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Rare semileptonic $b \to s\bar{\ell}\ell$ decays

Christoph Bobeth[*][†]
Institute for Advanced Study, Technische Universität München, 85748 Garching, Germany
E-mail: bobeth@ph.tum.de

Rare semileptonic $b \to s\bar{\ell}\ell$ decays are valuable probes of the flavour structure of the standard model and pose constraints on parameter spaces of its extensions. The first experimental analyses of angular distributions provide measurements of numerous new observables, among which many are free of form-factor normalisation. The global analyses of this data indicate deviations from standard model predictions for the short-distance coupling C_9, depending strongly on the chosen set of measurements and hadronic input.

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1. Introduction

The field of rare semileptonic $b \to s\bar{\ell}\ell$ decays has experienced a huge experimental progress in the last few years. The first detailed measurements of exclusive decay modes $B \to K\bar{\ell}\ell$ and $B \to K^*\bar{\ell}\ell$ come from the B-factory experiments Babar [1] and Belle [2], and were confirmed independently and extended later on at a hadronic machine by CDF [3]. Shortly after, the Run I $(2011+2012)$ of the LHC enabled also LHCb [4], CMS [5] and ATLAS [6] to contribute further new measurements. The number of events in different channels observed by each of the experiments is listed in table 1, showing that by now LHCb is able to enter a new era of precision measurements, which allow the determination of differential angular distributions [7, 8], CP asymmetries [9] and tests of lepton-universality among $\ell = e$ and μ [10].

Even by now, currently available data sets have not been yet fully analysed by Belle after reprocessing 711 fb^{-1}. LHCb is expected to update their angular analysis of $B \to K^*(\to K\pi)\bar{\ell}\ell$ soon for the combined data set from $(2011+2012)$ of 3 fb^{-1}, as well as for other channels: $B_s \to \phi\bar{\ell}\ell$, $\Lambda_b \to \Lambda\bar{\ell}\ell$ and $B^+ \to \pi^+\bar{\ell}\ell$. The same is true for the combined data sets from $(2011+2012)$ of 25 fb^{-1} for both, CMS and ATLAS, which is of concern for $B \to K^*(\to K\pi)\bar{\ell}\ell$ and $B \to K\bar{\ell}\ell$. In the future, the size of experimental data sets is expected to increase further during Run II of LHC $(2015-2018)$ at center of mass energies of 14 TeV. It is estimated that LHCb will be able to collect another 5 fb^{-1}. With the start of the Belle II experiment, presumably 2016, another large data sample with entirely different systematic uncertainties will provide after 5 years of running additional measurements, among which isospin related channels with neutral particles (π, K) in the final states, but also the inclusive channel $B \to X_s\bar{\ell}\ell$ for $\ell = e, \mu$ and perhaps tighter limits on channels with $\ell = \tau$.

These prospects have triggered a large amount of phenomenological and theoretical works with several different objectives:

- identification and study of new observables in angular distributions with enhanced sensitivity to new physics and reduced dependence on form factor normalisations at low q^2 [11, 12, 13] and high q^2 [14, 13, 15];
- study of subleading corrections in expansions in $1/m_b$ [16] and resonance contributions from $b \to s\bar{q}q \to s\bar{\ell}\ell$ processes with non-local operator product expansions (OPE) [17] or light-cone sum rules (LCSR) [18];
- first calculations of $B \to K$ ($B_s \to K$) [19] ([20]), $B_s \to K^*$, ϕ [21] and $\Lambda_b \to \Lambda$, p^+ [22] form factors from lattice QCD;
- model-independent fits of short-distance couplings of $|\Delta B| = |\Delta S| = 1$ effective theory and interpretation in extensions of the standard model (SM) [23, 24, 25, 26, 27].

Currently, all measurements are in the ball park of SM expectations, with some larger tensions, the most prominent found by LHCb in 1 fb^{-1} in the angular observable P_5' of $B \to K^*\bar{\ell}\ell$ [7] at low dilepton invariant mass[1] q^2 and P_4' at high q^2. The very recent measurement (3 fb^{-1}) of the

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[1] Throughout, q^2 denotes the dilepton invariant mass. The notation $\langle X\rangle_{[q^2_{\rm min}, q^2_{\rm max}]}$ implies q^2-integration of a observable X in the interval $q^2 \in [q^2_{\rm min}, q^2_{\rm max}]$ as defined in [14].

