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An iterative unfolding method dedicated to the measure of cosmic-ray fluxes with AMS-02

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The measurement of cosmic-ray fluxes gives an indirect access to the source spectra, the abundances of different species and to the propagation processes in the Galaxy. Due to the finite energy resolution of the detectors, the measured fluxes differ from the physical ones. The process to compute the true flux from the measured one is called unfolding. The method presented here was developed in the context of the AMS-02 experiment, in particular for the analysis of the proton spectrum.

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1. Introduction

Recent measurements [6] of the cosmic ray proton flux at Earth show a power-law energy spectrum with a spectral index of 2.8 which results from the combination of the source spectrum and propagation effects in the Galaxy. Any structure or change of the flux spectral index can be understood as a new feature in those causes.

The AMS-02 detector is a particle physics detector installed on the ISS since may 2011. It is composed of several instruments including a magnet which makes possible the separation between negative and positive particles. The trajectory of particles passing through the detector is measured and some physical properties can be deduced, like the rigidity ($R = \frac{pc}{Ze}$) or the absolute value of the charge of the particle. Its sign is determined by the orientation of the curvature of the trajectory in the magnetic field. As with all detectors, it has a finite rigidity resolution. Therefore, every particle detected has a measured rigidity, R_{meas}, which is different from the true one, R_{true}.

The number of particles having R_{meas} lower than R_{true} is the same as the number of particles with R_{meas} higher than R_{true}. Given the power-law shape of cosmic-ray fluxes, migration to higher energy has a more important impact than migration to lower energy. This is illustrated by figure 1, where migration leads to a distorted measured flux which needs to be corrected.

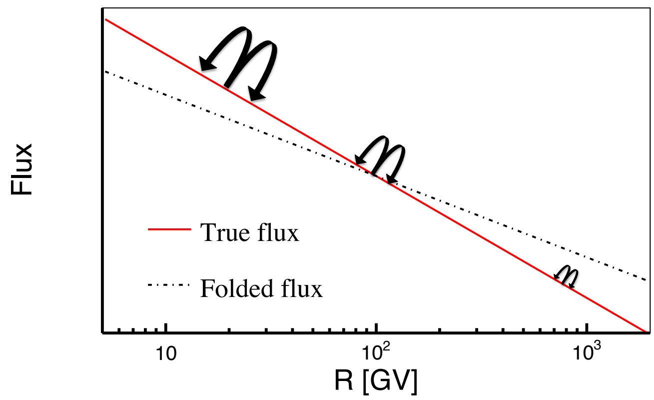

Figure 1: Sketch of the impact of the limited rigidity resolution on the flux. The arrows show that for every rigidity, events can migrate left or right, but due to the power-low shape of the flux the effect is much more important on the high rigidity side. The folding effect tends to decrease the spectral index.

Figure 2 illustrates this effect with a simulated flux (taken as a power law with a slope of 2.7) and the measured associated flux. The folding effect could lead to strong bias in the measurement of the fluxes (local structures and change of the spectral index). The goal of the unfolding process is to inverse the problem and to compute the true flux given the measured one.

2. Unfolding and regularisation

The AMS-02 chain of analysis is composed of several steps. First, the events to analyse have

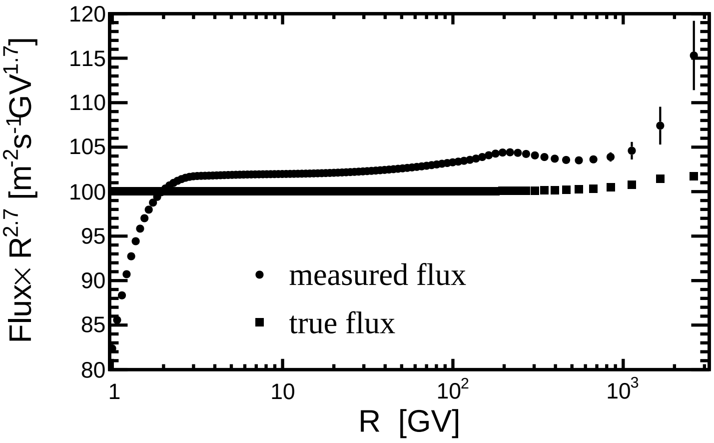

Figure 2: Shown are the toy flux (pure power law, black squares) and the associated measured flux (black dots) as a function of the rigidity of the measured particles ($\mathrm{R} = \frac{\mathrm{pc}}{\mathrm{Ze}}$). The measured flux is the product of a migration matrix and the true flux. The matrix corresponds to the probability for a particle to be detected at a rigidity given the true one. This matrix (see 3) has been taken from the Monte-Carlo simulation of the AMS-02 experiment.

to be correctly selected using the physical properties measured by the detector (*e.g.* $\mathrm{Z} = +1$ for protons). Then the number of events must be corrected with the effiencies of the different selections applied. The geometric acceptance has to be estimated using a Monte-Carlo simulation. The final step is to deconvolve the response of the detector; this is the unfolding. Here, we focus on this last step.

