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Hadron Freeze-Out and Unruh Radiation

Paolo Castorina*
Dipartimento di Fisica, Università di Catania, Via Santa Sofia 64, I-95123 Catania, Italy.
INFN, Sezione di Catania, I-95123 Catania, Italy.
E-mail: paolo.castorina@ct.infn.it

Alfredo Iorio
Faculty of Mathematics and Pysics, Charles University, V Holešovičkách 2, 18000 Prague 8, Czech Republic.
E-mail: alfredo.iorio@mff.cuni.cz

Helmut Satz
Fakultät für Physik, Universität Bielefeld, Germany .
E-mail: satz@physik.uni-bielefeld.de

In high energy collisions the description of newly formed hadrons as a Unruh radiation phenomenon applies at vanishing baryonchemical potential, $\mu_B \simeq 0$. It has already been found to correctly yield the freeze-out universal temperature, T_h, and the strangeness suppression in elementary collisions. Moreover, the Unruh mechanism leads, for $\mu_B \simeq 0$, to the freeze-out conditions on the average energy per hadron, $\langle E\rangle/\langle N\rangle \simeq 1.08$ GeV and on the entropy density, $s/T_h^3 \simeq 7.4$, by considering an area law for the entanglement entropy associated with the string breaking. These agree with phenomenological and lattice results, $s/T_h^3 \simeq 7$, $\langle E\rangle/\langle N\rangle \simeq 1.09$ GeV, that lack a basic theoretical justification.

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*Speaker.

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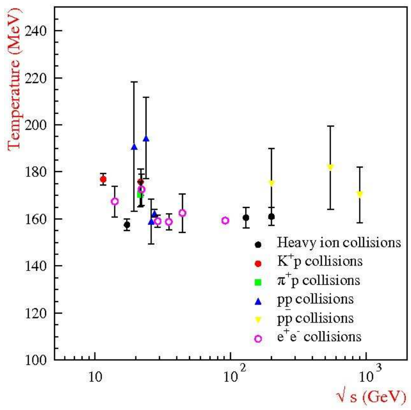

Figure 1: Hadronization temperature for different initial scattering configurations as a function of $\sqrt{s}$.

1. Introduction

The relative abundances of the produced hadrons in high energy e^+e^- annihilation, in hadron-hadron interaction as well as in the collisions of heavy ions over an energy range from around 10 GeV up to the TeV range, appear to be those of an ideal hadronic resonance gas at a quite universal temperature $T_h \approx 160 - 170$ MeV (see fig.1) [1, 2, 3, 4, 3, 5, 6].

However the production of strange hadrons in elementary collisions is suppressed relative to an overall equilibrium and this is usually taken into account phenomenologically by introducing an overall strangeness suppression factor $\gamma_s < 1$ [7], which reduces the predicted equilibrium abundances by γ_s^n for hadrons containing n strange quarks (or antiquarks), respectively.

In high energy heavy ion collisions, strangeness suppression becomes less, i.e. $\gamma_s \to 1$ at high energies, [8] and, moreover, another parameter , the baryochemical potential μ_B, takes into account the finite baryon density.

It is also found that, for a large range of baryon densities, chemical freeze-out occurs for a constant value of the dimensionless ratio $s/T_h^3 \simeq 7$ of the entropy density over T_h^3. An alternative parametrization gives a constant ratio of the average energy per hadron, $\langle E \rangle / \langle N \rangle \simeq 1.09$ GeV as freeze-out condition (see fig.2). Extensions to larger baryon density have led to various phenomenological proposals for freeze-out conditions [9, 10, 11, 12, 13, 14, 15].

The success of the statistical hadronization model raises many questions: why is the hadronization temperature universal? Why do even elementary, e^+e^- and hadron-hadron, collisions show thermal behaviour? Why is there in such interactions a suppression of strange particle production? Why does the strangeness suppression almost disappear in relativistic heavy ion collisions? Why does the thermal freeze-out curve correspond to $s/T_h^3 \simeq 7$ and to $\langle E \rangle / \langle N \rangle \simeq 1.09$ GeV?

