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Rare leptonic B-meson decays

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Loop-mediated rare decays $B_{s,d} \to \ell^+\ell^-$ provide sensitive tests of the Standard Model and constraints on its extensions. Measurements by LHCb and CMS of the $B_{s,d} \to \mu^+\mu^-$ branching ratios and prospects for their improvements in the future imply that the theory predictions must become more precise, too. For this purpose, three-loop strong interaction corrections and two-loop electroweak corrections to the relevant Wilson coefficient in the SM have recently been evaluated. In effect, non-parametric theory uncertainties have gone down from around $\pm 8\%$ to around $\pm 1.5\%$. At such a level of accuracy, special care must be devoted to the treatment of soft photon radiation.

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Rare B-meson decays are well known as probes of the Standard Model (SM) quantum structure, as well as a source of constraints on new physics models. Since they occur at scales $\mu \ll M_W$, it is convenient to describe them in the framework of an effective theory that arises after decoupling of the W-boson and all the heavier particles. The effective theory Lagrangian has the following generic form

$$\mathscr{L}_{\rm eff} \;=\; \mathscr{L}_{\rm QCD\times QED}(\text{leptons \& quarks} \neq t) \;+\; N\sum_n C_n(\mu) Q_n, \tag{1}$$

where Q_n are local interaction terms (operators), and C_n are the corresponding coupling constants (Wilson coefficients) that depend on the renormalization scale μ. Information on the electroweak-scale physics is encoded in the values of C_n. An advantage of such a description is the possibility of resumming large logarithms $\left(\alpha_s \ln M_W^2/m_b^2\right)^n$ using renormalization group techniques, as well as an easier account for symmetries.

The present article is devoted to the rare decays $B_q \to \ell^+\ell^-$ with $q = s, d$ and $\ell = e, \mu, \tau$, in particular to the $B_s \to \mu^+\mu^-$ mode which belongs to the flavour-physics highlights of the LHC. It is a strongly suppressed, loop-generated process in the SM. Its average, time-integrated branching ratio (with the final-state photon bremsstrahlung included) reads [1]

$$\overline{\mathscr{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}. \tag{2}$$

The above SM prediction is based on the recent perturbative calculations of the two-loop electroweak [2] and three-loop QCD [3] corrections to the relevant Wilson coefficient. It is in agreement with the current experimental world average [4]

$$\overline{\mathscr{B}}(B_s \to \mu^+\mu^-)_{\rm exp} = \left(2.8^{+0.7}_{-0.6}\right) \times 10^{-9} \tag{3}$$

that has been obtained by combining the measurements of CMS [5] and LHCb [6]. These results have a significant impact on parameter spaces of various beyond-SM theories. In the case of the Minimal Supersymmetric Standard Model (MSSM), they exclude a large part of the region with large $\tan\beta$ (the ratio of the two Higgs doublet vacuum expectation values). However, the moderate $\tan\beta$ region is hardly affected, especially when the superpartners are heavy enough to satisfy the Higgs mass constraints and the direct search bounds.

The operators in Eq. (1) that matter for $B_s \to \mu^+\mu^-$ in the SM and beyond read

$$Q_A = (\bar{b}\gamma^\alpha\gamma_5 s)(\bar{\mu}\gamma_\alpha\gamma_5\mu), \qquad Q_S \;=\; (\bar{b}\gamma_5 s)(\bar{\mu}\mu), \qquad Q_P \;=\; (\bar{b}\gamma_5 s)(\bar{\mu}\gamma_5\mu). \tag{4}$$

The normalization constant in Eq. (1) in this case can be written as $N = V_{tb}^\star V_{ts} G_F^2 M_W^2/\pi^2$, where G_F is the Fermi constant (extracted from the muon decay), M_W is the W-boson on-shell mass, and V_{ij} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements

