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Symmetry, Physical Theories and Theory Changes

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We discuss the problem of theory change in physics. We characterize a physical theory based through its kinematical symmetries. The progressions of theories is mathematically described as Inönü-Wigner extensions of the associated kinematical groups. This is compatible with the modern ideas in philosophy of science – e.g. the semantic approach to a scientific theory, with the advantage of remaining conceptually simple and admitting a well defined mathematical structure, with hints at a logic of discovery. This is also linked to the Bargmann-Wigner program. We propose extensions of our idea in the final discussion.

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1. Introduction

This work presents a perspective for the characterization of scientific (physical) theories based on the existence of *symmetry groups*, and of their progression through the notion of Inönü-Wigner contraction/extension, which may be seen as a path towards *simplicity*.

Section 2 introduces the mathematical notions of Inönü-Wigner contraction and extension, for groups and algebras. Section 3 applies to kinematical groups (and algebras) leading from Galileo group to conformal group, via Poincaré group and deSitter/anti-deSitter (cosmological) groups. Section 4 shortly presents some analogies with quantization and non-commutive geometry, and concluding remarks.

A linear representation of a group G is defined (functorially) as a group homomorphism from G to the group of endomorphisms of the linear space. We can then handle the elements of a group as transformations acting on the underlying linear space associated to a physical system. In the *classical domain*, the kinematic group acts as canonical transformations on the "phase space" endowed with a symplectic structure [7]. In the *quantum domain* it acts (unitarily) on a Hilbert space of states. This is also known as (anti-)unitary representation.

2. Contractions and Extensions of Groups

2.1 Group contractions

The Inönü-Wigner contraction of a Lie group [10] is best described in terms of its associated Lie algebra which can be seen as its infinitesimal counterpart. The process allows one to construct a new Lie algebra, not isomorphic to the initial one but preserving some of its structure. It proceeds by singular transformations of the infinitesimal elements (the generators) and, in this sense, it can be generalized to other algebraic structures [11]. Starting from a Lie algebra $\mathfrak{g}$, one constructs a parametrized family of new algebras, $\mathfrak{g}_\varepsilon$, which are isomorphic to $\mathfrak{g}$ for $\varepsilon \neq 0$, but not for the singular value $\varepsilon = 0$.

The algebras $\mathfrak{g}_\varepsilon$, for $\varepsilon \neq 0$, are obtained by reparametrizations of $\mathfrak{g}$. Then, the new Lie algebra emerges as the singular limit $\varepsilon \to 0$ of the parameter (in turn its generates a new Lie group via the exponential map). Such a contraction may be seen as a special case of *degeneration* [3].

We assume that $\mathfrak{g}$ contains a subalgebra $\mathfrak{h} \subset \mathfrak{g}$, with complement $\mathfrak{p}$ in $\mathfrak{g}$, i.e., $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{p}$ (direct sum of vector spaces). The commutators can then be schematically decomposed as

$$[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h},\mathfrak{p}] \subset \mathfrak{p}, \qquad [\mathfrak{p},\mathfrak{p}] \subset \mathfrak{h}+\mathfrak{p}. \tag{2.1}$$

The reparametrization replaces each generator $J \in \mathfrak{p}$ by a generator $J' = \varepsilon\, J$, with $\varepsilon \neq 0$. In abbreviated notation, $\mathfrak{p}$ becomes $\mathfrak{p}' = \varepsilon\, \mathfrak{p}$. The algebra remains the same, but with the reparametrized commutation relations

$$[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h},\mathfrak{p}'] = [\mathfrak{h},\varepsilon\,\mathfrak{p}] \subset \varepsilon\,\mathfrak{p} = \mathfrak{p}', \qquad [\mathfrak{p}',\mathfrak{p}'] = [\varepsilon\,\mathfrak{p},\varepsilon\,\mathfrak{p}] \subset \varepsilon^2\,(\mathfrak{h}+\mathfrak{p}). \tag{2.2}$$

The singular limit $\varepsilon \to 0$ gives a well-defined but different Lie algebra $\mathfrak{g}_0$ obeying

$$[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h},\mathfrak{p}'] \subset \mathfrak{p}', \qquad [\mathfrak{p}',\mathfrak{p}'] = 0. \tag{2.3}$$

The new Lie algebra is the *semi-direct product* of Lie algebras $\mathfrak{g}' = \mathfrak{h} \ltimes \mathfrak{p}'$. If the second relation in (2.3) were $[\mathfrak{h}, \mathfrak{p}'] = 0$, then the new Lie algebra would be a *direct product* $\mathfrak{g}' = \mathfrak{h} \times \mathfrak{p}'$.

