PROCEEDINGS OF SCIENCE

On the modified Yamaguchi-type functions for the Bethe-Salpeter equation

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The modification of the so-called Yamaguchi-type functions for the Bethe-Salpeter equation with a separable kernel is discussed. Such functions are introduced to calculate observables in reactions with interacting np-pair in the final state (electro- and photo-disintegration of the deuteron, $pd \to p(pn)$ and so on).

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1. Introduction

One of the most consistent nucleon-nucleon interaction theories is based on the solution of the Bethe-Salpeter (BS) equation [1]. In this case, we have to deal with a nontrivial integral equation for the bound state (*np* pair). There is no method to get its exact solution. So, various approximations are worked out.

An effective and solvable approach based on the exact solution of the BS equation is to use the separable ansatz for the interaction kernel in the BS equation [2]. In this case one can transform an initial integral equation into a system of linear equations. Parameters of the kernel are obtained by fitting of phase shifts, inelasticity and low-energy parameters for respective partial-wave states.

First separable parametrizations were worked out within nonrelativistic models. The separable functions (called form factors) of the interaction kernel used in these models had no poles on the real axis in the relative energy complex plane [3, 4]. However, such poles appeared when the interaction kernel was relativistically generalized.

In some cases they do not prevent to perform the calculations. However, at high energies, one would have to deal with several thresholds corresponding to the production of one, two and more mesons of different types. Which is clearly not feasible to deal with. The more practical approach is to employ a phenomenological covariant separable kernel, who do not exhibit the meson-production thresholds and can even be constructed in a singularity-free fashion, using separable form factors and Wick-rotation prescription as it is done in the present paper. Thus, an accurate description of on-shell nucleon-nucleon data is possible up to quite high energies. One then can hope that the obtained separable interaction kernels have also a reasonable off-shell behavior, so that they can be applied to other reactions.

2. Bethe-Salpeter formalism

We start with the partial-wave decomposed Bethe-Salpeter equation for the nucleon-nucleon (*NN*) scattering matrix T (in the rest frame of two-nucleon system):

$$\begin{aligned} t_{L'L}(p'_0,p',p_0,p;s) &= v_{L'L}(p'_0,p',p_0,p;s) \\ &+ \frac{i}{4\pi^3}\sum_{L''}\int dk_0 \int k^2\,dk\, \frac{v_{L'L''}(p'_0,p',k_0,k;s)t_{L''L}(k_0,k,p_0,p;s)}{(\sqrt{s}/2-E_k+i\varepsilon)^2-k_0^2}. \end{aligned} \tag{2.1}$$

Here t is the partial-wave decomposed T matrix and v is the kernel of the NN interaction, $E_k = \sqrt{k^2+m^2}$. There is only one term in the sum for the singlet (uncoupled triplet) case ($L=J$) and there are two terms for the coupled triplet case ($L = J \mp 1$). We introduce square of the total momentum $s = P^2 = (p_1+p_2)^2$ and the relative momentum $p = (p_1-p_2)/2$ [$p' = (p'_1-p'_2)/2$] (for details, see reference [2]).

Assuming the separable form (rank I) for the partial-wave decomposed kernels of NN interactions:

$$v_{L'L}(p'_0,p',p_0,p;s) = \lambda g^{[L']}(p'_0,p')g^{[L]}(p_0,p), \tag{2.2}$$

we can solve Eq. (2.1) and write for the T matrix:

$$t_{L'L}(p'_0,p',p_0,p;s) = \tau(s) g^{[L']}(p'_0,p')g^{[L]}(p_0,p), \tag{2.3}$$

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with function $\tau(s)$ being:

$$\tau(s) = 1/(\lambda^{-1} + h(s)). \tag{2.4}$$

Function $h(s)$ has the following form:

$$h(s) = \sum_L h_L(s) = -\frac{i}{4\pi^3} \int dp_0 \int p^2\, dp \sum_L \frac{[g^{[L]}(p_0,p)]^2}{(\sqrt{s}/2 - E_p + i\varepsilon)^2 - p_0^2}. \tag{2.5}$$

The simplest separable function $g(p_0, p)$ which can be used, is a covariant generalization of the non-relativistic *Yamaguchi*-type [5] function:

$$g(p_0,p) = \frac{1}{p_0^2 - p^2 - \beta^2 + i\varepsilon}, \tag{2.6}$$

where β is a parameter.

