PROCEEDINGS OF SCIENCE

Damping of gravitational waves in the nonperturbative spinor vacuum

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The propagation of gravitational waves in the presence of a nonperturbative vacuum of a spinor field is considered. The differences between this case and the case of waves propagating through empty space are demonstrated.

Frontiers of Fundamental Physics 14 - FFP14,
15-18 July 2014
Aix Marseille University (AMU) Saint-Charles Campus, Marseille

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†The talk is based on Ref.[1]

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1. Introduction

Gravitational waves (GWs) are probably the most suitable object for studying the deep space. It is usually assumed that GWs propagate in a classical vacuum, i.e., in empty space. But a quantum vacuum possesses the energy associated with the unavoidable quantum fluctuations of various fields when the vacuum expectation value of any quantum field is zero but the expectation value of the square of fluctuations is nonzero. This energy turns out to be infinite that results in a number of fundamental problems, including the problem of ultraviolet divergences arising in attempting to quantize gravity [2] and the well-known "cosmological constant problem" for the Universe [3]. This motivates one to consider other approaches allowing to resolve these problems [2].

One of the possibilities in this direction is to take into account nonperturbative effects arising in quantization of various fundamental fields. In this framework, of special interest is to study the question of the propagation of GWs in the case where fluctuations of a quantum spinor field are taken into account. The reason is that the energy-momentum tensor of a spinor field contains the spin connection, which in turn contains first derivatives of tetrad components with respect to the coordinates. As a result, the Einstein equations yield the wave equation for a GW which contains second derivatives of the tetrad components on the lefthand side and their first derivatives on the righthand side. The latter may play a role of friction, giving damped GWs and other interesting effects which could in principle be traced in experiments. Below we will describe some of such effects.

2. Perturbed Einstein equations

In order to describe an exact formulation of the problem of gravitational waves propagation in a spinor vacuum, we need to consider both a metric and a spinor field as quantum objects. For the nonperturbative quantization, we have to write the following equations (for details, see Ref. [4]):

$$\hat{R}_{\bar{a}\mu} - \frac{1}{2}\hat{e}_{\bar{a}\mu}\hat{R} = \varkappa \hat{T}_{\bar{a}\mu}, \tag{2.1}$$

$$\gamma^\mu \nabla_\mu \hat{\psi} - m\hat{\psi} = 0, \tag{2.2}$$

where $\hat{R}_{\bar{a}\nu}$ and $\hat{R}$ are correspondingly the operators of the Ricci tensor and the Ricci scalar; $\hat{e}_{\bar{a}\mu}$ is the vierbein operator; $\hat{T}_{\mu\nu}$ is the operator of the energy-momentum tensor; $\hat{\psi}$ is the operator of the spinor field; $\bar{a} = \bar{0}, \bar{1}, \bar{2}, \bar{3}$ is the vierbein index; $\mu = 0, 1, 2, 3$ is the coordinate index; $\nabla_\mu \hat{\psi} = \partial_\mu \hat{\psi} - \Gamma_\mu \hat{\psi} = \partial_\mu \hat{\psi} + \frac{1}{4}\hat{\omega}_{\bar{a}\bar{b}\mu}\gamma^{\bar{a}}\gamma^{\bar{b}}\hat{\psi}$ is the covariant derivative for the spinor with the operator of the spin connection $\hat{\omega}_{\bar{a}\bar{b}\mu}$; $\gamma^{\bar{a}}$ are the Dirac matrices in flat Minkowski spacetime; $\varkappa = 8\pi\kappa/c^4$, κ is the gravitational constant.