Indeed, there is a still ongoing debate about the interpretation of the observed thermal behavior [16]: in high energy heavy ion collisions multiple parton scattering could lead to kinetic thermalization, but e^+e^- or elementary hadron interactions do not readily allow such a description. Moreover, the universality of the observed temperatures, suggests a common origin for all high energy collisions.

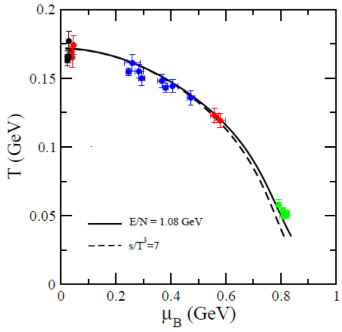

Figure 2: Chemical freeze-out curve compared with the conditions $s/T_h^3 \simeq 7$ and to $\langle E\rangle/\langle N\rangle \simeq 1.09$ GeV

It has been recently proposed [17] that thermal hadron production is the QCD counterpart of Hawking-Unruh (H-U) radiation [18, 19], emitted at the event horizon due to colour confinement. In the case of approximately massless quarks, the resulting universal hadronization temperature is determined by the string tension σ, with $T \simeq \sqrt{\sigma/2\pi} \simeq 165$ MeV [17]. Moreover in ref.[20] it has been shown that strangeness suppression in elementary collisions naturally occurs in this framework, without requiring an ad-hoc suppression factor, due to the non-negligible strange quark mass, which modifies the acceleration and therefore the emission temperature for such quarks.

In the next sections, after a general discussion concerning the thermal spectrum of the particles radiated from an event horizon, it is shown that the freeze-out conditions, $s/T_h^3 \simeq 7$ and to $\langle E\rangle/\langle N\rangle \simeq 1.09$ GeV, have a natural interpretation , for $\mu_B = 0$, in the proposed Unruh hadronization approach.

2. Event Horizon and Thermal Spectrum

Before considering the QCD case, let us recall that for a black-hole the Hawking radiation has a thermal spectrum (with the limits discussed in ref.[21]), with the Hawking temperature[18] $T_{haw} = k/2\pi = 1/8\pi GM$, where k is the surface gravity and G is the Newton constant.As shown in ref. [22, 23] this result can be understood in terms of tunneling through the event horizon.

On the other hand, for an observer in uniform acceleration a (a Rindler observer), with a space-time hyperbolic motion, there is an event horizon: the observer in the accelerating rocket in fig.3 can send informations to but cannot (classically) receive any signals from a Minkowski observer. For the accelerated observer any n-point Green function of an interacting field theory corresponds to the n-point Green function for the Minkowski observer evaluated in a thermal bath with the Unruh temperature $T_U = a/2\pi$ [19, 24]. Moreover the Unruh temperature can be evaluated by considering particle tunneling through the event horizon [25].

The correspondence between black-hole and uniformly accelerated observer is much more than an analogy. Indeed for a Rindler observer the space-time metric in spherical coordinates $(\tau, \chi, \theta, \phi)$ is given by $ds^2 = \chi^2 a^2 dt^2 - d\chi^2 - \chi^2 cosh^2(a\tau)(d\theta^2 + sin^2\theta d\phi^2)$ and the metric for a

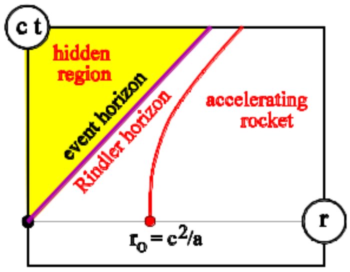

Figure 3: Event Horizon for Rindler observer

Schwarzschild black-hole in the near horizon approximation turns out to be $ds^2 = \eta^2 k^2 dt^2 - d\eta^2 - R^2(d\theta^2 + sin^2\theta d\phi^2)$. A direct comparison shows that the Rindler metric corresponds to the near horizon black-hole metric if the acceleration a is equal to the surface gravity k. Indeed the Unruh formula, $T_U = a/2\pi$, for $a = k = 1/4GM$ gives the Hawking temperature.

