PROCEEDINGS OF SCIENCE

PICARD: A new Code for The Galactic Cosmic Ray Propagation Problem

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We present PICARD, our newly developed code for the numerical solution of the cosmic ray transport equation with a focus on Galactic cosmic rays. This code allows for the computation of cosmic ray spectra and the resulting Galactic gamma-ray emission. Shortcomings present in other propagation codes were overcome by the development of PICARD: we introduced contemporary numerical solvers that allow efficient computation on very high resolution (decaparsec) scales. As emphasized in recent studies, we also allow for locally anisotropic spatial diffusion using a full diffusion tensor. The capabilities of PICARD are illustrated by investigating the transition from axially symmetric cosmic ray source distributions to spiral arm cosmic ray source distribution, including consequences for the various observables related to cosmic rays.

Cosmic Rays and the InterStellar Medium
24-27 June 2014
Montpellier, France

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PoS(CRISM2014)046

1. Introduction

The propagation of Galactic cosmic rays from their sources to the observer can be approximated as a convection diffusion process in both configuration and momentum space (see, e.g., [15]). For a particular cosmic ray species i the resulting transport equation reads in the most general form:

$$\begin{aligned}\frac{\partial \psi_i}{\partial t} =& q(\vec{r},p) + \nabla \cdot (\mathscr{D}\nabla\psi_i - \vec{v}\psi_i) + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p}\frac{1}{p^2}\psi_i \\ & - \frac{\partial}{\partial p}\left\{\dot{p}\psi_i - \frac{p}{3}(\nabla\cdot\vec{v})\psi_i\right\} - \frac{1}{\tau_f}\psi_i - \frac{1}{\tau_r}\psi_i\end{aligned} \tag{1.1}$$

where q is the parametrisation of the cosmic ray source density, $\mathscr{D}$ is the spatial diffusion tensor, $\vec{v}$ is the spatial advection velocity, D_{pp} represents diffusive re-acceleration (i.e., Fermi II acceleration), the subsequent term represents adiabatic energy changes, and τ_f and τ_r are the time scales for fragmentation and radiative decay. This equation describes the time evolution of the distribution function ψ_i of cosmic ray species i, where the interaction between different species is parametrised via the source and the loss terms. Since an analytical solution of this transport equation requires significant simplifications of the propagation parameters and possibly a change of the structure of the transport equation, in the general case a numerical solution to this transport equation is sought.

To this end, different groups use a variety of propagation codes that range from fast semi-analytical models like USINE (see, e.g, [14; 13]) to fully numerical approaches like GALPROP (see, e.g., [18; 12]) or DRAGON (see [7]). Apart from these there are other specialised solution approaches as discussed, e.g., in [5], [4] or [9]. Here we will focus on those codes that take the full nuclear network into account and in principle allow the most detailed description of the Galaxy. Mainly, we will be referring to GALPROP since it is the most widely used of these codes, with many comments related to GALPROP also applicable to other codes.

Codes like GALPROP aim to include the full physics of the cosmic ray transport problem. Thus, they include a full model of our Galaxy and additionally, e.g., the different cross-sections necessary to describe the interaction between the cosmic rays and the interstellar medium. Such models allow the computation of cosmic ray spectra at any position in the Galaxy. Thus, their results can not only be compared to cosmic ray data measured at Earth but also to observations of secondaries related to the interaction of cosmic rays with the interstellar medium, like gamma rays.

When GALPROP was introduced to the cosmic ray community, corresponding simulations were still very demanding with the computational resources available at that time. Currently, however, similar problems can be solved in such a short time as to allow for extended parameter studies (see, e.g., [21]). There are also efforts to improve the quality of such numerical models, e.g., by extending the models from two to three spatial dimensions. Many recent works, however, are still using very similar propagation models as in the early GALPROP versions. Taking into account the much higher computational power available today, we feel that there are several aspects that should be improved in a modern cosmic ray propagation code. Here, we will address these aspects specifically, where we distinguish between improvements of the propagation physics and the underlying numerics. Finally, we introduce PICARD, our new Galactic cosmic ray propagation code, and show corresponding first model results.

