{B_q}$ denotes RGI B-parameter, and we show statistical (stat) and systematic (sys) errors. Note that $O(1/m_b)$ uncertainty is not included in the systematic errors above. The error budget is shown in Tab. 2. Our results have $\sim 10\%$ larger value for decay constants f_B and f_{B_s} from other works

Table 2: Error budget [%] for B meson decay constants and mixing matrix elements [4].

	f_B	f_{B_s}	f_{B_s}/f_B	$f_B\sqrt{\hat{B}_B}$	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	ξ
Statistics	2.99	1.81	1.65	6.34	3.12	3.36
Chiral/continuum extrapolation	3.54	1.98	2.66	2.55	2.13	3.08
Finite volume effect	0.82	0.0	1.00	0.76	0.00	1.07
Discretization	1.0	1.0	0.2	1.0	1.0	0.2
One-loop renormalization	6.0	6.0	0.0	6.0	6.0	1.2
$g_{B^*B\pi}$	0.24	0.00	0.00	0.14	0.00	0.25
Scale	1.82	1.85	0.04	1.84	1.86	0.05
Physical quark mass	0.05	0.01	0.06	0.06	0.19	0.20
Off-physical sea s quark mass	0.84	0.69	0.79	0.20	0.39	0.91
Fit-range	0.44	2.31	0.26	0.10	1.74	0.58
Total systematic error	7.38	7.09	2.97	6.90	6.94	3.66
Total error (incl. statistical)	7.96	7.32	3.40	9.37	7.61	4.97

at physical b quark mass point, which would be plausibly understood by the static approximation ambiguity. (See Ref. [2] for the comparison.) On the other hand, the results of $f_B\sqrt{\hat{B}_B}$, $f_{B_s}\sqrt{\hat{B}_{B_s}}$, $\hat{B}_B$ and $\hat{B}_{B_s}$ at the physical b quark point and the static limit are consistent within the somewhat largish errors. For the SU(3) breaking ratios, significant deviation from others is not seen, which is consistent with the estimate of the systematic error of these ratios in the static approximation $\sim 2\%$ [4].

4. Statistical improvement using all-mode-averaging

Dominant source of uncertainties includes statistical error, we thus try to reduce it using all-mode-averaging (AMA) procedure [10], by which good statistics is achievable with relatively low computational cost. Fig. 1 represents chiral and continuum extrapolations of f_{B_s} and $\mathscr{M}_B$ comparing results from the AMA with those in Ref. [4] (without AMA). While some data points in the AMA currently show larger statistical error than Ref. [4], the AMA data present better statistics as a whole. Especially, a peculiar light quark mass dependence in the B_s sector, where f_{B_s} of Ref. [4] in Fig. 1 increases as the light quark mass becomes small, is not seen in the new data. Fig. 2 gives the chiral and continuum extrapolated values of decay constants and mixing matrix elements only with statistical error, presenting the statistical effectiveness of the AMA results. We can see results from the AMA are consistent with those in Ref. [4], while the statistics is largely improved.

5. Future direction to reduce uncertainties

Other than the statistics, we have significant systematic errors. Dominant sources of the systematic uncertainty are chiral extrapolation and renormalization.

The lightest pion mass in the current calculation is about 290 [MeV], which leaves large uncertainty from the chiral extrapolation. This error would be significantly reduced by the physical point simulation, where the pion mass is about 135 [MeV]. The $2+1$ flavor dynamical ensembles at