	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 9.6 fb^{-1}	LHCb 2011 (+2012)$^{*)}$ 1 (+2) fb^{-1}	CMS 2011 (+2012)$^{*)}$ 5 (+20) fb^{-1}	ATLAS 2011 5 fb^{-1}
$B^0 \to K^{*0}\bar{\ell}\ell$	$137 \pm 44^{\dagger)}$	$247 \pm 54^{\dagger)}$	288 ± 20	$2361 \pm 56^{*)}$	415 ± 70	426 ± 94
$B^+ \to K^{*+}\bar{\ell}\ell$			24 ± 6	$162 \pm 16^{*)}$		
$B^+ \to K^+\bar{\ell}\ell$	$153 \pm 41^{\dagger)}$	$162 \pm 38^{\dagger)}$	319 ± 23	$4746 \pm 81^{*)}$	not yet	not yet
$B^0 \to K_S^0\bar{\ell}\ell$			32 ± 8	$176 \pm 17^{*)}$		
$B_s \to \phi\bar{\ell}\ell$			62 ± 9	174 ± 15		
$B_s \to \bar{\mu}\mu$				emerging$^{*)}$	emerging$^{*)}$	limit
$\Lambda_b \to \Lambda\bar{\ell}\ell$			51 ± 7	78 ± 12		
$B^+ \to \pi^+\bar{\ell}\ell$		limit		25 ± 7		
$B_d \to \bar{\mu}\mu$			limit	limit	limit	limit

Table 1: Number of observed events by the experiments in various exclusive $b \to (s,d)\bar{\ell}\ell$ channels. All results are CP-averaged samples of B and $\bar{B}$ mesons with vetoed regions in the dilepton invariant mass around the J/ψ and ψ' resonances. For CDF, LHCb, CMS and ATLAS $\ell = \mu$, whereas Babar and Belle are lepton-flavour averaged for $\ell = e, \mu$. Moreover, $^{\dagger)}$ is an unknown mixture of B^0 and B^+ mesons and $^{*)}$ is the combined data set of 2011 and 2012.

ratio $R_K = Br[B \to K\bar{\mu}\mu]/Br[B \to K\bar{e}e]$, points towards lepton-non-universality[2] [10]. Also, lattice predictions of $B \to K^*$ and $B_s \to \phi$ form factors predict at high q^2 branching fractions systematically above measurements [21].

Phenomenological studies of angular observables J_i [29] in $B \to K^*(\to K\pi)\bar{\ell}\ell$

$$\frac{d^3\langle\Gamma\rangle}{d\cos\theta_\ell \, d\cos\theta_K \, d\phi} \sim \big(\langle J_{1s}\rangle + \langle J_{2s}\rangle\cos 2\theta_\ell + \langle J_{6s}\rangle\cos\theta_\ell\big)\sin^2\theta_K$$
$$+\big(\langle J_{1c}\rangle + \langle J_{2c}\rangle\cos 2\theta_\ell + \langle J_{6c}\rangle\cos\theta_\ell\big)\cos^2\theta_K + \big(\langle J_3\rangle\cos 2\phi + \langle J_9\rangle\sin 2\phi\big)\sin^2\theta_K\sin^2\theta_\ell \tag{1.1}$$
$$+\big(\langle J_4\rangle\cos\phi + \langle J_8\rangle\sin\phi\big)\sin 2\theta_K\sin 2\theta_\ell + \big(\langle J_5\rangle\cos\phi + \langle J_7\rangle\sin\phi\big)\sin 2\theta_K\sin\theta_\ell$$

have revealed many useful tests of the SM and specific new physics couplings in these twelve observables. Including the CP-conjugated decay, one has at disposal twelve CP-averaged and twelve CP-asymmetric observables [30]. There are combinations of J_i that are free of form-factor normalisations at low q^2 [11, 12, 13] and high q^2 [13, 14, 15] up to subleading corrections in $1/m_b$ expansions. Moreover, there are combinations of J_i at high q^2 that allow to measure ratios of form factors (in the absence of certain new physics couplings) [14, 12, 31, 26]. And further, at high q^2 there are relations among different combinations predicted by the adapted OPE such that strong violations by measurements would indicate either a breakdown of the OPE and/or the presence of scalar and/or tensor-like NP contributions [14].

