Matter Bounce Scenario in F(T) gravity

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It is shown that teleparallel $F(T)$ theories of gravity combined with holonomy corrected Loop Quantum Cosmology (LQC) support a Matter Bounce Scenario (MBS) which is a potential alternative to the inflationary paradigm. The Matter Bounce Scenario is reviewed and, according to the current observational data provided by PLANCK's team, we have summarized all the conditions that it has to satisfy in order to be a viable alternative to inflation, such as to provide a theoretical value of the spectral index and its running compatible with the latest PLANCK data, to have a reheating process via gravitational particle production, or to predict some signatures in the non-gaussianities of the power spectrum. The calculation of the power spectrum for scalar perturbations and the ratio of tensor to scalar perturbations has been done, in the simplest case of an exact matter dominated background, for both holonomy corrected LQC and teleparallel $F(T)$ gravity. Finally, we have discussed the challenges (essentially, dealing with non-gaussianities, the calculation of the 3-point function in flat spatial geometries for theories beyond General Relativity) and problems (Jeans instabilities in the case of holonomy corrected LQC or local Lorentz dependence in teleparallelism) that arise in either bouncing scenario.

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1. Introduction

It is well-known that inflation suffers from several problems (reviewed for instance in [1]), like the initial singularity which is usually ignored, or the fine-tuning of the degree of flatness required for the potential in order to achieve successful inflation [2].

An alternative scenario to the inflationary paradigm, called *Matter Bounce Scenario* (MBS), has been developed in order to explain the evolution of our Universe [3] avoiding those problems. Essentially, it presents at very early times a matter dominated Universe in a contracting phase, evolving towards the bounce after which it enters an expanding phase. This model, like inflation, solves the horizon problem that appears in General Relativity (GR) and improves the flatness problem in GR (where spatial flatness is an unstable fixed point and fine tuning of initial conditions is required), because the contribution of the spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding one (see for instance [4]).

The aim of our work is to show viable bouncing cosmologies where the matter part of the Lagrangian is composed of a single scalar field and, therefore, have to go beyond General Relativity, since GR for flat spatial geometries forbids bounces when one deals with a single field. Hence, theories such as holonomy corrected Loop Quantum Cosmology (LQC) [5], where a big bounce appears owing to the discrete structure of space-time [6] or teleparalellism, that is $F(T)$ gravity, [7] must be taken into account.

The units used in the paper are: $\hbar = c = 8\pi G = 1$.

2. $F(T)$ gravity in flat FLRW geometry

Teleparallel theories are based in the *Weitzenböck space-time*. This space is $\mathscr{R}^4$, with a Lorentz metric, in which a global, orthonormal basis of its tangent bundle given by four vector fields $\{e_i\}$ has been selected, that is, they satisfy $g(e_i, e_j) = \eta_{ij}$ with $\eta = \text{diag}(-1, 1, 1, 1)$. The Weitzenböck connection ∇ is defined by imposing that the basis vectors e_i be absolutely parallel, i.e. that $\nabla e_i = 0$.

The Weitzenböck connection is compatible with the metric g, and it has zero curvature because of the global parallel transport defined by the basis $\{e_i\}$. The information of the Weitzenböck connection is carried by its torsion, and its basic invariant is the *scalar torsion T*. The connection, and its torsion, depend on the choice of orthonormal basis $\{e_i\}$, but if one adopts the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric and selects as orthonormal basis $\{e_0 = \partial_0, e_1 = \frac{1}{a}\partial_1, e_2 = \frac{1}{a}\partial_2, e_3 = \frac{1}{a}\partial_3\}$, then the scalar torsion is $T = -6H^2$, where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, and this identity is invariant with respect to local Lorentz transformations that only depend on the time, i.e. of the form $\tilde{e}_i = \Lambda_i^k(t)e_k$ (see [8, 9]).

With the above choice of orthonormal fields, the Lagrangian of the $F(T)$ theory of gravity is

$$\mathscr{L}_T = \mathscr{V}(F(T) + \mathscr{L}_M), \tag{2.1}$$

where $\mathscr{V} = a^3$ is the element of volume, and $\mathscr{L}_M$ is the matter Lagrangian density.

The Hamiltonian of the system is

$$\mathscr{H}_T = \left(2T\frac{dF(T)}{dT} - F(T) + \rho\right)\mathscr{V}, \tag{2.2}$$

where ρ is the energy density. Imposing the Hamiltonian constrain $\mathcal{H}_T = 0$ leads to the modified Friedmann equation

$$\rho = -2\frac{dF(T)}{dT}T + F(T) \equiv G(T) \tag{2.3}$$

which, as $T = -6H^2$, defines a curve in the plane (H, ρ).