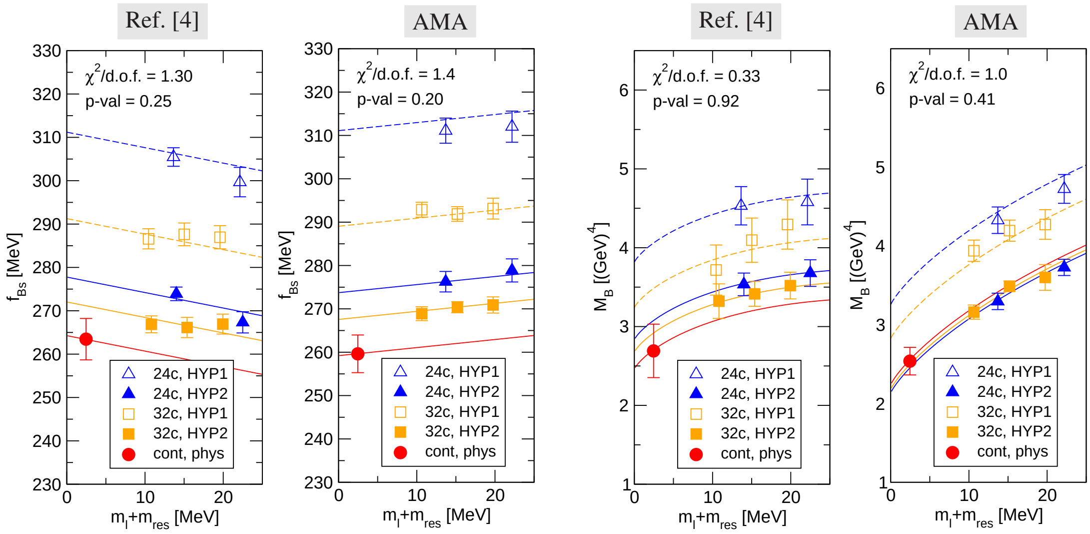

Figure 1: Chiral and continuum extrapolation of f_{B_s} and $\mathscr{M}_B$ comparing Ref. [4] and AMA results. (Preliminary)

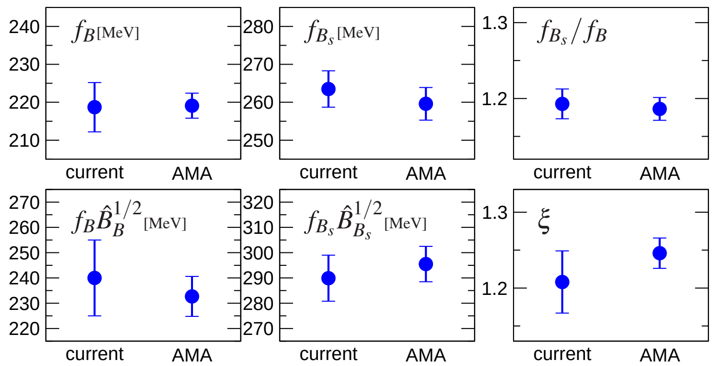

Figure 2: Comparison of chiral and continuum extrapolated physical quantities between current and AMA results. The error denotes only statistical one. (Preliminary)

almost physical pion mass were generated by RBC/UKQCD Collaborations using Möbious DWF (MDWF) keeping almost the same lattice spacings as those in this work, but with doubled physical volume [11, 12]. Their parameters are listed in Tab. 3. It would increase computational cost by a large amount, hence the AMA technique previously mentioned or all-to-all propagator strategy would be crucial.

While one-loop renormalization uncertainty is 0% or quite small for SU(3) braking ratios, it is estimated to be, at the most, 6% for non-ratio quantities. Non-perturbative renormalization is, hence, required for the non-ratio quantities to reduce the large uncertainty. The renormalization would be accomplished by RI-MOM scheme [13] or coordinate space renormalization method [14].

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Table 3: Dynamical $2+1$ flavor domain-wall fermion ensembles produced by the RBC and UKQCD collaborations [8, 11, 12].

gluon	fermion	$L^3 \times T \times L_s$	am_l	am_h	$am_{\rm res}$	m_π [MeV]	size [fm]
Iwasaki	DWF	$24^3 \times 64 \times 16$	0.01	0.04	0.00308	420	2.7
$\beta = 2.13$	DWF	$24^3 \times 64 \times 16$	0.005	0.04	0.00308	330	2.7
	MDWF	$\mathbf{48^3 \times 96 \times 24}$	**0.00078**	**0.0362**	**0.000610**	**139**	**5.5**
Iwasaki	DWF	$32^3 \times 64 \times 16$	0.008	0.03	0.000664	420	2.6
$\beta = 2.25$	DWF	$32^3 \times 64 \times 16$	0.006	0.03	0.000664	370	2.6
	DWF	$32^3 \times 64 \times 16$	0.004	0.03	0.000664	310	2.6
	MDWF	$\mathbf{64^3 \times 128 \times 12}$	**0.000678**	**0.02661**	**0.000312**	**139**	**5.4**

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