3. Modified Yamaguchi-type functions

Let us consider integral $h(s)$ (Eq. 2.5). Taking into account the pole structure of the propagators:

$$p_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_p \pm i\varepsilon \tag{3.1}$$

and of g functions:

$$p_0^{(3,4)} = \mp E_\beta \pm i\varepsilon \tag{3.2}$$

and using the Cauchy theorem, $h(s)$ function can be written as follows:

$$\frac{1}{2\pi^2} \int p^2 dp \frac{1}{(s/4 - \sqrt{s}E_p + m^2 - \beta^2)^2} \frac{1}{\sqrt{s} - 2E_p + i\varepsilon}. \tag{3.3}$$

To calculate integral Eq. (3.3) one should analyze numerator $f = (s/4 - \sqrt{s}E_p + m^2 - \beta^2)$ as a function of s:

- if $2(m-\beta) < \sqrt{s} < 2(m+\beta)$ then always $f < 0$ and function $1/f^n$ *is integrable* for any integer n and any E_p;
- for a bound state $\sqrt{s} = M_d = (2m - \varepsilon_D)$. Since for minimal $\beta_{min} = 0.2$ GeV always $\beta_{min} > \varepsilon_D/2$ then function $1/f^n$ *is integrable* for any integer n and any E_p;
- if $\sqrt{s} < 2(m-\beta)$ or $\sqrt{s} > 2(m+\beta)$ then f can be positive and negative and $1/f^n$ *is non-integrable* for even n at any E_p.

Critical value $s^c = 4(m+\beta)^2$ corresponds to laboratory kinetic energy of np-pair $T^c_{lab} = 4\beta + 2\beta^2/m \simeq 4\beta$. If $\beta_{min} = 0.2$ GeV then $T^{min}_{lab} = 0.8$ GeV.

So, if we consider disintegration processes of the deuteron such as photo- or electro-breakup Yamaguchi-functions can be used if only laboratory kinetic energy of the np-pair is less then T^{min}_{lab}.

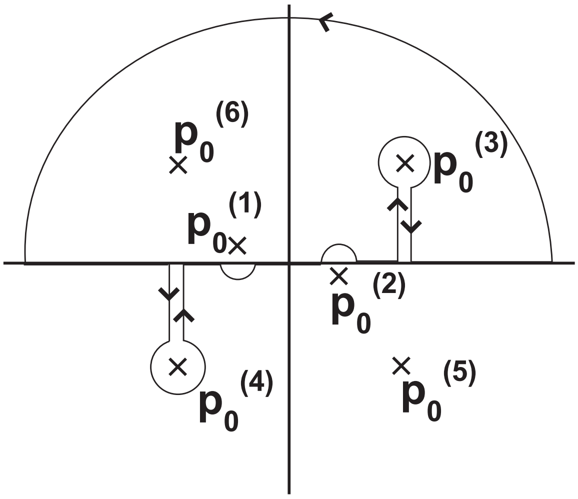

Figure 1: Contour for integration over p_0 according to the Cauchy theorem.

To avoid this restriction we suggest to use Yamaguchi-type functions modified in the following way:

$$g_{\rm Y}(p_0,p) = 1/(p_0^2-p^2-\beta^2) \longrightarrow g_{\rm MY}(p_0,p) = 1/((p_0^2-p^2-\beta^2)^2+\alpha^4),$$

here Y stands for Yamaguchi-like and MY - for Modified Yamaguchi.

To work with the modified Yamaguchi-type functions the procedure of p_0 integration should be modified, too. This procedure is worthy of a special discussion. The poles of the $h(s)$ integral with modified Yamaguchi-type functions are:

$$p_0^{(3,4)} = \pm\sqrt{p^2+\beta^2+i\alpha^2}, \\ p_0^{(5,6)} = \pm\sqrt{p^2+\beta^2-i\alpha^2}. \tag{3.4}$$

All poles and the contour of integration are pictured in Fig.1,2. The idea how to choose the contour appeared owing to [6, 7]. It is:

1. the contour must envelope the poles of g form factors which will be inside the standard contour in $\alpha \to 0$ limit. "Standard" means contour used in the quantum field theory calculations with a propagator which has poles only on the real axis in the p_0 complex plane; one of them is circled from below and the other - from above. So, the path of integration is defined by an appropriate contour for the propagator;

2. the calculation over the presented path leads to the pure real contribution from the form factor poles and, therefore, to the unitary S matrix (or corresponding unitarity condition for T matrix). We also obtain a correct transition to ordinary form factors of type $g \sim 1/(p_0^2 - p^2-\beta^2)^2$ in $\alpha \to 0$ limit.