Angular analysis have been extended to $B \to K\pi\bar{\ell}\ell$ for $K\pi$-invariant masses off the K^* resonance [32] in order to understand S-wave contributions and to explore complementarity of constraints on short-distance couplings to $B \to K^*(\to K\pi)\bar{\ell}\ell$ (on-resonance) at low and high q^2 [33].

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[2] This is also seen in inclusive $B \to X_s\bar{\ell}\ell$ [28], but with less significance.

Very recently also baryonic decays $\Lambda_b \to \Lambda(\to N\pi)\bar{\ell}\ell$, $(N = p, n)$, have been considered at low and high q^2 [34], showing also complementary constraints on short-distance couplings to $B \to K^*(\to K\pi)\bar{\ell}\ell$ (on-resonance).

Although we witness an impressive increase of the size of experimental data sets, it is currently not sufficient to carry out all the proposed phenomenological ideas designed to test the SM and to further tighten constraints on new physics. In this respect, experimental and theoretical communities already work closely together to be well prepared for the optimal interpretation and utilisation of future data.

2. Theory of exclusive $b \to s\bar{\ell}\ell$ decays

Theoretical predictions of exclusive $b \to s\bar{\ell}\ell$ decays are based on the effective theory of electroweak interactions

$$\mathscr{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\left(V_{tb}V_{ts}^*\sum_i C_i(\mu)\mathscr{O}_i + V_{ub}V_{us}^*\sum_j C_j(\mu)\mathscr{O}_j\right) \tag{2.1}$$

with different types of $|\Delta B| = |\Delta S| = 1$ higher-dimensional flavour-changing operators $\mathscr{O}_i$. The according short-distance couplings C_i are known in the SM up to NNLO [35], including renormalisation group evolution [36] from the matching scale of the order of electroweak symmetry breaking $\mu_0 \sim m_W$ down to the scale of bottom quark masses $\mu \sim m_b$. CP violation in $b \to s$ transitions is due to current-current 4-quark operators and doubly-Cabibbo suppressed by the quark-mixing combination $V_{ub}V_{us}^*$.

The semileptonic operators $\mathscr{O}_{9\,(10)} \sim [\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]$ give numerically leading contributions to most of the observables in $B \to K^{(*)}\bar{\ell}\ell$ decays in regions of q^2 away from J/ψ and ψ' resonances. They factorize into hadronic and leptonic currents (at lowest order in QED), requiring "only" the knowledge of hadronic $B \to$ light-meson form factors and can be calculated without further complications. The same applies to all other non-standard semileptonic operators ($\sim [\bar{s}\Gamma b][\bar{\ell}\Gamma'\ell]$) with right-handed, scalar or tensor Lorentz structures $\Gamma \otimes \Gamma'$.

The contribution of other operators — (up- and charm) current-current ($i = 1,2$), QCD-penguins ($i = 3,4,5,6$) and electro- and chromo-magnetic dipole ($i = 7,8$) — involve nonperturbative physics that can be dealt with only in particular regions of q^2 using various theoretical approaches. At low q^2 the large recoil of the $K^{(*)}$ allows to factorize hard spectator scattering (HS) contributions out of form factors [37] and calculate HS and weak annihilation contributions to the exclusive $b \to s\bar{\ell}\ell$ amplitudes at leading order in a $1/m_b$ expansion and systematically to higher orders in QCD (α_s) using QCD factorisation [38] or soft-collinear effective theory [39]. In particular the role of unknown subleading corrections in $1/m_b$ to form-factor relations are debated [16] concerning there impact on predictions of optimised observables. A second class of corrections are due to 4-quark operators that contribute via intermediate resonant structures $(\bar{q}q)$ with $q = u,d,s,c$, which decay electromagnetically to the same final state $b \to s(\bar{q}q) \to s\bar{\ell}\ell$. For $q = c$, these contributions induce huge "backgrounds" for $\sqrt{q^2}$ close to the J/ψ and ψ' masses compared to the contributions from $\mathscr{O}_{7,9,10}$. The "tails" of these contributions affect both, the low q^2 region and the whole high q^2 region with higher charmonia resonances. At $q^2 \ll 4m_c^2$ a non-local OPE [17] can be combined with dispersion relations to extrapolate up to $\sqrt{q^2}$ close to the J/ψ and ψ' masses, involving some modelling of the hadronic resonant structure. These studies showed that charm-quark contributions become important at around $q^2 = 6$ GeV^2. Alternatively, at low q^2 also LCSR calculations have been performed [18]. At high q^2 — preferably above the open charm threshold $q^2 = 15$ GeV^2 — a local OPE of these contributions is used [40]. With regard to tests of the SM or searches for new physics, the region 6 GeV$^2 \leq q^2 \leq 15$ GeV^2 is currently not under theoretical control.