Both Q_S and Q_P can be expressed in terms of the axial quark current

$$Q_{S(P)} \;\sim\; [\bar{b}\gamma^\alpha\gamma_5 s]\,\partial_\alpha[\bar{\mu}(\gamma_5)\mu] \;+\; \boxed{T} \;+\; \boxed{E}, \tag{5}$$

up to total derivatives $\boxed{T}$ and terms that vanish by the equations of motion $\boxed{E}$. In effect, the B_s-meson decay constant f_{B_s} defined by

$$\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s\rangle = ip^\alpha f_{B_s} \tag{6}$$

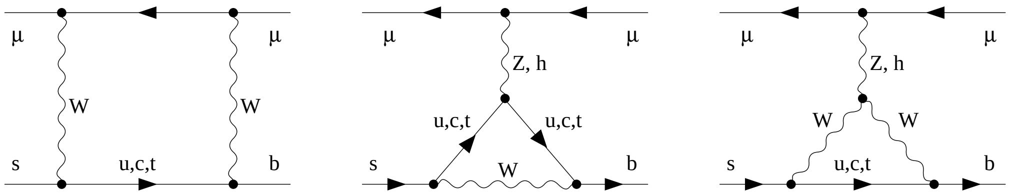

Figure 1: One-loop diagrams that determine the leading-order Wilson coefficients in the SM (unitary gauge).

is the only non-perturbative quantity that one needs for evaluation of $\overline{\mathscr{B}}(B_s \to \mu^+\mu^-)$ in the SM and beyond. For the numerical value of f_{B_s}, the average [7]

$$f_{B_s} = (227.7 \pm 4.5)\,\text{MeV} \tag{7}$$

of the $N_f = (2+1)$ lattice results [8, 9, 10] is going to be used.

Starting from Eq. (1), the following result for the average time integrated branching ratio can be derived

$$\overline{\mathscr{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\,\Gamma_H^s}\,\beta\left[|rC_A - uC_P|^2 F_P + |u\beta C_S|^2 F_S\right] \;+\; \mathscr{O}(\alpha_{em})\,, \tag{8}$$

where M_{B_s} is the B_s meson mass, and Γ_H^s stands for the total width of the heavier mass eigenstate in the $B_s\bar{B}_s$ system. The Wilson coefficients should be evaluated at the scale $\mu_b \sim m_b$. The quantities r, β and u are given by

$$r = \frac{2m_\mu}{M_{B_s}}, \qquad \beta = \sqrt{1-r^2}, \qquad u = \frac{M_{B_s}}{m_b + m_s}\,. \tag{9}$$

In the absence of beyond-SM sources of CP-violation, we have $F_P = 1$ and $F_S = 1 - \Delta\Gamma_s/\Gamma_L^s$, where Γ_L^s is the lighter eigenstate width, and $\Delta\Gamma^s = \Gamma_L^s - \Gamma_H^s$. In a generic case, from the results of Ref. [11] one derives

$$\begin{aligned} F_P &= 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\sin^2\left[\frac{1}{2}\phi_s^{\text{NP}} + \arg(rC_A - uC_P)\right], \\ F_S &= 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\cos^2\left[\frac{1}{2}\phi_s^{\text{NP}} + \arg C_S\right], \end{aligned} \tag{10}$$

where ϕ_s^{NP} describes the CP-violating "new physics" contribution to $B_s\bar{B}_s$ mixing, i.e., $\phi_s^{c\bar{c}s} \simeq \arg[(V_{ts}^* V_{tb})^2] + \phi_s^{\text{NP}}$ (see Sec. 2.2 of Ref. [12]).

In the SM, the Wilson coefficients are determined at the leading order by the one-loop diagrams shown in Fig. 1. Extra diagrams with the pseudo-Goldstone scalars need to be added in non-unitary gauges. C_A is given by the W-boxes and Z-penguins alone, while the Higgs (and pseudo-Goldstone) penguins need to be included for $C_{S,P}$. With our normalization, C_A turns out to be a dimensionless order-unity function of m_t^2/M_W^2. On the other hand, $C_{S,P}$ receive an additional suppression by $m_\mu m_b/M_W^2$. This suppression factor should be compared to r = $2m_\mu/M_{B_s}$ that multiplies C_A in