Equations 2.1 and 2.2 cannot be solved explicitly, and we have to use some approximation. Within our approximation, we will consider the following set of equations:

$$\delta R_{\bar{a}\bar{b}} - \frac{1}{2}\eta_{\bar{a}\bar{b}}\delta R = \varkappa \left\langle Q \left| \widehat{\delta T}_{\bar{a}\bar{b}} \right| Q \right\rangle, \tag{2.3}$$

$$\left\langle Q \left| \widehat{\delta T}_{\bar{a}}^{\ \mu} \right| Q \right\rangle_{;\mu} = 0, \tag{2.4}$$

where $\delta R_{\bar{a}\bar{b}}$, δR, and $\widehat{\delta T}_{\bar{a}}^{\ \mu}$ are the gravitational wave approximation for the Ricci tensor, the Ricci scalar, and the energy-momentum tensor. In turn, the righthand side of Eq. 2.3 is calculated in subsequent sections. Here $|Q\rangle$ is a quantum state describing the propagation of a GW through a spinor vacuum. To simplify the notation we will hereafter use $\langle\cdots\rangle$ instead of $\langle Q|\cdots|Q\rangle$. Let us introduce the vierbein perturbations $\phi_{\bar{a}}{}^{\bar{b}}$ defined in the following manner:

$$e^{\bar{a}}{}_{\mu} = \left(\delta^{\bar{a}}_{\bar{b}} - \phi^{\bar{a}}{}_{\bar{b}}\right) \overset{0}{e}{}^{\bar{b}}{}_{\mu}, \tag{2.5}$$

$$e_{\bar{a}}{}^{\mu} = \left(\delta^{\bar{b}}_{\bar{a}} + \phi_{\bar{a}}{}^{\bar{b}}\right) \overset{0}{e}{}_{\bar{b}}{}^{\mu}, \tag{2.6}$$

where $\overset{0}{e}{}^{\bar{a}}{}_{\mu}$ is the unperturbed tetrad; $\overset{0}{e}{}_{\bar{a}}{}^{\mu}$ is the unperturbed inverse tetrad; $e^{\bar{a}}{}_{\mu}$ is the perturbed tetrad; $e_{\bar{a}}{}^{\mu}$ is the perturbed inverse tetrad; $-\phi^{\bar{a}}{}_{\bar{b}} \overset{0}{e}{}^{\bar{b}}{}_{\mu}$ is the perturbation of the tetrad. Introducing the tetrad perturbations

$$\phi_{\bar{a}\bar{b}} = \begin{pmatrix} \psi & \partial_x w & w_{\bar{2}} & w_{\bar{3}} \\ \partial_x \tilde{w} & \Phi + \partial_x^2 h & \partial_x h_{\bar{2}} & \partial_x h_{\bar{3}} \\ \tilde{w}_{\bar{2}} & \partial_x \tilde{h}_{\bar{2}} & \Phi + h_{\bar{2}\bar{2}} & h_{\bar{2}\bar{3}} + \partial_x \tilde{h} \\ \tilde{w}_{\bar{3}} & \partial_x \tilde{h}_{\bar{3}} & h_{\bar{3}\bar{2}} - \partial_x \tilde{h} & \Phi - h_{\bar{2}\bar{2}} \end{pmatrix}, \tag{2.7}$$

the perturbations of the Einstein tensor then are:

$$\delta G_{\bar{a}\bar{b}} = \begin{pmatrix} -2\partial_x^2 \Phi & 2\partial_x \dot{\Phi} & -\frac{1}{2}\partial_x^2 W_y & -\frac{1}{2}\partial_x^2 W_z \\ 2\partial_x \dot{\Phi} & -2\ddot{\Phi} & \frac{1}{2}\partial_x \dot{W}_y & \frac{1}{2}\partial_x \dot{W}_z \\ -\frac{1}{2}\partial_x^2 W_y & \frac{1}{2}\partial_x \dot{W}_y & -2\ddot{\Phi} - \partial_x^2\left(\Psi - \Phi\right) + \Box h_+ & \Box h_\times \\ -\frac{1}{2}\partial_x^2 W_z & \frac{1}{2}\partial_x \dot{W}_z & \Box h_\times & -2\ddot{\Phi} - \partial_x^2\left(\Psi - \Phi\right) - \Box h_+ \end{pmatrix}, \tag{2.8}$$