2.1.1 A First Example of Inönü-Wigner Contraction

The group $SO(3)$, of rotations in three-dimensional Euclidean space, admits the Lie algebra $\mathfrak{so}(3)$, generated by J_i, $i = 1,2,3$, with commutation relations

$$[J_i, J_j] = \sum_{k=1}^{3} \varepsilon_{ijk} \, J_k. \tag{2.4}$$

Here ε_{ijk} is the Levi-Civita symbol. We define $\mathfrak{h}$ as the subalgebra generated by J_3 only (with $[J_3, J_3] = 0$), so that J_1, J_2 generate $\mathfrak{p}$. We rescale the elements of $\mathfrak{p}$ by Λ as

$$j_1 = \Lambda \, J_1, \qquad j_2 = \Lambda \, J_2, \qquad j_3 = J_3. \tag{2.5}$$

For $\Lambda \neq 0$, the algebra remains the same, although with the new expression for the commutators

$$[j_1, j_2] = \Lambda^2 \, j_3, \qquad [j_2, j_3] = j_1, \qquad [j_3, j_1] = j_2. \tag{2.6}$$

The limit $\Lambda \to 0$ provides a new Lie algebra with three generators obeying the relations

$$[j_1, j_2] = 0, \qquad [j_2, j_3] = j_1, \qquad [j_3, j_1] = j_2. \tag{2.7}$$

They characterize the Lie algebra $\mathfrak{e}(2) = \mathfrak{so}(2) \ltimes \mathbb{R}^2$ of the *two-dimensional Euclidean group* E(2). The (special) orthogonal group $SO(2)$ is the group of rotations of the two-dimensional Euclidean plane; it is natural to associate to it the group of translations of the same space, to complete the isometries. This corresponds to the natural *augmentation* $SO(2) \overset{augm}{\to} ISO(2) = E(2)$.

2.2 Extensions

A group that cannot be written as a (semi-)direct product is a *simple group*. The Inönü-Wigner contraction diminishes simplicity. There is an inverse procedure, the *Inönü-Wigner extension*, which achieves simplicity. It extends a Lie group which is a (semi-)direct product, towards a simpler group: with less (semi-)direct products of groups. For instance, the extension associated with the previous contraction may be written as $ISO(2) \overset{ext}{\to} SO(3)$, so that we may write the diagram

$$SO(2) \overset{augm}{\to} ISO(2) \overset{ext}{\to} SO(3).$$

It expresses the transition from pre-Newtonian to Newtonian physics, through the introduction of *isotropic* Euclidean space [15]. We will see below further extensions of this diagram.

3. Study of Case of Theory Change: from Newton to Cosmology

The kinematic group of a theory is the group of isometries of the space-time of that theory. We will consider the sequence of theory change

$$\text{Galilei} \to \text{Einstein} \to \text{(anti-)deSitter} \to \text{conformal}. \tag{3.1}$$

3.1 Newtonian Kinematic: the Galilei Group

The Galilei group $\mathscr{G}$ is the group of isometries of the Newtonian space-time $\mathbb{R}\times\mathbb{R}^3$, where the first component stands for the time direction. A generic element $g = (b,\vec{a},\vec{v},R)$ combines a time translation b, a three-dimensional rotation R; a three-dimensional spatial translation $\vec{a} = (a_x,a_y,a_z)$, and a Galilei transform (or boost) $\vec{v} = (v_x,v_y,v_z)$ which exchanges inertial frames.

The Galilei group admits the maximal Abelian subgroup $\mathscr{U}$, generated by spatial translations and Galilei boosts. The quotient $\mathscr{G}/\mathscr{U}$ (the group of classes of equivalence) is not a simple group: it still contains a (maximal) Abelian subgroup $\mathscr{T}$ generated by time translations. The factor group $(\mathscr{G}/\mathscr{U})/\mathscr{T}$ is the simple group $\mathscr{R}$ of three-dimensional rotations.

The Lie algebra of $\mathscr{G}$ admits then ten generators (H,P_i,C_i,L_{ij}), $i,j = 1,2,3$: the Hamiltonian H generates time-translations; the momenta P_i generate spatial-translations, the C_i generate the Galilei boosts and the $L_{ij} = -L_{ji}$ spatial rotations.

We will consider the subgroup $ISO(3)$ of the Galilei group $\mathscr{G}$, generated by the spatial rotations $SO(3)$ and spatial translations $\mathbb{R}^3$.