Eq. (8). In consequence, contributions of $C_{S,P}$ to the branching ratio are suppressed by $M_{B_s}^2/M_W^2$ with respect to those of C_A. Thus, we can neglect $C_{S,P}$ in the SM, which simplifies Eq. (8) to

$$\overline{\mathscr{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\Gamma_H^s}\beta\, r^2|C_A|^2 \;+\; \mathscr{O}(\alpha_{em})\,. \tag{11}$$

The above expression depends only on the heavier eigenstate width Γ_H^s. It is so because $\overline{\mathscr{B}}(B_s \to \mu^+\mu^-)$ in the SM is practically saturated by the heavier eigenstate decays. This fact can easily be understood in the limit of no CP-violation (real CKM matrix), adopting a phase convention in which the heavier (lighter) eigenstate is CP-odd (-even). Since the muons produced by Q_A are CP-odd, the lighter eigenstate cannot contribute via this operator. Interestingly, the latter statement remains true also after turning on CP-violation in the CKM matrix. One observes a cancellation of CP-violating phases in the mixing and decay amplitudes, up to tiny contributions from CP-violation in the absorptive part of the mixing amplitude.

The numerical result in Eq. (2) originating from Ref. [1] is based on Eq. (11). It includes complete corrections of order $\mathscr{O}(\alpha_{em})$ to the Wilson coefficient $C_A(\mu_b)$, but the $\mathscr{O}(\alpha_{em})$ term in Eq. (11) has been neglected. Such an approach can be justified by observing that some of the $\mathscr{O}(\alpha_{em})$ corrections to $C_A(\mu_b)$ get enhanced by $1/\sin^2\theta_W$, powers of m_t^2/M_W^2 or logarithms $\ln^2 M_W^2/\mu_b^2$, as explained in Ref. [2]. None of these enhancements is possible for the $\mathscr{O}(\alpha_{em})$ term in Eq. (11). This term is μ_b-dependent and contains contributions from operators like $(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\ell}\gamma^\alpha\ell)$ or $(\bar{b}\gamma_\alpha P_L c)(\bar{c}\gamma^\alpha P_L s)$, with photons connecting the quark and lepton lines. It depends on non-perturbative QCD in a way that is not described by f_{B_s} alone. Its part that does depend on f_{B_s} must compensate the μ_b-dependence of $C_A(\mu_b)$ which amounts to about 0.3% when μ_b is varied from $m_b/2$ to $2m_b$. This is much less than the two-loop electroweak corrections to $|C_A(\mu_b)|^2$ that can reach a few percent level [2].

The only other possible enhancement of QED corrections might be due to soft photon bremsstrahlung. Let us consider $B_s \to \mu^+\mu^-(n\gamma)$ with $n = 0,1,2,\ldots$. The dimuon invariant-mass spectrum in this process is obtained by summing the two distributions shown in Fig. 2. The dotted (blue) curve corresponds to the direct emission, i.e., real photon emission from the quarks. It has been estimated using Eq. (25) of Ref. [13]. The solid (red) curve is understood to describe all the other contributions to the considered process. Its tail is dominated by soft photon radiation from the muons. Interference between the two types of contributions has been neglected, as it gets suppressed by another power of r. The vertical dashed and dash-dotted (green) lines indicate the CMS [5] and LHCb [6] blinded signal windows, respectively. In the displayed region below the windows (i.e., between 5 and 5.3 GeV), each of the two types of contributions integrates to around 5% of the total rate.

The branching ratio determination on the experimental side includes a correction due to photon bremsstrahlung from the muons. For this purpose, both CMS [5] and LHCb [6] apply PHOTOS [14]. Given the current experimental uncertainties, such an approach can be understood as equivalent to extrapolating along the solid curve in Fig. 2 down to zero. In the resulting quantity, all the soft QED logarithms cancel out, and we arrive at Eq. (11), up to $\mathscr{O}(\alpha_{em})$ terms that undergo no extra enhancement.