where the dot denotes differentiation with respect to $\tau = ct$; $\Box = \frac{\partial^2}{\partial \tau^2} - \nabla^2$ is the d'Alembertian and h_+ and $h_\times$ are the two polarizations of gravitational waves

$$h_+ = h_{yy} = -h_{zz}, \; h_\times = h_{yz} = h_{zy}. \tag{2.9}$$

Here $\Phi, W_{\bar{i}}, \Psi$ are vierbein components given by the formulae 2.7 and $\Psi = \psi - \partial_t\left(w + \tilde{w} - \partial_t h\right), W_i = w_i + \tilde{w}_i - \partial_t\left(h_i + \tilde{h}_i\right)$, where $i = 1,2,3$ are the spacelike world indices. To calculate the expectation value of the energy-momentum tensor of the spinor field, we state the following assumptions concerning the spinor field:

- The vacuum expectation value of the spinor field is zero:

$$\langle \hat{\psi}_{\mathfrak{a}} \rangle = 0. \tag{2.10}$$

- The vacuum expectation value of the product of the spinor field in two points x, y is nonzero:

$$\langle \hat{\psi}^*_{\mathfrak{a}}(x) \hat{\psi}_{\mathfrak{b}}(y) \rangle = \Upsilon_{\mathfrak{a}\mathfrak{b}}(x,y) \neq 0. \tag{2.11}$$

  Here $\hat{\psi}$ is the operator of the spinor field; $\mathfrak{a}, \mathfrak{b}$ are the spinor indices; $\Upsilon_{\mathfrak{a}\mathfrak{b}}$ is the 2-point Green's function.

- Every component

$$|\Upsilon_{\mathfrak{a}\mathfrak{b}}(x,y)| = \text{const}. \tag{2.12}$$

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- As a consequence of Eq. 2.12 we have

$$\langle \hat{\psi}^*_{\mathfrak{a}}(x) \partial_{y^\mu} \hat{\psi}_{\mathfrak{b}}(y) \rangle = 0. \tag{2.13}$$

The energy-momentum tensor contains the following unperturbed and perturbed contributions:

$$\hat{T}_{\bar{a}\bar{b}} = \overset{0}{\hat{T}}_{\bar{a}\bar{b}} + \widehat{\delta T}_{\bar{a}\bar{b}}, \tag{2.14}$$

where $\overset{0}{\hat{T}}_{\bar{a}\bar{b}}$ is calculated for unperturbed Minkowski spacetime with zero spin connection, $\omega_{\bar{a}\bar{b}\mu} = 0$.

3. Gravitational wave propagating on the background of the spinor vacuum

Consider here a particular case of GWs for which $\Phi = \Psi = W_i = 0$. Below we employ two different ansätzs for the spinor field.

3.1 Case I

Let us consider the ansätz

$$\hat{\psi}^T = e^{-i(\omega t - kx)} \left(\hat{A}, \hat{B}, \hat{B}, \hat{A} \right), \tag{3.1}$$

where ω is the frequency and k is the x-component of the wave vector. We assume the following values of the 2-point Green's functions of the spinor field ψ:

$$\Upsilon = \langle \psi^*_1 \psi_2 \rangle = \langle \psi^*_1 \psi_3 \rangle = \langle \psi^*_4 \psi_2 \rangle = \langle \psi^*_4 \psi_3 \rangle = \langle A^* B \rangle = \Upsilon_1 + i\Upsilon_2, \tag{3.2}$$

$$\Upsilon^* = \langle \psi^*_2 \psi_1 \rangle = \langle \psi^*_2 \psi_4 \rangle = \langle \psi^*_3 \psi_1 \rangle = \langle \psi^*_3 \psi_4 \rangle = \langle B^* A \rangle = \Upsilon_1 - i\Upsilon_2, \tag{3.3}$$

