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The exact Foldy-Wouthuysen transformation for a Dirac Theory with the complete set of CPT-LORENTZ Violating terms - The Torsion Field Case

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In this work, we consider the combined action of torsion and magnetic field on the massive spinor field. In this case, the Dirac equation is not straightforward solved. We suppose that the spinor has two components. The equations have mixed terms between the two components. The torsion field is described by the field S_μ. The main purpose of the work is to get an explicit form to the equation of motion that shows the possible interactions between the external fields and the spinor in a Hamiltonian that is independent to each component. We consider that S_0 is constant and is the unique non-vanishing term of S_μ. This simplification is taken just to simplify the algebra, as our main point is not to describe the torsion field itself. We perform the Exact Foldy-Wouthuysen transformation and a transformed Hamiltonian that describes a half spin field in the presence of electromagnetic and torsion external fields is presented. We get an explicit form to the equation of motion that shows the possible interactions between the external fields and the spinor in a Hamiltonian that is independent to each component. In order to get a possible experimental perspective, we perform the calculation of the bound state in the last part of the work.

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1. Introduction

Our starting point is the action of a Dirac fermion theory, including the complete set of CPT-Lorentz symmetry breaking terms presented in [1]. In order to condense the text we do not present this action here. In such work the authors present a complete table which contains the 80 cases of CPT-Lorentz violating terms in the modified Dirac equation that admits Exact Foldy Wouthuysen Transformation (EFWT) [2, 3]. However, in the present work we are interested only in the torsion field case and we shall perform, in this situation, the corresponding EFWT as well as the calculation of the equations of motion and the bound state. The corresponding Hamiltonian is given by the following relation

$$H = c\vec{\alpha}\cdot\vec{p} - e\vec{\alpha}\cdot\vec{A} + \eta_1\gamma_5 S_0 + mc^2\beta\,. \tag{1.1}$$

Here we used notations $A_\mu = (\Phi, \mathbf{A})$ and $S_\mu = (S_0, \mathbf{S})$. We consider the magnetic and torsion fields which can only vary with time, but do not depend on the space coordinates. As one can check, the Hamiltonian described by (1.1) obeys the relation $JH + HJ = 0$. Such relation is a condition to perform EFWT [2, 3, 4]. The quantity J is known as the involution operator. For the sake of completeness, we detach that relation (1.1) does not contain a term of the kind $\eta_1\vec{\alpha}\vec{S}\gamma_5$ which represents the Dirac field interaction with the vector torsion part. It is possible to note that with this additional term the corresponding Hamiltonian would not obey the above mentioned anti commutation relation. However the main point in discarding the S_i-term is the fact that we are interested in finding the possible experiment by the bound state (see [5]) that would enable the measure of this field. Therefore, the S_0 part of torsion can describe the interaction with the Dirac particle.

Furthermore, due to the weakness of the torsion field, we are really interested only in the linear order in torsion while the magnetic field should be treated exactly. For this reason we consider that S_0 is constant and is the unique non-vanishing term of S_μ.

2. Exact Foldy-Wouthuysen transformation for scalar Torsion

Now, according to the standard prescription [2], the next step is to obtain H^2. Direct calculations give the result

$$H^2 = (c\vec{p} - e\vec{A} - \eta_1\vec{\Sigma}S_0)^2 + m^2c^4 + \hbar ce\vec{\Sigma}\cdot\vec{B}\,. \tag{2.1}$$

In order to get the transformed Hamiltonian H^{tr} we rewrite H^2 as $H^2 = A^2 + B$ with A being m-dependent terms in H^2 and B the ones that do not depend on mass. In this case we present $A = mc^2$. Then, we search for an operator K in the form

$$K = A + \frac{1}{A}K_1 + K_1\frac{1}{A} + \vartheta(\frac{1}{A^2})\,, \tag{2.2}$$

such that $K^2 = A^2$. Finally, using (2.1) and the fact that

$$H^{tr} = UHU^* = \beta[\sqrt{H^2}]^{EVEN} + J[\sqrt{H^2}]^{ODD}\,, \tag{2.3}$$

where the even (odd) terms in (2.3) are the ones that commute (anti-commute) with the matrix β, we get

$$H^{tr} = \beta mc^2 + \frac{\beta}{2mc^2}(c\vec{p} - e\vec{A} - \eta_1 \vec{\Sigma} S_0)^2 + \beta \frac{\hbar e}{2mc} \vec{\Sigma} \cdot \vec{B} - \beta \frac{(\eta_1)^2}{mc^2}(S_0)^2 . \tag{2.4}$$

The next step is to present the Dirac fermion ψ in the bi-spinor (described by φ and χ) form with the kinetic term $-imc^2\hbar^{-1}t$ and use the equation $i\hbar\partial_t \psi = H\psi$ to derive the Hamiltonian for the two-spinor φ. Inserting the first into the second, we obtain the two-component equation. Using the fact that the transformed Hamiltonian is an even function, we obtain, in the φ sector, the nonrelativistic Hamiltonian

$$H^{tr}_{\varphi} = \frac{1}{2m}(\vec{\Pi})^2 + B_0 + \vec{\sigma} \cdot \vec{Q} ,$$

$$\vec{\Pi} = \vec{p} - \frac{e}{c}\vec{A} - \frac{\eta_1}{c} S_0 \vec{\sigma} \quad , \quad B_0 = -\frac{(\eta_1)^2}{mc^2}(S_0)^2 \quad , \quad \vec{Q} = \frac{\hbar e}{2mc}\vec{B} . \tag{2.5}$$

The expressions above are exactly the same as derived in [6] and in [7] through the usual perturbative Foldy-Wouthuysen transformation.