The direct emission is infrared safe by itself because the decaying meson is electrically neutral. It survives in the limit $m_\mu \to 0$, which explains its considerable size in Fig. 2. It should be treated

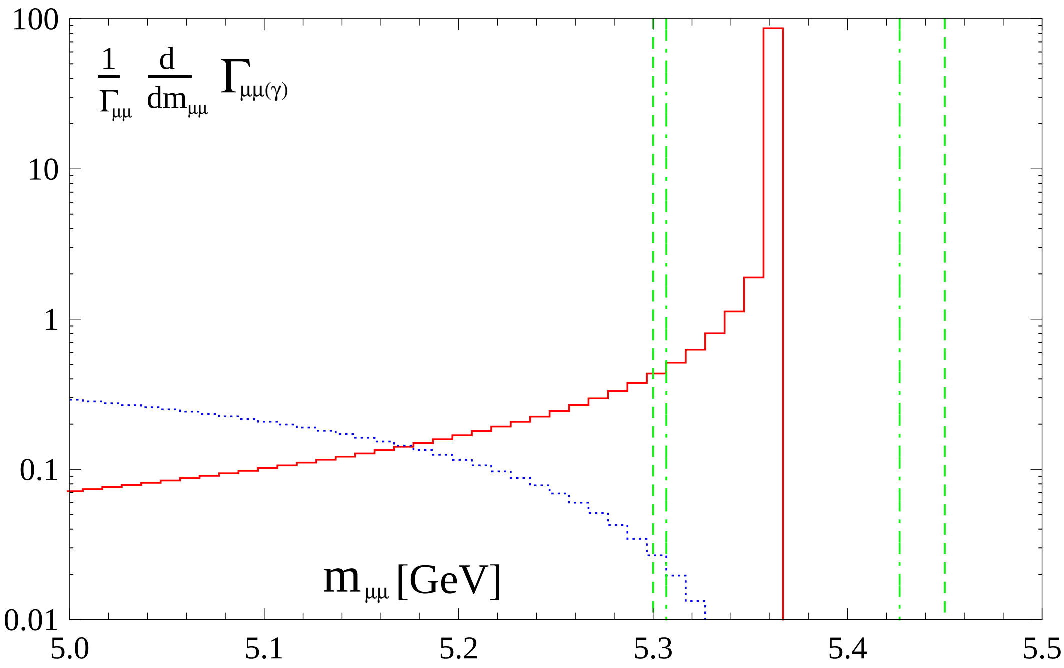

Figure 2: Contributions to the dimuon invariant-mass spectrum in $B_s \to \mu^+\mu^-(n\gamma)$ with $n = 0, 1, 2, \ldots$ (see the text). Both of them are displayed in bins of 0.01 GeV width.

as background on both the experimental and theoretical sides. On the theory side, it is just excluded from $\overline{\mathscr{B}}(B_s \to \mu^+\mu^-)$ by definition. On the experimental side, the current situation is somewhat more complex. Monte-Carlo routines are used to simulate the direct emission inside and outside the blinded windows. For the purpose of future measurements, one should either render such simulations precise (a difficult task), or restrict the actual signal windows to become as narrow as the current blinded ones, which would make the direct emission negligible. The latter solution seems to be a preferred choice, given that our knowledge of the blue curve in Fig. 2 is model-dependent and very rough.

All the input parameters that are necessary to evaluate the branching ratio in Eq. (11) are collected in Table 1 of Ref. [1]. The CKM matrix element $|V_{cb}|$ is treated in a special manner because it is responsible for the largest parametric uncertainty. One should be aware of a long-lasting tension between its determinations from the inclusive and exclusive semileptonic decays [15]. The recent inclusive fit from Ref. [16] is adopted for our present purpose. It is the first one where both the semileptonic data and the precise quark mass determinations from flavor-conserving processes have been taken into account. Once $|V_{cb}|$ is fixed, we evaluate $|V_{tb}^\star V_{ts}|$ using the accurately known ratio $|V_{tb}^\star V_{ts}/V_{cb}|$.