with $|\Upsilon_{1,2}| = \text{const}$. Equations 2.3 give the following set of equations for the components 2.9:

$$h''_{\bar{y}\bar{y}} - \ddot{h}_{\bar{y}\bar{y}} = -2\varkappa \left(\langle \hat{A}^* \hat{B} \rangle + \langle \hat{A} \hat{B}^* \rangle \right) \dot{h}_{\bar{y}\bar{z}}, \tag{3.4}$$

$$h''_{\bar{y}\bar{z}} - \ddot{h}_{\bar{y}\bar{z}} = 2\varkappa \left(\langle \hat{A}^* \hat{B} \rangle + \langle \hat{A} \hat{B}^* \rangle \right) \dot{h}_{\bar{y}\bar{y}}, \tag{3.5}$$

where the prime denotes differentiation with respect to x, and the appearance of the derivatives of the components $h_{\bar{y}\bar{y}}, h_{\bar{y}\bar{z}}$ on the righthand side of these equations is connected with the presence of the spin connection on the righthand side of Einstein's equations. We are looking for the x-plane wave solution in the form

$$h_{\bar{y}\bar{y}} = -h_{\bar{z}\bar{z}} = A_1 e^{-i(\omega t - kx)}, \tag{3.6}$$

$$h_{\bar{y}\bar{z}} = h_{\bar{z}\bar{y}} = A_2 e^{-i(\omega t - kx)}. \tag{3.7}$$

Substituting the solutions (3.6) and (3.7) into the wave equations (3.4) and (3.5) and using the expressions (3.2) and (3.3), we obtain the following relations (hereafter we work in natural units where $\hbar = c = 1$):

$$A_1 \left(k^2 - \omega^2 \right) = -4i\varkappa A_2 \Upsilon_1 \omega, \tag{3.8}$$

$$A_2 \left(k^2 - \omega^2 \right) = 4i\varkappa A_1 \Upsilon_1 \omega. \tag{3.9}$$

This gives us the phase difference of $\pm\pi/2$ between $\bar{y}\bar{y}, \bar{z}\bar{z}$ and $\bar{y}\bar{z}$ components of the GW:

$$A_2 = \pm i A_1 = A_1 e^{\pm i \frac{\pi}{2}}. \tag{3.10}$$

The dispersion relation is

$$k^2 = \omega^2 \pm 4\varkappa \Upsilon_1 \omega. \tag{3.11}$$

The phase velocity of the GW is given by

$$v_p = \frac{\omega}{k} = \sqrt{\frac{1}{1 \pm \frac{4\varkappa\Upsilon_1}{\omega}}} \neq 1. \tag{3.12}$$

The group velocity of the GW is

$$v_g = \frac{d\omega}{dk} = \frac{\sqrt{1 \pm \frac{4\varkappa\Upsilon_1}{\omega}}}{1 \pm \frac{2\varkappa\Upsilon_1}{\omega}} \neq 1. \tag{3.13}$$

3.2 Case II

Consider here the following ansätz for the spinor field:

$$\hat{\psi} = e^{-i(\omega t - kx)} \left(\hat{A}, \hat{A}, \hat{V}, \hat{V} \right), \tag{3.14}$$

where $\left\langle \hat{A}\hat{A}^* \right\rangle, \left\langle \hat{V}\hat{V}^* \right\rangle$ are taken to be constant. For the ansätz (3.14), we assume the following values of the 2-point Green's functions of the spinor field ψ:

$$\langle \psi_1^* \psi_2 \rangle = \langle \psi_1 \psi_2^* \rangle = \langle \psi_1^* \psi_1 \rangle = \langle \psi_2^* \psi_2 \rangle = \left\langle \hat{A}^* \hat{A} \right\rangle = \Upsilon_1, \tag{3.15}$$