The purely leptonic decays $B_q \to \bar{\ell}\ell$ are free of such long-distance contributions and can be predicted with highest precision of all exclusive decays. After the inclusion of higher order radiative corrections [41], the largest uncertainties are of parametric origin from lattice determinations of the decay constants f_{B_q} and the quark-mixing elements, especially V_{cb} and V_{td} [42].

3. Data analyses and New Physics constraints

Last years measurement of angular observables in $B \to K^*(\to K\pi)\bar{\ell}\ell$ from LHCb [7] triggered model-independent studies of $b \to s(\gamma, \bar{\ell}\ell)$ data. A strong deviation of the angular observable P_5' at low q^2 from SM expectations raised hopes of a sign of non-standard physics in $b \to s\bar{\ell}\ell$. A common approach is to fit the effective couplings C_i in (2.1) from the data, also called "model-independent" analysis, where different scenarios correspond to non-zero new physics contributions to a particular set of short-distance couplings C_i.

The first model-independent frequentist analysis [24] restricted the set of measurements to exclusively LHCb for $B \to K^*\bar{\ell}\ell$ optimised observables – besides some others like, $B \to X_s(\gamma, \bar{\ell}\ell)$, $B \to K^*\gamma$ and $B_s \to \bar{\mu}\mu$ — and discarded branching fraction measurements from $B \to K^{(*)}\bar{\ell}\ell$ as well as measurements from other experiments. This might be motivated by the desire to be independent of the form-factor normalisation and to reduce form-factor dependences in general. Moreover, the q^2 bins up to $q^2 = 8.7$ GeV^2 have been included. This way, indeed large negative deviations of the order of 30% from the SM in C_9 at the level of 3.9σ have been found, which reduce to 3.2σ when restricting to measurements with $q^2 \leq 6$ GeV^2. It is found that allowing for new physics in other short-distance couplings C_i does not improve significantly the fit.

The follow-up analysis [25] uses instead of optimised observables the CP-averaged angular observables J_i and includes also branching fraction measurements of $B \to K^{(*)}\bar{\ell}\ell$, as well as all available measurements from different experiments Babar, Belle, CDF, CMS and ATLAS. At lowq^2, only measurements with $q^2 \leq 6$ GeV^2 are taken. The general finding is confirmed that the new physics contribution to C_9 is negative. However, it is also found that scenarios with chirality-flipped short-distance couplings $C_{9',10'}$ can better reduce the total χ^2 then C_9 alone once including branching fraction measurements of $B \to K^{(*)}\bar{\ell}\ell$. In this case a positive new physics contribution $C_{9'}$ is required.

A set of measurements of only high q^2 observables in $B \to K^*\bar{\ell}\ell$ and $B_s \to \phi\bar{\ell}\ell$ also confirms the pattern of negative new physics in C_9 and positive $C_{9'}$ [21]. This analysis is based on the novel results of $B \to K^*$ and $B_s \to \phi$ form factors from lattice calculations, which give larger predictions of branching fractions at high q^2 in both channels compared to the measurements. These form factor results had not been available to the previous two analyses [24, 25].