In general, the modified Yamaguchi-type functions can be written as:

$$g_i^{[a]}(p_0,p) = \frac{(p_{ci}-p_0^2+p^2)^{n_i}(p_0^2-p^2)^{m_i}}{((p_0^2-p^2-\beta_{1i}^2)^2+\alpha_{1i}^4)^{k_i}((p_0^2-p^2-\beta_{1i}^2)^2+\alpha_{1i}^4)^{l_i}}, \tag{3.5}$$

where parameters - n_i, m_i, k_i, l_i (integer), $p_{ci}, \beta_{1i}, \beta_{2i}, \alpha_{1i}, \alpha_{2i}$ (real) are depend on channel $[a]$ under consideration. Such g form factors are used to describe neutron-proton scattering observables (phase shifts, inelasticities, low-energy parameters and deuteron characteristics) for total angular momentum $J = 0, 3$ in a wide energy range (see [8]-[10]).

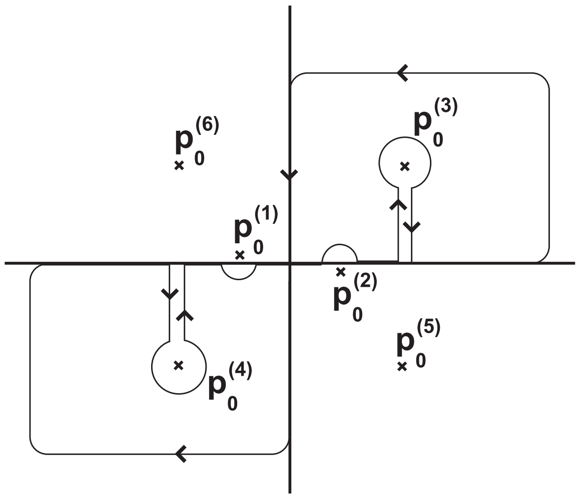

Figure 2: Contour for integration over p_0: the Wick rotation.

References

[1] E.E. Salpeter and H.A. Bethe, Phys. Rev. **84**, 1232 (1951).

[2] S.G. Bondarenko, V.V. Burov, A.V. Molochkov, G.I. Smirnov, and H. Toki, Prog. Part. Nucl. Phys. **48**, 449 (2002).

[3] L. Mathelitsch, W. Plessas, and W. Schweiger, Phys. Rev. **C26**, 65 (1982).

[4] J. Haidenbauer and W. Plessas, Phys. Rev. **C30**, 1822 (1984).

[5] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954); Y. Yamaguchi, Y. Yamaguchi, Phys. Rev. **95**, 1635 (1954).

[6] R.E. Cutkosky, P.V. Landshoff, D.I. Olive, and J.C. Polkinghorne, Nucl. Phys. **B12**, 281 (1969).

[7] T.D. Lee and G.C. Wick, Nucl. Phys. **B9**, 209 (1969).

[8] S.G. Bondarenko, V.V. Burov, W.-Y. Pauchy Hwang, and E.P. Rogochaya, Nucl. Phys. **A832**, 233 (2010); S.G. Bondarenko, V.V. Burov, W.-Y.Pauchy Hwang, and E.P. Rogochaya, Nucl. Phys. **A848**, 75 (2010).

[9] S.G. Bondarenko, V.V. Burov, and E.P. Rogochaya, Phys. Lett. **B705**, 264-268 (2011); Nucl. Phys. Proc. Suppl. **219-220**, 126-129 (2011).

[10] S.G. Bondarenko, V.V. Burov, and E.P. Rogochaya, Nucl. Phys. Proc. Suppl. **245**, 291-297 (2013); S.G. Bondarenko, V.V. Burov, S.E. Kemelzhanova, E.P. Rogochaya, N. Sagimbaeva, Proceedings of the 12th International Workshop "Relativistic Nuclear Physics from Hundreds of MeV to TeV", Slovak Republic, Stara Lesna, June 16-20, 2014. JINR, Dubna. P. 70.

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