2. Improving the physics in Galactic cosmic ray propagation

In many numerical models for Galactic cosmic ray propagation most of the propagation parameters are assumed to be constant in time and space. Such parameters are, e.g., spatial and momentum diffusion and also spatial convection, where the latter is neglected in many models. While such an approach is necessary in analytical or semi-analytical models (see [14]), a fully numerical code should also be able to handle variable transport parameters. Of course, a significant spatial variation in the transport parameters will make finding a parameter set that can fit the observations more difficult. But at the same time, in a simplified description of the transport parameters the resulting transport parameters might have not the desired physical meaningfulness.

There are, however, some recent efforts to establish more complex physics in the propagation models. [5] investigated the effect of anisotropic diffusion on Galactic cosmic ray protons, while [8] use a spatially variable diffusion tensor to explain the CR gradient problem, without invoking strong radial variation in the X_{CO} parameter. This shows, that more complex propagation parameters might even help to avoid additional assumptions necessary in more simplified propagation models. Additionally, for propagation within the heliosphere, anisotropic diffusion, even with different energy dependencies for diffusion along and perpendicular to the magnetic field, turns out to be necessary to explain the observations at different locations within the heliosphere.

Apart from that, the presence of a Galactic wind is discussed by different authors (see, e.g., [3; 26; 6]), where this wind is even assumed to be driven by the cosmic ray pressure. More importantly, a Galactic wind is often constrained to the central regions of a Galaxy. This would imply both a strong spatial variation of the advection velocity connected to a Galactic wind and also a magnetic field line structure advected vertical to the Galactic plane (see, e.g., [10; 16] for a discussion of the magnetic field in halos of other galaxies.). The latter could have, again, some impact on spatial diffusion, thus, showing that we can indeed expect spatially variable diffusion and advection.

To take such effects into account a spatially three-dimensional solver is necessary. While GALPOP also offers such simulations, they are still very demanding even with modern computers. This is due to the four dimensional nature of the transport problem: when taking the full nuclear network up to iron into account in such a four dimensional setup, the problem size can become very large. At the same time many numerical Galactic propagation solvers use a time-integration procedure to find a steady state solution of the transport problem. In the next section we will show that this can be error-prone and is also rather inefficient as compared to a solver that computes the steady state solution to the transport equation directly.

3. Improving the numerical solver

The standard approach to solve the cosmic ray transport equation is to integrate in time until a steady state solution has been found. Here, we will restrict the discussion to grid-based codes, thereby not including such methods as a solution with SDEs (see, e.g., [25]) or Monte Carlo methods (see, e.g., [2]).

In grid-based codes usually an implicit method is applied, since large time steps are desired. This is very important in Galactic cosmic ray transport, because the time to convergence turns out to be very long (on the order of several 100 million years at low energies), while the characteristic time is very short (depending on the specific model problem, characteristic time scales as low as 50 years are possible). Even with an implicit time integrator this still implies a very large number of time steps. In GALPROP this problem is avoided by starting the integration with the longest time steps, while successively decreasing the time step until the shortest time scales are reached (see [18]).

While this method makes a numerical solution possible in the first place, it still has some drawbacks: the user needs to be aware of the integration method and recheck the time integration parameters, whenever there is a significant change to the model setup, i.e., the propagation parameters. These time integration parameters can be changed and tuned by the user. A time integration procedure also results in the problem that it needs to be verified, whether a steady state solution has indeed be found. Realising that in the majority of Galactic cosmic ray propagation studies a steady state solution is desired, we introduced the PICARD code that computes a steady state solution of the transport equation directly, without using a time-integration approach.

4. The PICARD code

The specifics of the PICARD code are extensively discussed in [11]. The underlying idea is to solve the transport equation together with the constraint $\partial \psi_i / \partial t = 0$. Then, a discretisation of the remaining transport equation leads to a coupled matrix equation of band-diagonal form. While also an implicit time-integration method as used in GALPROP leads to a coupled matrix equation, the latter is of tri-diagonal form, that can be solved rather easily. There are, however, also various methods available for the efficient solution of a band diagonal matrix equation. In PICARD we apply an iterative multigrid method (see [22]) that quickly yields a solution of the steady state equation only subject to the discretisation error.

Apart from this steady state solver, PICARD also offers an explicit time-integration solver, where usually the steady state solution is used as the initial condition. PICARD is implemented in an MPI-parallel way, allowing the use of modern distributed memory supercomputers. With this, PICARD can be used for high-resolution 3D Galactic cosmic ray propagation simulations. So far, our highest resolution was $\sim$75 pc in all spatial dimensions, but when taking into account the entire nuclear network up to iron, we usually use a resolution of $\sim$150 pc due to the huge memory demand.