Apart from the masses and couplings, the branching ratio depends on two renormalization scales $\mu_0 \sim M_t$ and $\mu_b \sim m_b$ used in the calculation of the Wilson coefficient C_A. This dependence is very weak thanks to the new calculations of the two-loop electroweak and three-loop QCD corrections in Refs. [2, 3]. Here, we just fix here these scales to $\mu_0 = 160\,\text{GeV}$ and $\mu_b = 5\,\text{GeV}$. Allowing only the top-quark mass and the strong coupling constant to deviate from their central values, one finds the following fit for the relevant Wilson coefficient in the SM: $C_A(\mu_b) = 0.4690\, R_t^{1.53}\, R_\alpha^{-0.09}$,

	f_{B_q}	CKM	Γ_H^q	M_t	α_s	other parametric	non-parametric	Σ
$\overline{\mathcal{B}}(B_s \to \ell^+\ell^-)$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
$\overline{\mathcal{B}}(B_d \to \ell^+\ell^-)$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%

Table 1: Relative uncertainties from various sources in $\overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$. In the last column they are added in quadrature.

where $R_\alpha = \alpha_s(M_Z)/0.1184$ and $R_t = M_t/(173.1\,\text{GeV})$. The fit is accurate to better than 0.1% in C_A for $\alpha_s(M_Z) \in [0.11, 0.13]$ and $M_t \in [170, 175]\,\text{GeV}$. Inserting this fit into Eq. (11), and setting both R_t and R_α to unity, one arrives at the SM result given in Eq. (2).

All the $\overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$ branching ratios calculated along the same lines yield [1]

$$\begin{aligned}
\overline{\mathcal{B}}(B_s \to e^+e^-) &= (8.54 \pm 0.55) \times 10^{-14}, & \overline{\mathcal{B}}(B_d \to e^+e^-) &= (2.48 \pm 0.21) \times 10^{-15},\\
\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) &= (3.65 \pm 0.23) \times 10^{-9}, & \overline{\mathcal{B}}(B_d \to \mu^+\mu^-) &= (1.06 \pm 0.09) \times 10^{-10},\\
\overline{\mathcal{B}}(B_s \to \tau^+\tau^-) &= (7.73 \pm 0.49) \times 10^{-7}, & \overline{\mathcal{B}}(B_d \to \tau^+\tau^-) &= (2.22 \pm 0.19) \times 10^{-8},
\end{aligned} \tag{12}$$

A summary of their error budgets is presented in Table 1. It is clear that the main parametric uncertainties come from f_{B_q} and the CKM angles. The non-parametric uncertainty has been estimated at the level of around $\pm 1.5\%$ of the branching ratio [1], as compared to around $\pm 8\%$ prior to the calculations in Refs. [2, 3]. As far as the parametric uncertainties are concerned, their future reduction is not unlikely. In the $B_s \to \mu^+\mu^-$ case, we could already reduce the overall uncertainty to the $\pm 4.7\%$ level by being less conservative, i.e., taking the most optimistic results for $|V_{cb}|$ (weighted average of the inclusive and exclusive determinations) and for f_{B_s} (the HPQCD self-average [17] with a $\pm 3\,$MeV error). The experimental prospects are equally bright, with the projected overall uncertainties for $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ reaching around $\pm 8\%$ at LHCb [18] and around $\pm 12\%$ at CMS [19].

As far as $\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)$ is concerned, the SM result in Eq. (12) can be compared to the experimental determination of this quantity [4]

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)_{\text{exp}} = \left(3.9^{+1.6}_{-1.4}\right) \times 10^{-10}. \tag{13}$$

One observes that the measurement is around 2σ above the SM prediction, and the experimental uncertainties are dominant. Although statistically insignificant at present, such a deviation might be interpreted as a hint for beyond-SM theories with non-minimal flavour violation.

To conclude, the rare leptonic B-meson decay modes provide important constraints on beyond-SM physics, and require precise perturbative calculations within the SM. Recently calculated corrections to the Wilson coefficients significantly improve the accuracy in the $B_q \to \ell^+\ell^-$ case, reducing the non-parametric uncertainty to around $\pm 1.5\%$. At such a level of accuracy, special care must be devoted to the treatment of soft photon radiation.

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