One can also perform the canonical quantization of the theory in a way similar to [6]. In order to do this we introduce the operators of coordinate $\hat{x}_i$, momenta $\hat{p}_i$ and spin $\hat{\sigma}_i$ and implement the equal-time commutation relations of the following form

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \quad , \quad [\hat{x}_i, \hat{\sigma}_j] = [\hat{p}_i, \hat{\sigma}_j] = 0 \quad , \quad [\hat{\sigma}_i, \hat{\sigma}_j] = 2i\varepsilon_{ijk}\hat{\sigma}_k . \tag{2.6}$$

The Hamiltonian operator $\hat{H}$ which corresponds to the energy (2.5) is easily constructed in terms of the operators $\hat{x}_i, \hat{p}_i, \hat{\sigma}_i$ and then these operators yield the equations of motion

$$i\hbar\frac{d\hat{x}_i}{dt} = [\hat{x}_i, H] \quad , \quad i\hbar\frac{d\hat{p}_i}{dt} = [\hat{p}_i, H] \quad , \quad i\hbar\frac{d\hat{\sigma}_i}{dt} = [\hat{\sigma}_i, H] . \tag{2.7}$$

The straightforward calculations lead to the equations [1]

$$\frac{dx_i}{dt} = \frac{1}{m}\left(p_i - \frac{e}{c}A_i - \frac{\eta_1}{c}\sigma_i S_0\right) = v_i ,$$

$$\frac{dp_i}{dt} = \frac{1}{m}\left(p^j - \frac{e}{c}A^j - \frac{\eta_1}{c}\sigma^j S_0\right)\frac{e}{c}\frac{\partial A_j}{\partial x^i} ,$$

$$\frac{d\sigma_i}{dt} = \left[\vec{R} \times \vec{\sigma}\right]_i \quad , \quad \vec{R} = \frac{2\eta_1}{\hbar}\left[-\frac{1}{c}\vec{v}S_0\right] + \frac{e}{mc}\vec{B} . \tag{2.8}$$

Using the first and second of equations (2.8) it is possible to obtain

$$m\frac{dv_i}{dt} = -\frac{e}{c}\frac{\partial A_i}{\partial t} + \frac{e}{c}\left[\vec{v} \times \vec{B}\right]_i - \frac{\eta_1}{c}\sigma_i\frac{\partial S_0}{\partial t} - \frac{\eta_1}{c}S_0\frac{d\sigma_i}{dt} . \tag{2.9}$$

This equation is the correction to the very well known expression or the Lorentz force. Unfortunately, it doesn't show us an explicit interaction between the torsion and electromagnetic field (as for the gravitational waves, for example, [8]), the last two terms in the right hand side of the equation shows the possible interaction of scalar field torsion with the Dirac particle.

[1] At this point we can omit all the terms which vanish when $\hbar \to 0$.

3. Possible experimental tests

The main point in the experimental tests involving torsion field is the weakness of the this external field. A natural question arises here. Is it possible to get a torsion field experimental data? In order to get some indication about the possibility of such experimental data it is necessary to know, first of all, the bound state of this theory [9]. Such bound gives an indication about which atomic experiment should be performed in order to get possible measurements of the space time torsion field. In order to calculate the bound state, the starting point is the transformed Hamiltonian, given by the equation (2.5).

In order to calculate the bound state associated with the torsion field in the Dirac Theory, let us take into account the Lorentz violating potential V, which obeys the relation presented in [5], that is $V = -\tilde{b}_j \sigma^j$, where σ represents the spin matrices. After some algebra, one can write the corresponding bound in the following way

$$\tilde{b}_j = b_j + \frac{e\hbar}{2mc} B_j - \frac{\eta_1 S_0}{mc} \left(p_j - \frac{e}{c} A_j \right). \tag{3.1}$$

The bound in the last equation enables us to consider the possibility about getting an indication of possible atomic experiments on the table presented in [5].

4. Conclusions and discussions

The EFWT was here considered and performed in the context of all the scalar torsion field, in the Dirac equation. The first result of the work is given by the equation (2.4), which presents the diagonal transformed Hamiltonian, for this case. We also derived the semi-classical equations of motion for $\hat{x}_i$, $\hat{p}_i$ and $\hat{\sigma}_i$. We have shown that it is possible to combine equations of motion to get a generalized Lorentz force corrected by the scalar Torsion term, given by the equation (2.9).

In the last section we have highlighted that the bound state equation is important in order to get a better understanding about which kind of atomic experiments should be performed in order to try to measure the possible modification in the physical trajectory of the Dirac particle that the interaction with torsion could possible cause.

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