Beyond that, the implementation of propagation physics in PICARD is ongoing. It is already possible, e.g., to treat anisotropic diffusion in a similar way as discussed in [5]. Principle physical quantities like nuclear cross-sections [see, e.g., 19] were taken from a public version (v54) of GALPROP (See `http://sourceforge.net/projects/galprop/`).

The PICARD code will be publically released, when the technical implementation is sufficiently documented and robust.

5. Application: Milkyway as spiral galaxy

In a first study we investigated the effect of different spiral arm source distributions on cosmic ray electrons and protons. Such a distribution of the cosmic ray sources is motivated by the most

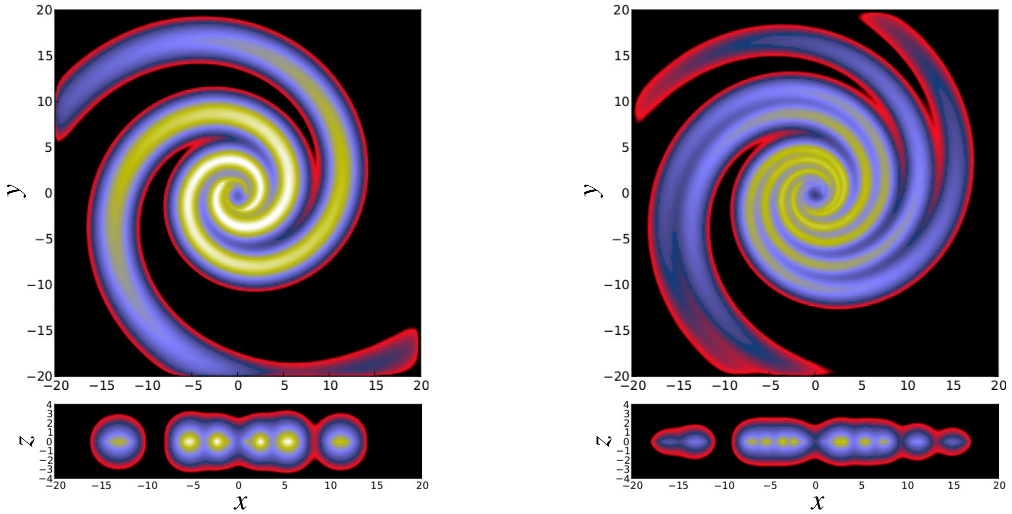

Figure 1: Flux of 1 TeV Galactic cosmic ray electrons in the Galactic plane (top) and in the $x-z$-plane (bottom). Results are shown for a two-arm (left) and for a four-arm (right) source distribution. Spatial scales are given in kpc. The Earth is located at $x = 8.5$ kpc, $y = z = 0$ kpc.

relevant cosmic ray sources, like supernova remnants and short-period pulsars, being rather young objects. Thus, they should mostly be related to star-forming regions, which occur most frequently in the vicinity of the Galactic spiral arms or the Galactic bar. Here, we investigated different spiral arm models, since the specific spiral arm structure of our Galaxy is still under discussion (see, e.g., in [17]). Further details of the specific spiral arm models investigated in our study can be found in [24]. For the cosmic ray transport parameters we used the plain diffusion parameter set ’z04LMPDS’ from [20].

As was to be expected, electrons are affected most strongly by such localised cosmic ray source distributions. As an example, we show the spatial distribution of 1 TeV electrons for a two-arm and a four-arm source distribution in Fig. 1. There, the strong localisation of the high energy electrons becomes apparent together with the ability of PICARD to capture small scale structures, where the corresponding results were computed with a resolution of ∼150 pc.

While for cosmic ray nuclei the most significant effect is a change in the total flux from an on-arm to an inter-arm position, for electrons we also observe significant changes in the spectral slope. Especially for the model with only two spiral arms, we find a much steeper slope for the electron spectrum at the nominal position of the Earth than in other models. This can be attributed to the Earth being located in an inter-arm region in the two-arm model. Thus, the Earth is very distant from the next cosmic ray sources. In connection with the strong energy losses at high energies this means that the flux of high energy electrons is significantly decreased at the nominal position of the Earth, while the low energy electron flux is increased. This explains why we find a significantly steeper electron spectrum than in the vicinity of the cosmic ray sources.