Equation (2.3) may be inverted, so a curve of the form $\rho = G(T)$ defines an $F(T)$ theory with

$$F(T) = -\frac{\sqrt{-T}}{2} \int \frac{G(T)}{T\sqrt{-T}} dT. \tag{2.4}$$

To produce a cyclically evolving Universe, let us take the $F(T)$ theory arising from the ellipse that defines the holonomy corrected Friedmann equation in Loop Quantum Cosmology

$$H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right), \tag{2.5}$$

where ρ_c is the so-called *critical density*.

To obtain a parametrization of the form $\rho = G(T)$, the curve has to be split in two branches

$$\rho = G_\pm(T) = \frac{\rho_c}{2}\left(1 \pm \sqrt{1 + \frac{2T}{\rho_c}}\right), \tag{2.6}$$

where the branch $\rho = G_-(T)$ corresponds to $\dot{H} < 0$ and $\rho = G_+(T)$ is the branch with $\dot{H} > 0$. Applying Eq. (2.4) to these branches produces the model ([7, 10])

$$F_\pm(T) = \pm\sqrt{-\frac{T\rho_c}{2}}\arcsin\left(\sqrt{-\frac{2T}{\rho_c}}\right) + G_\pm(T). \tag{2.7}$$

3. Matter Bounce Scenario

Matter Bounce Scenarios (MBS) [3] are essentially characterized by the Universe being nearly matter dominated at very early times in the contracting phase (to obtain an approximately scale invariant power spectrum), and evolving towards a bounce where all the parts of the Universe become in causal contact [10], solving the horizon problem, to enter into a expanding regime, where it matches the behavior of the standard hot Friedmann Universe. They constitute a promising alternative to the inflationary paradigm.

According to the current observational data, in order to obtain a viable MBS model, the bouncing model has to satisfy some conditions that we have summarized as follows:

1. The latest Planck data constrain the value of the spectral index for scalar perturbations and its running, namely n_s and α_s, to 0.9603 ± 0.0073 and -0.0134 ± 0.009 respectively [11]. The analysis of these parameters provided by Planck makes no slow roll approximation (for example, the first year WMAP observations was done considering the ΛCDM model [12]), and thus, the parameters n_s and α_s can be used to test bouncing models. A nearly scale

invariant power spectrum of perturbations with running is obtained either as a quasi de Sitter phase in the expanding phase or as a nearly matter domination phase at early times, in the contracting phase [13]. Then, since for an exact matter domination in the MBS the power spectrum is flat, to improve the model so as to match correctly with the observational data, one has to consider, at early times in the contracting phase, a quasi-matter domination period characterized by the condition $\left| w \equiv \frac{P}{\rho} \right| \ll 1$, with P and ρ being respectively the pressure and the energy density of the Universe.

2. The Universe has to reheat creating light particles that will thermalize matching with a hot Friedmann Universe. Reheating could be produced due to the gravitational particle creation in an expanding Universe [14]. In this case, an abrupt phase transition (a non adiabatic transition) is needed in order to obtain sufficient particle creation that thermalizes producing a reheating temperature that fits well with current observations. It is shown in [15] that gravitational particle production could be applied to the MBS, assuming a phase transition from the matter domination to an ekpyrotic phase, which also maintains the isotropy of the bounce, i.e., solves the Belinsky-Khalatnikov-Lifshitz instability [16], in the contracting regime. In [17], dealing with massless nearly conformally coupled particles, a reheating temperature compatible with current data has recently been obtained.

3. The data of the seven-year survey WMAP ([18]) constrains the value of the power spectrum for scalar perturbations to be $\mathscr{P}_S(k) \cong 2 \times 10^{-9}$. The theoretical results calculated with bouncing models have to match with that observational data.

4. The constrain of the tensor/scalar ratio provided by WMAP and Planck projects ($r \leq 0.11$) is obtained indirectly assuming the *consistency* slow roll relation $r = 16\varepsilon$ (where $\varepsilon = -\frac{\dot{H}}{H^2} \cong \frac{1}{2}\left(\frac{V_\varphi}{V}\right)^2$ is the main slow roll parameter) [19], because gravitational waves turn out not to have been detected by those projects. This means that the slow roll inflationary models must satisfy this constrain, but not the bouncing ones, where there is not any consistency relation. Actually, to check if the MBS models provide a viable value of the tensor/scalar ratio, first of all gravitational waves must be clearly detected in order to determine the observed value of this ratio.

5. Some non-gaussianities has been detected by PLANCK's team [20]. A theoretical viable bouncing model has to take into account these non-gaussianities. However, as shown by the recent calculation of the 3-point function in bouncing cosmologies [21], this seems to be a very difficult challenge in MBS.