2.1 The unfolding algorithm

To perform the unfolding and compute the true flux, a migration matrix is needed. It makes the link between the measured flux and the true one. It is usually computed from a Monte-Carlo simulation of the detector. It corresponds to the probability for a particle with a true rigidity $\mathrm{R}_{\mathrm{true}}$ to be detected at a given measured one $\mathrm{R}_{\mathrm{meas}}$.

If we consider a measured flux $\mathbf{F}^{\mathrm{m}}$, an unfolded one $\mathbf{U}^{\mathrm{unf}}$ and a migration matrix M, the link between all those quantities is:

$$\mathbf{F}^{\mathrm{m}} = \mathrm{M} \times \mathbf{U}^{\mathrm{unf}}. \tag{2.1}$$

A first naive idea to unfold $\mathbf{F}^{\mathrm{m}}$ could be to invert M, such as $\mathrm{M}^{-1}\mathbf{F}^{\mathrm{m}} = \mathbf{U}^{\mathrm{unf}}$. Due to the statistical fluctuations in $\mathbf{F}^{\mathrm{m}}$, the unfolded flux obtained this way usually presents some unacceptable structures [1]. Therefore, some methods with regularisation have been developed [2, 3, 4, 7] to unfold the measured fluxes. The method described here is iterative and a regularisation process is also proposed.

The process consists of applying several times the matrix M to a flux $\mathbf{U}$, corrected each time by a weighting factor (since the process is iterative the index k will denotes the iteration we are considering):

$$\begin{cases} \mathbf{F}^k &= M \times \mathbf{U}^k; \\ W^k &= \dfrac{F^k}{U^k}; \\ U^{k+1} &= \dfrac{F^m}{\tilde{W}^k}. \end{cases} \tag{2.2}$$

The quantity $\tilde{\mathbf{W}}^{\mathrm{k}}$ denotes the regularised version of $\mathbf{W}^{\mathrm{k}}$ (see subsection 2.2).

Since the process is iterative, a convergence criterion is necessary to stop the algorithm. One possibility is to demand the maximum difference in rigidity between two successive unfolded fluxes to be lower than a given value ρ_{min}. However, there is no standard value for ρ_{min}: it depends on the migration matrix and the precision requirements, but also on the initial condition of the unfolding process. Taking as a starting point an initial flux $\mathbf{U}^0$ as close as possible to the true flux (see 2.2) ensures a rapid convergence of the algorithm.

2.2 The regularisation process

The goal of the regularisation process is to limit the propagation of statistical fluctuations from $\mathbf{F}^{\mathrm{m}}$ into the unfolding process. If no regularisation is applied, the statistical fluctuations will be considered as fine structures and will be amplified by the unfolding procedure. The different methods in the literature [2, 3, 4, 7] usually rely on strong hypotheses and can introduce a bias on the estimation of the true flux. The method proposed here has only one hypothesis which is the smoothness of the weight factors $\mathbf{W}$. This hypothesis is natural since the weight factors represent the folding effect at a given iteration which is expected to be smooth (see figure 2).

The principle is to fit $\mathbf{W}$ with a spline function before applying it to $\mathbf{F}^{\mathrm{m}}$ (see Eq. 2.2). The spline function used is a cubic interpolation between nodes with a fixed x position. Assuming a continuity of the three first derivatives, the only degree of freedom left is the y position of the nodes. One advantage of this regularisation process is that, assuming that $\mathbf{U}^0$ is not too different from the true flux, the unfolding process will be quick and all the $\mathbf{W}^{\mathrm{k}}$ will be very similar. Then, fitting them all with the same procedure will ensure a fast and robust method.

To control the systematic errors due to the regularisation process, the number of nodes and node positions are varied. The dispersion is calculated from all the resulting unfolded fluxes, which gives an estimate of the error.