In the four-arm model depicted in Fig. 1 the electron spectrum at the position of the Earth is much more similar to the results from an axially symmetric model. In the case of the four-arm model the Earth is located at the rim of a spiral arm. When assuming that the chosen propagation parameter set is representative for the cosmic ray transport even in a model with such localised

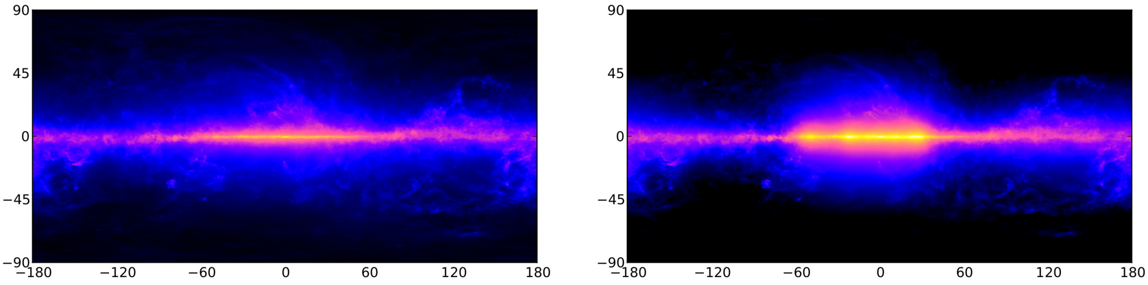

Figure 2: Gamma-ray flux at an energy of ∼100 GeV for the model with an axially-symmetric source distribution (left) and for the model with a two-arm source distribution (right).

sources, it turns out that the Earth should indeed be located at the rim of a spiral arm to allow a successful fit to the observational cosmic ray data. That is, when taking into account also heavier nuclei, we find a satisfying fit of the different secondary to primary ratios only in the immediate vicinity of the spiral arms.

This, however, does not mean that the two-arm model needs to be discarded, even though currently also other observations point in that direction (see, e.g., [23]): either the propagation parameters need to be very different within such a model, or the orientation of the spiral arms would need to be revised. As is also obvious from Fig. 1, the flux at the position of the spiral arms becomes very high in case of the two-arm model, investigated here. This is related to the flux-normalisation applied in PICARD, that is also usual in other cosmic ray propagation models. In such models the intensity of the cosmic ray sources is determined *after* the simulation by fixing the electron and the proton flux at a chosen energy to corresponding observations. This implies that the source intensity and also the flux at the location of the spiral arms decreases, when Earth is located nearer to the position of the spiral arms in such a model.

Thus, it is important to investigate the global distribution of cosmic rays. This is of course only possible via an observation of secondaries arriving at Earth, of which currently gamma-rays are the only channel with a sufficiently high statistics. In the diffuse gamma-ray emission (see, e.g., [1]) from the models discussed here, we find, as expected, significant differences between the two-arm model and a model with an axially symmetric source distribution.

Corresponding all-sky gamma-ray fluxes are shown in Fig. 2 at an energy of 100 GeV. Apparently, the diffuse gamma-ray emission is more concentrated to the Galactic plane in case of the two-arm model. Also, the structure of the spiral arms is actually visible in the Galactic centre. Finally, IC emission is more dominant than for an axially symmetric source distribution, visible by the smoother structure of the gamma-ray emission. This was to be expected since the electron flux is locally very high in the two-arm model. While these results are are not yet compared to observations, they show that the investigation of the diffuse gamma-ray emission computed from Galactic cosmic ray propagation models offers unique prospects to assess the quality of the models and to obtain some idea about the global distribution of the cosmic rays.

6. Conclusion

Here, we introduced our new Galactic cosmic ray propagation code PICARD. This code is optimised for the solution of spatially 3D Galactic propagation models, where it efficiently computes the resulting steady state distribution. Using the specific example of different spiral arm cosmic ray source distributions we have verified the capability to solve very high resolution models. With this code we have the power to distinguish the effects of such different source models and compute both the resulting cosmic ray flux and also the ensuing Galactic diffuse gamma-ray emission.

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