It is rather interesting to see if some of the previous considerations apply to QCD dynamics.

Indeed lattice simulations and phenomenological analyses indicate that the large distances behavior of the potential,V, between two static color charges increases linearly with the separation r, $V = \sigma r$.

Therefore QCD at large distances has a typical Rindler force, i.e. a constant acceleration, usually associated with a flux tube (a string) between a quark and an antiquark. Incidentally, it has been recently shown that gravity at large distances can be described by a Rindler force [26].

The phenomenological consequences of a constant acceleration and of the breaking of the flux tube in the hadronization process have been analyzed in ref.[17, 20, 27]) with the conclusion that:1) there is a universal Unruh temperature associated with the hadronization, $T_h \simeq 165$ MeV; 2) a small difference in the acceleration due to quark masses ($m_s \neq m_u = m_d$) explains the strangeness suppression in elementary collisions; 3) this suppression is almost removed in heavy ion collisions.

In this framework a possible understanding [28] of the freeze-out conditions $s/T_h^3 \simeq 7$, $\langle E \rangle / \langle N \rangle \simeq 1.09$ GeV, for $\mu_B \simeq 0$, is proposed in the next section.

3. Rindler force, string breaking and freeze-out conditions

3.1 String breaking and $\langle E \rangle / \langle N \rangle \simeq 1.09$ GeV

In the Unruh scenario, the fundamental mechanism of hadronization is quark acceleration, leading to string breaking with the resulting pair production: to separate ends for an initial $q\bar{q}$ pair at a distance R, when each quark hits the confinement horizon, i.e., when it reaches the end of the binding string, a further quark-antiquark system is excited from the vacuum. Although the new pair $q_1\bar{q}_1$ is at rest in the overall center of mass system, each of its constituents has a transverse momentum k_T, determined by the uncertainty relation in terms of the transverse dimension of the string flux tube. String theory [29] gives for the basic thickness $r_T = \sqrt{2/\pi\sigma}$,leading to $k_T = \sqrt{\pi\sigma/2}$. The maximum separation distance R is thus specified by

$$\sigma R = 2\sqrt{m_q^2 + k_T^2} = 2k_T, \tag{3.1}$$

where we have taken $m_q = 0$ for the quark mass. From this we obtain

$$R = \sqrt{2\pi/\sigma} \tag{3.2}$$

as the string breaking distance.

The Unruh phenomenon, in high energy collisions, is responsible for the production of *newly formed* hadrons, therefore, it cannot address the role of the nucleons *already present* in the initial state of heavy ion collisions. As such, it captures the whole of the freeze-out process only as long as there are no significant baryon-density effects, i.e., only for $\mu_B \simeq 0$, corresponding to high energy collisions where the large part of produced hadrons are $q\bar{q}$ mesons.

The energy of the pair produced by string breaking, i.e., of the newly formed hadron, is, from previous equation, given by

$$E_h = \sigma R = \sqrt{2\pi\sigma}. \tag{3.3}$$

In the central rapidity region of high energy collisions, one has $\mu_B \simeq 0$, so that E_h is in fact the average energy $\langle E \rangle$ per hadron, with an average number $\langle N \rangle$ of newly produced hadrons.

Hence we obtain

$$< E > / < N >= \sqrt{2\pi\sigma} \simeq 1.09 \pm 0.08,\ GeV \tag{3.4}$$

for $\sigma = 0.19 \pm 0.03$ GeV^2, in accord with the phenomenological fit obtained from the species abundances in high energy collisions [9, 10, 11].

It is, of course, an intriguing open question why phenomenological conditions, well accounted for by the Unruh mechanism, appear to remain as valid also for increasing μ_B.

3.2 Freeze-out for $s/T_h^3 \simeq 7$ and string breaking

In the hadronization process one deals with quantum particles, relativistically accelerated by the strong Rindler force, which experience a Rindler spacetime with an event horizon. From this point of view, since there is no gravitational interaction, the corresponding entropy has to be considered as an entanglement entropy due to causally disconneted regions.