3. Test and example

In this section, we give more details about the unfolding of the toy flux already shown in Fig. 2. Shown in figure 3 is the migration matrix as provided by the AMS-02 Monte-Carlo simulation of the detector. This matrix gives some information about the detector resolution. The spread at high rigidity is typical from magnet working detectors. In AMS-02, a silicon tracker is used to detect the particle trajectories in the magnetic field of the instrument. High rigidity particles have a trajectory which is almost a straight line. Since the differentiation of two straight trajectories is very hard, the resolution of the detector at this rigidity is lower than at low rigidity. Therefore, the spread of the matrix at 1 TV is bigger than the one at 10 GV.

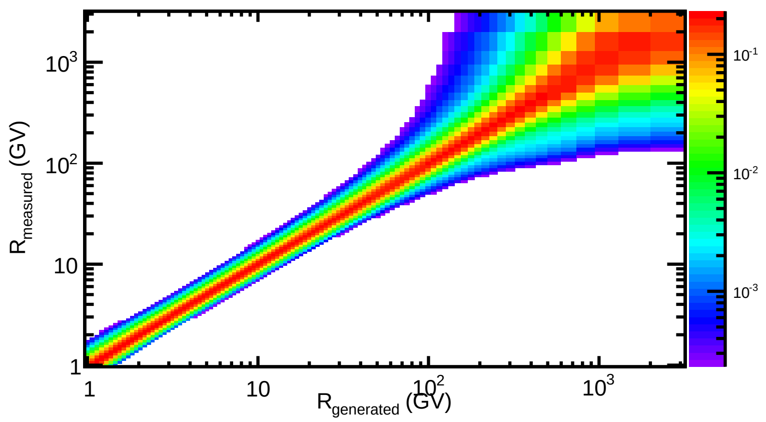

Figure 3: Migration matrix. The true rigidity is on the x-axis and the reconstructed one on the y-axis. A given bin is the probability for a particle to be detected at the corresponding rigidity given a true one.

The test procedure consists of (i) start with a true proton flux of power-law spectral index 2.8; (ii) compute the measured flux with the matrix and Equation 2.1; (iii) test the unfolding algorithm on this measured flux using the same matrix. This way, we are able to compare directly the unfolded flux to the true one. This comparison is only possible with simulated fluxes since in real measurement, the true flux is unknown. A good quantity to test the method is the ratio between the unfolded flux and the true one. In order to better see the convergence, this ratio is displayed for different iterations (see figure 4). In this case, the convergence criterion used is $\rho_{\min} = 0.01\%$, which is a very strong one.

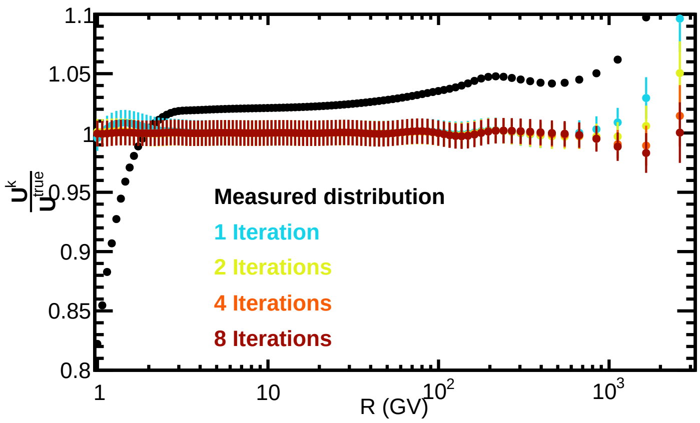

Figure 4: Ratio between the unfolded flux and the true one for different iterations. The black dots show the ratio between the measured distribution and the true one.

Figure 4 shows that the algorithm converges quickly where the folding effect is small and constant (from 3 GV to 500 GV) and takes more iterations to converge at higher rigidity. The wave structure is due to the regularisation process. We can see that after 8 iterations (the algorithm stopped after the ninth) the difference between the true flux and the one obtained is always below 2%.

4. Conclusions and overview

Unfolding processes are important in high energy physics to properly compare the results from different experiments and from theoretical predictions. The method presented here is inspired from the convergent weights method [5] for which a regularisation process is proposed. It is easy to implement and fast. The method relies on only one hypothesis, that is the smoothness of the folding effect at each iteration, and the regularisation process is done on the weight factors. It does not suppress any structure from the data, and it keeps the statistical fluctuations at the level present in the measured fluxes. This unfolding procedure is not sensitive to fine folding effects, happening at small rigidity scale, but it unfolds global effects (like the change of slope in a measured flux). This method has been developed and tested for AMS-02 data, and it is one of the methods used in the forthcoming AMS-02 protons flux analysis.

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