3.2 Special Relativity: the Poincaré Group

The Poincaré group $\mathscr{P}$ (we consider its *proper orthochronous* component, which does not include time-reversal and parity) acts as isometries of the Minkowski space-time. It includes the space-time translations and rotations, together with their combinations. The space-time translations generate its maximal Abelian subgroup $\mathbb{R}^{1,3}$. The quotient (group of classes of equivalence) $\mathscr{L} = \mathscr{P}/\mathbb{R}^{1,3}$ is the Lorentz group $\mathscr{L} = SO(3,1)$, which comprises the space-time rotations, which combine three-dimensional spatial rotations with Lorentz boosts. It is a subgroup of the Poincaré group and the latter is also known as the inhomogeneous Lorentz group. This is a simple group and the Lie algebra $\mathfrak{p}$ of $\mathscr{P}$ may be written as the semi-direct product $\mathfrak{p} = \mathfrak{l}\ltimes\mathbb{R}^{1,3}$ where $\mathfrak{l}$ stands for the Lie algebra of $\mathscr{L}$.

The general element $g = (a,R)$, where $a\in\mathbb{R}^4$ is a 4-translation and $R\in\mathscr{L}$ a four-dimensional (Lorentz) rotation. The Lie algebra $\mathfrak{p}$ admits ten generators, $(P_\mu,J_{\mu\nu})$, $\mu,\nu = 0,1,2,3$. The four 4-momenta P_μ generate the space-time translations; the six $J_{\mu\nu} = -J_{\nu\mu}$ generate space-time rotations.

The above kinematics were discussed by A. Einstein in 1905, where he proposed a kinematics covariant under the symmetries of the electromagnetism. Before A. Einstein there was an astonishment among physicists due to the fact that the symmetries of the Maxwell equations describing electrodynamics (i.e., the Poincaré group) differed from those of the Newton-Galilei kinematics, i.e., the Galilei group.

3.3 From Galilean Kinematics to Special Relativity

The group $ISO(3)$ is not simple. It admits a natural Inönü-Wigner extension to the Lorentz group $ISO(3) \overset{ext}{\to} SO(3,1)$, with parameter $1/c$. The Lorentz group is stable, i.e., admits no further similar extension. This lifts to an extension of their prolongations: from the Galilei group to the Poincaré group, i.e., from Newtonian kinematics to special relativity. The Poincaré group is however not simple and the process may be continued.

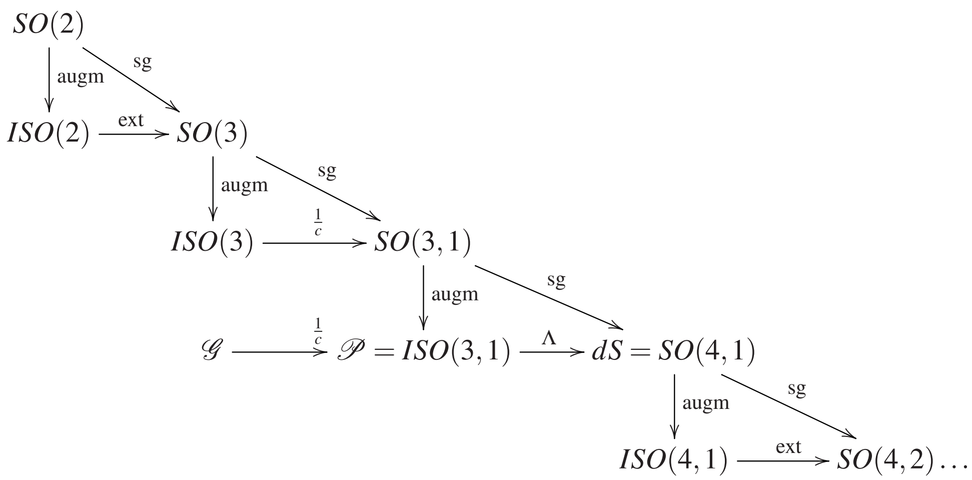
3.4 From Special Relativity to Cosmology

The natural augmentation $SO(3,1) \overset{augm}{\to} \mathscr{P} = ISO(3,1)$ completes the space-time translations with the space-time rotations. Then, $ISO(3,1)$ admits also a family of natural extensions with a parameter Λ called the *cosmological constant*. This gives the anti-deSitter group $AdS = SO(3,2)$ ($\Lambda < 0$) or the deSitter group $dS = SO(4,1)$ ($\Lambda > 0$). These *simple* kinematic groups act as isometries on the respective cosmological space-times of constant non zero curvatures $\pm\Lambda^{-2}$, the deSitter and anti–de Sitter space-times (embedded in $R^{4,1}$ and $R^{3,2}$ respectively).