A quantum field near an event horizon has modes belonging to both sides, inside and outside the horizon, whose entanglement entropy can be computed to give

$$S_{\text{ent}} = \alpha \frac{A}{r^2}\ , \tag{3.5}$$

see, e.g., [30], [31]. Here A is the area of the event horizon, r is the scale of characteristic quantum fluctuations (UV cut-off), α is an undetermined numerical constant.

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This expression shares the holographic structure[1] with the Bekenstein-Hawking entropy [34] of gravitational origin

$$S_{\rm BH} = \frac{1}{4}\,\frac{A}{r_P^2}\;, \tag{3.6}$$

with $r_P = \sqrt{\hbar G/c^3}$, the Planck length, i.e. the scale of quantum gravity fluctuactions. It is an important open question of fundamental physics whether the entanglement entropy can fully account for the Bekenstein-Hawking's[2].

The Rindler metric can be written also as

$$ds^2_{Rindler} = e^{2a\xi}(d\eta^2 - d\xi^2) - R^2 d\Omega^2\;, \tag{3.7}$$

where and η and ξ are Rindler time and space coordinates, respectively, related to the Minkowskian ones, t and x, through the usual transformations

$$t = a^{-1}e^{a\xi}\sinh a\eta\;,\quad x = a^{-1}e^{a\xi}\cosh a\eta\;. \tag{3.8}$$

The other two dimensions have the topology of a sphere [37], as this is the case of interest for the hadronization mechanism [17], thus, the area of the event horizon, $A_h = \int dydz$, that is a surface of constant proper acceleration (i.e., of constant Rindler coordinate ξ), and of constant Rindler time η, is given by $A_h = 4\pi R^2$, where R is the radius of the spherical Rindler horizon.

As well known [38], the Bekenstein-Hawking formula also holds for the Rindler spacetime, when the latter is the near-horizon approximation of a black hole spacetime.

Therefore let us take a phenomenological point of view and extend the validity of the formula to the case in point, where a constant, non gravitational, acceleration is involved and where the scale of the characteristic quantum fluctuations is given by r_T, (that is $r_P \to r_T$), and, noticeably, the whole entropy is of the entanglement type. If for the entropy associated with hadron production at the string breaking the formula

$$S_h = \frac{1}{4}\,\frac{A_h}{r_T^2} = \frac{1}{4}\,\frac{4\pi R^2}{r_T^2} \tag{3.9}$$

is used , with R given by eq.(3.2) and $T \simeq \sqrt{\sigma/2\pi}$ [17], one gets

$$S_h = \pi^3, \tag{3.10}$$

i.e., the entropy is a pure number with an exact cancellations in the ratio of dimensionfull quantities.

The entropy *density* divided by T_h^3 at freeze out (for $\mu_B \simeq 0$) turns out to be

$$\frac{s}{T_h^3} = \frac{S_h}{(4\pi/3)R^3T_h^3} = \frac{3\pi^2}{4} \simeq 7.4. \tag{3.11}$$

This result is in good accord with the value obtained for s/T_h^3 in terms of the ideal resonance gas model [14, 15] and with the most recent lattice QCD studies [39].

[1] Holography of entanglement entropy is a widely general result, see, e.g., [32], and the review [33].

[2] Standard quantum field theory computations leading to (3.5) cannot reproduce $\alpha = 1/4$, have to face the issue of the UV cut-off r, that cannot be fixed by the theory to be r_P, and the so called, “species puzzle” issue. On this see, e.g., [33], and the recent [35].

4. Conclusions

The formula (3.9) deserves a full theoretical explanation, that would bring us right in the core of the long-standing debate on the entanglement nature of the Bekenstein-Hawking entropy. We hope to come back to this in future work. Nonetheless, let us stress here that is of paramount importance, for fundamental questions of physics, to have a Unruh phenomenon as the one we have here. Therefore, the phenomenological regime of zero chemical potential, $\mu \simeq 0$, is perhaps the one to focus on, to extract from it precious experimental informations on the Unruh radiation, a phenomenon that has eluded direct observation since decades.

Acknowledgements

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