$$\langle \psi_3^* \psi_4 \rangle = \langle \psi_4^* \psi_3 \rangle = \langle \psi_3^* \psi_3 \rangle = \langle \psi_4^* \psi_4 \rangle = \left\langle \hat{V}^* \hat{V} \right\rangle = \Upsilon_2, \tag{3.16}$$

with $|\Upsilon_{1,2}| = \text{const}$. Substituting these expressions into Eq. (2.3), looking for the x-plane wave solution in the form (3.6) and (3.7) and taking into account (3.15) and (3.16), we obtain the following relations:

$$A_1 \left(k^2 - \omega^2 \right) = 2i\varkappa A_2 \left[k \left(\Upsilon_1 - \Upsilon_2 \right) - \omega \left(\Upsilon_1 + \Upsilon_2 \right) \right], \tag{3.17}$$

$$A_2 \left(k^2 - \omega^2 \right) = -2i\varkappa A_1 \left[k \left(\Upsilon_1 - \Upsilon_2 \right) - \omega \left(\Upsilon_1 + \Upsilon_2 \right) \right], \tag{3.18}$$

which immediately give

$$A_2 = \pm i A_1 = A_1 e^{\pm i \frac{\pi}{2}}. \tag{3.19}$$

That is, the phase difference between $\bar{y}\bar{y}, \bar{z}\bar{z}$ and $\bar{y}\bar{z}$ components of the GW is again $\pm\pi/2$, as in the case I. In turn, the dispersion relation takes the form

$$k^2 \pm 2\varkappa k \left(\Upsilon_1 - \Upsilon_2 \right) + \left[-\omega^2 \mp 2\varkappa\omega \left(\Upsilon_1 + \Upsilon_2 \right) \right] = 0. \tag{3.20}$$

Here we have two cases:

(1) For $A_2 = iA_1$, the wave vector is

$$k_{+,1,2} = -\varkappa \left(\Upsilon_1 - \Upsilon_2 \right) \pm \sqrt{\varkappa^2 \left(\Upsilon_1 - \Upsilon_2 \right)^2 + \omega^2 + 2\varkappa\omega \left(\Upsilon_1 + \Upsilon_2 \right)}. \tag{3.21}$$

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(2) For $A_2 = -iA_1$, the wave vector is

$$k_{-,1,2} = \varkappa(\Upsilon_1 - \Upsilon_2) \pm \sqrt{\varkappa^2(\Upsilon_1 - \Upsilon_2)^2 + \omega^2 - 2\varkappa\omega(\Upsilon_1 + \Upsilon_2)}. \quad (3.22)$$

An analysis of these equations indicates that there are two GWs with different wave vectors for the same frequency ω. But in the case (2) a situation may occur where the GW is damped, that happens if the expression under the square root in (3.22) is negative.

4. Conclusions

We have considered the process of propagation of GWs on the background of the nonperturbative vacuum of spinor fields. Using the simplifying assumptions from Sec. 2, it was shown that there are several distinctive features in comparison with the propagation of GWs through empty space:

- There exists the fixed phase difference of $\pm\pi/2$ between components $h_{yy,zz}$ and h_{yz}.
- The phase and group velocities of GWs are not equal to the velocity of light. Moreover, the group velocity is always less than the velocity of light.
- The components $h_{yy,zz}$ and h_{yz} exist together only.
- Depending on the properties of the spinor vacuum, the damping of GWs may occur for some frequencies ω of the spinor field, or no GW may exist.
- For given frequency ω, there exist two waves with different wave vectors k.

All features mentioned above can in principle be verified after the experimental detection of GWs. Then the simplest test will be to verify the existence of the phase difference. In addition, one might expect that GWs could be a fruitful tool for studying *nonperturbative* quantum field theories.

Acknowledgments

This work was partially supported by grants #1626/GF3 and #139 in fundamental research in natural sciences by the Science Committee of the Ministry of Education and Science of Kazakhstan and by a grant of the Volkswagen Foundation.

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