4. Perturbations in Matter Bounce Scenario

The Mukhanov-Sasaki equations for $F(T)$ gravity and LQC are given by [22, 23]

$$\zeta''_{S(T)} - c_s^2 \nabla^2 \zeta_{S(T)} + \frac{Z'_{S(T)}}{Z_{S(T)}} \zeta'_{S(T)} = 0, \tag{4.1}$$

where ζ_S and ζ_T denote the amplitude for scalar and tensor perturbations.

In teleparallel $F(T)$ gravity one has [24]

$$Z_S = \frac{a^2|\Omega|\dot{\varphi}^2}{c_s^2 H^2}, \quad Z_T = \frac{a^2 c_s^2}{|\Omega|}, \quad c_s^2 = |\Omega| \frac{\arcsin\left(2\sqrt{\frac{3}{\rho_c}}H\right)}{2\sqrt{\frac{3}{\rho_c}}H}, \quad \text{with} \quad \Omega = 1 - \frac{2\rho}{\rho_c}. \quad (4.2)$$

In contrast, for holonomy corrected LQC one has [25]

$$Z_S = \frac{a^2\dot{\varphi}^2}{H^2}, \quad Z_T = \frac{a^2}{\Omega}, \quad c_s^2 = \Omega. \quad (4.3)$$

Dealing with the simplest model of MBS, i.e., a background depicted by an exact matter domination, whose scale factor is given by $a(t) = \left(\frac{3}{4}\rho_c t^2 + 1\right)^{1/3}$ one obtains the following power spectrum for scalar perturbations, given by [26]

$$\mathscr{P}_S(k) = \frac{3\rho_c^2}{\rho_{pl}}\left|\int_{-\infty}^{\infty} Z_S^{-1}(\eta)d\eta\right|^2. \quad (4.4)$$

In the particular case of teleparalell $F(T)$ gravity one has $\mathscr{P}_S(k) = \frac{16}{9}\frac{\rho_c}{\rho_{pl}}\mathscr{C}^2$, [24] where $\mathscr{C} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - ... = 0.915965...$ is Catalan's constant, and for holonomy corrected LQC $\mathscr{P}_S(k) = \frac{\pi^2}{9}\frac{\rho_c}{\rho_{pl}}$ [27].

In the same way, the ratio of tensor to scalar perturbations in MBS is given by

$$r = \frac{8}{3}\left(\frac{\int_{-\infty}^{\infty} Z_T^{-1}(\eta)d\eta}{\int_{-\infty}^{\infty} Z_S^{-1}(\eta)d\eta}\right)^2, \quad (4.5)$$

obtaining for teleparallel $F(T)$ gravity $r = 24\left(\frac{Si(\pi/2)}{\mathscr{C}}\right)^2$, where $Si(x) \equiv \int_0^x \frac{\sin y}{y}dy$ is the Sine integral function. In contrast, for holonomy corrected LQC this ratio is very close to zero.

A final remark is in order: *All of the current models that depict the MBS have some problem. For example, in holonomy corrected LQC the square of the velocity of sound becomes negative in the super-inflationary phase ($\rho_c/2 < \rho \leq \rho_c$). In spite of the fact that in holonomy corrected LQC, in order to calculate the power spectrum, only modes that satisfy the long-wavelength condition $|c_s^2 k^2| \ll \left|\frac{z''}{z}\right|$ are used, it is important to realize that, in the super-inflationary regime, the ultraviolet modes (modes that satisfy the condition $|c_s^2 k^2| \gg \left|\frac{z''}{z}\right|$) will suffer Jeans instabilities, and thus undesirable cosmological consequences could appear. This is a serious problem that needs to be addressed in holonomy corrected LQC. In $F(T)$ gravity, the square of the velocity of sound is always positive. However, teleparallelism suffers from the problem that the main invariant, the scalar torsion T, is not at all an invariant, in the sense that it depends on the choice of the orthonormal basis in the tangent bundle (the tetrad). The scalar curvature R is a real invariant, but the current bouncing models in modified $F(R)$ gravity, do not support a matter domination in the contracting phase, and thus their power spectrum is not nearly scale invariant [28].*

Another possibility is to consider the MBS for a flat FLRW geometry in the context of GR. In this case one needs more than one field, one of them a ghost condensate field [29] or a Galileon type field [30] which violates the Null Energy Condition, to obtain a non-singular bounce. In this case the scenario is very complicated, and it is not clear at all how to compute the theoretical quantities such as the spectral index and its running, to be compared with the current observational data.

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