The comprehensive bayesian analysis [26] includes in the fit besides the short-distance couplings C_i also the most relevant other "nuisance"-parameters, like quark masses and mixings, form-factor parameters and a naive parameterization of subleading contributions in $1/m_b$, also adopted in

[24, 25]. In consequence prior knowledge of these parameters will be updated in the fit, providing some helpful insights. The chosen data set comprises optimised observables, branching fractions of $B \to K^{(*)}\bar{\ell}\ell$ at low and high q^2 as well as complementary $B \to X_s(\gamma, \bar{\ell}\ell)$, $B \to K^*\gamma$ and $B_s \to \bar{\mu}\mu$ and includes measurements from all experiments. The fit of the SM, i.e. only nuisance parameters, has a satisfactory p value[3] of 0.12, which decreases to 0.06 when including lattice results of $B \to K^*$ form factors from [21]. Some parameters of subleading corrections in $B \to K^*\bar{\ell}\ell$ at low q^2 are shifted by order Λ/m_b w.r.t. their prior value. Besides the SM, also scenarios with real short-distance couplings, i.e., aligned with the SM quark mixing phase of $V_{tb}V_{ts}^*$ are considered: $C_{7,9,10}$ and chirality-flipped $C_{7',9',10'}$, as well as the variant with only $C_{9,9'}$. In all considered scenarios, pull values of observables are small ($\leq 2\sigma$) with a few exceptions that can not be addressed independently of the scenario: ATLAS and Babar measurements of $\langle F_L \rangle_{[1,6]}$ (see also [25]), the $\langle P_4' \rangle_{[14,16]}$ from LHCb, $\langle Br \rangle_{[16,19]}$ from Belle and $\langle A_{\rm FB} \rangle_{[16,19]}$ from ATLAS. The need for new physics in C_9 is strongly reduced compared to [24], usually to a $(1-2)\sigma$ deviation from the SM, depending on the scenario and 1D- versus 2D-marginalised posterior distributions. The lattice results of $B \to K^*$ form factors [21] tend to increase the deviation from the SM. In the future better prior information on subleading corrections is required in order to disentangle them from new physics contributions of chirality-flipped operators. A comparison of Bayes factors shows that the scenario with only $C_{9,9'}$ comes close to describe data as efficiently as the SM, whereas other scenarios are punished by the increased dimension of the parameter space.

Concerning explicit models, the challenge consists in explaining rather large new physics only in C_9, without modifying to much other effective couplings like $C_{7,10}$ etc. A qualitative discussion for the MSSM has been given in [25] and a quantitative analysis for SUSY-scenarios CMSSM(5), NUHM(6) and pMSSM(19) in [43], which both conclude that within the MSSM large contributions $|C_{9,9'}| \sim 1$ are not possible. However, $B \to K^*\bar{\ell}\ell$ measurements now provide complementary constraints to $B \to X_s\gamma$ and $B_s \to \bar{\mu}\mu$ in SUSY searches. Models with partial compositeness require a large degree of compositeness and cancellations for $C_{10,10'}$ in order to have large new physics in $C_{9,9'}$ [25], whereas constraints from the lepton sector are not yet taken into account. Models with flavour-changing transitions at tree-level are the simplest candidates that can accommodate new physics in C_9 without changing too much C_{10}, according Z and Z' models have been discussed in the literature [44]. In this respect, also QED mixing of $b \to s\bar{q}q$ operators into $\mathcal{O}_{7,9}^{(\prime)}$ provides a mechanism that prevents changes in $C_{10^{(\prime)}}$. While many operators for $q = u, d, s, c$ might be strongly constrained, depending on their chirality structure, for example the scenario of scalar $b \to s\bar{b}b$ operators [45] is able to explain larger new physics contributions to C_9 without being in conflict with $\Delta B = 2$ constraints.

In summary, all global analyses point towards a negative nonstandard contribution to C_9, the size depending on the chosen set of measurements, the prior assumptions on subleading contributions and form factor input. At present it can not be fully excluded that this is due to not understood QCD effects — usually contributing via $C_{7,9} \to C_{7,9}^{\rm eff}(q^2)$ — nor due to fluctuations in measurements. In the future, the experimental progress at LHCb and Belle II will provide more data, also in additional channels, which will allow for some cross checks. Theoretical progress concerning the resonant as well as subleading contributions will be of highest importance in order to be able

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[3] See latest arXiv version 3.

to detect small deviations from the standard model predictions.

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