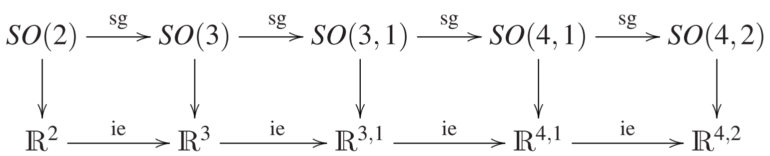
The limit $\Lambda \to 0$ corresponds to their common contraction, the Poincaré group $\mathscr{P}$. Both dS and AdS are stable, in the sense that they admit no further similar extension. For each of them, the Lie algebras admit ten generators $J_{ab} = -J_{ba}$, $a,b = 0,1,...,4$.

Going further, the natural augmentations of dS and AdS (by adding " translations ") give respectively $ISO(4,1)$ and $ISO(3,2)$. Both admit a common extension under the form of the *conformal group* $SO(4,2)$. The latter admits the ten generators of dS (or AdS), augmented by five new generators, one for *scaling transformations*, and four generating the so called *special conformal transformations*. This group plays an important role in physics. For instance it preserves electromagnetism. As proposed initially by Weyl [18] it may constitute the symmetry group of a *conformal theory of gravitation*.

We resume the sequence through the following diagram. Horizontal arrows indicate *extensions*; oblique arrows *subgroup inclusion* (sg); and vertical ones *natural augmentations*. It may be continued, but without straightforward applications to physics. A similar version applies with dS $= SO(4,1)$ and $ISO(4,1)$ replaced by AdS $= SO(3,2)$ and $ISO(3,2)$, respectively.



Furthermore, the groups in the above series act as isometries of the space-times indicated in the diagram below, where the symbol ie stands for isometrical embedding.



The notations $\mathbb{R}^2, \mathbb{R}^3$ stand respectively for Euclidean plane and Euclidean space; $\mathbb{R}^{p,q}$ for the pseudo–Euclidean space of signature p,q.

4. Discussion and Conclusion

Comparable procedures also account for the transition from the classical to the quantum domain: the process of quantization corresponds to the replacement (with some conditions) of a commutative *Poisson algebra* A of classical observables (functions on a symplectic manifold Γ) by a non-commutative algebra $\mathscr{A}$ of quantum observables (seen as operators acting on an Hilbert space). This can be accomplished by an *algebra deformation* procedure, which associates to A a parametrized family of non-commutative algebras. The quantum algebra of interest is obtained with the value $\hbar$ of the parameter. This procedure bears some similarities with the extension of Lie algebras (see, e.g., [17]).

The geometrical interpretation of algebra extension (diagram B) considers each algebra in the chain as the isotropy algebra of a manifold, and so provides a progression chain of these manifolds. The deformation quantization $A \to \mathscr{A}$ above also admits a geometrical interpretation, at the basis of *non-commutative geometry* (NCG) [6]. The Gel'fand duality — in fact a categorial equivalence [13][14] — associates a space with the commutative algebra of functions on it. NCG associates a *non-commutative space* (not made of points) to a non-commutative C^*-algebra $\mathscr{A}$. Quantization is so interpreted geometrically as an upgrading of a *phase space* manifold to a non commutative space. A similar procedure may also be applied to space-time, with the goal of constructing a new physics in the frame of a *non-commutative space-time* [16]. This intends for instance to give a phenomenological description of the effects of quantum gravity which are thought to destroy the manifold structure of space-time at small scales. Another original possibility, developed by A. Connes and collaborators [4, 5], considers an *internal* (discrete) non-commutative space: the product of this internal space by the (commutative) space-time manifold provides the geometrical framework for the matter fields. Application of a *spectral action principle* led Connes and his collaborators to a pure geometrical derivation of the physics of both standard model and gravity.

We have applied the perspective of extensions of (kinematical) groups and their associated space-times, with contraction providing the way back to the problem of theory evolution — and hence of scientific discovery [12]. This is connected with the Bargmann-Wigner program [19, 1, 2]. We have achieved, in our sense, a precision which is not obtained by other exclusively logic approaches like for instance the semantic one. The same conclusion is given by Halvorson [8] when he states that despite their adequateness, the logic approaches are too general to usefully characterize specific disciplines.

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