

Determination of classes of events in multiplicity and its relevance to centrality for heavy and light ion collisions at high energy in MC models

Drozhzhova Tatiana^{*†}

Saint-Petersburg State University

E-mail: tatiana.drozhzhova@cern.ch

We present the critical review of estimates of centrality in nucleus-nucleus and proton-nucleus collision. We construct a consequential method of centrality determination in terms of the multiplicity and the impact parameter. Using Monte-Carlo simulations we analyze the consistency of the concept of centrality in the cases of nucleus-nucleus and proton-nucleus collisions for heavy and light ions. We discuss the results of this analysis and consider a universal procedure to obtain certain centrality classes, where the fluctuations related to event-by-event variance in the impact parameter and/or the number of nucleons-participants are minimized.

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^{*}Speaker.

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1. Introduction

The concept of centrality of the collision plays an important role in every heavy-ion collisions experiment. An event is meant to be more central than the smaller the value of impact parameter is. Due to the fact that it is not possible to measure the impact parameter experimentally, centrality of collisions has to be determined indirectly. For more central collisions the number of nucleons-participants is greater. To determine the centrality of collisions in a real experiment it is sufficient to relate the centrality to a real observable. Multiplicity is taken for this purpose in majority of experiments. It turns out that using such procedure of centrality determination it is not possible to determine the number of nucleons-participants with enough precision for each event.

The accuracy in measurements of observables is critical in the experiments aimed at finding the effects associated with fluctuations of such quantities. Large fluctuations of observables may become a signal of quark-gluon plasma (QGP) formation. Hot and cold strong-interactive matter formed in ultra-relativistic heavy ions collisions in framework of thermodynamical approach can be described in terms of bariochemical potential, temperature, heat capacity, compressibility, etc. [1], [2], [3] Fundamental properties of a system can be traced in such a global observables, as multiplicity of produced particles, average transverse momentum, net electric and barionic charge, etc. Measurements of fluctuations and correlations of such quantities allows us to study border region of the phase diagram of QCD, where hadronic gas and QGP are converted into each other. Experimental data [4], [5] and theoretical calculations [6], [7], [8], [9] points to existence of the triple critical point for the phase transition, but any direct experimental observation had not been made yet.

It is expected for the critical point on the phase diagram to be accompanied by large fluctuations of the aforementioned quantities [10], [11], [12]. For the first order phase transition it is considered that supercooling may cause the density fluctuations, and further the formation of a hot drop. In turn, it can cause large fluctuations of observables. Modern experiments NA61/SHINE (SPS), program of beam energy scanning RHIC are currently executed to find the critical point of transition from hadronic gas to QGP, development onset of deconfinement [13], [14]; facilities under construction will allow us to make more detailed scanning of the phase diagram of strong-interactive matter and study the properties of QCD matter in the area of significant net baryon density (future experiments CBM at FAIR and MPD at NICA) [15], [16], and accumulate the data with large statistics.

Observation of critical fluctuations is fraught with all sorts of difficulties, connected with a complex evolution process of nuclei-nuclei collisions and large uncertainties in its theoretical description [17] (in particular, absence of direct application of perturbation theory to QCD). To interpret the data including such a delicate effects correctly it is necessarily to minimize the fluctuations related to event-by-event variance in the impact parameter and/or the number of nucleons-participants, appearing as a background for the desired signal. This problem is very important for understanding and analysis of the initial state effect in case of long-correlation study, being currently implemented by the Laboratory of Ultra-High Energy Physics of SPbSU with reference to two experiments (ALICE, NA61/SHINE) at CERN.

In this work we performed an analysis in determining the quality of nucleons-participants number in different centrality classes. In most experiments, where it is impossible to measure the num-

ber of nucleons-participants directly, centrality is determined via multiplicity of charged particles. We did the same matching. To carry out such an analysis all the necessary information (including the number of nucleons-participants) can be extracted from the output of the event-by-event modeling of collisions by event generators. For this purpose the data was calculated by generator based on the Glauber model (created by our group) and event generator HIJING. Both generators are based on the Monte-Carlo method. The ion-ion and proton-ion collisions at LHC energies (2.76TeV and 5.02TeV respectively) and heavy ions (Pb-Pb) and light ions ($Be^7 - Be^9$) collisions at SPS energies were modeled to analyze the dependence of the effect on energy and number of nucleons in colliding nuclei.

In the main part of the paper the basic features and parameters of generators are described (Section 2), the method for determination of centrality classes is described in details (Section 3) and illustrated on examples of Pb-Pb (2.76 TeV , Section 4.1), Pb-Pb (17.3 GeV , Section 4.2), $Be^7 - Be^9$ (16.8 GeV , Section 4.3), p-Pb (5.02 TeV , Section 4.4) collisions. Results of the analysis are discussed in the conclusion (Section 5).

2. Parameters of Generators

2.1 Glauber model

Our group has developed an event generator based on Glauber model. Results obtained by this generator are consistent with theoretical calculations [18] and were published in our previous work [19].

Glauber model is based on a picture of an independent nucleon-nucleon collisions with inelastic cross-section value taken from p-p experimental data [20], [21]. To describe the distribution of nucleons inside the nucleus the nuclear matter density is used in form of Woods-Saxon distribution for heavy ions:

$$\rho(r) = \rho_0 \left\{ 1 + \exp\left(\frac{r - R_A}{a}\right) \right\}^{-1} \quad (2.1)$$

Here the nucleus radius:

$$R_A = R_0 \cdot A^{\frac{1}{3}}, R_0 = 1.07\text{ fm}, a = 0.545\text{ fm}. \quad (2.2)$$

And for light nuclei the density of nuclear matter is taken in form of harmonic oscillator:

$$\rho(r) = \frac{4}{\pi^{\frac{3}{2}} C^3} \left[1 + \frac{A-4}{6} \cdot \left(\frac{r}{C}\right)^2 \right] \exp\left(\frac{-r^2}{C^2}\right) \quad (2.3)$$

Here:

$$C^2 = \left(\frac{5}{2} - \frac{4}{A}\right)^{-1} \cdot \left(\langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_p\right) \quad (2.4)$$

Later the generator was extended to implementation of the multiplicity of charged particles based on a two-component model. The results shown by this generator in calculation of normalized variance at SPS energies were consistent with experiment [22], [23]. In the model multiplicity of charged particles can be obtained according to the Poisson distribution with the mean proportional to the number of strings formed after nuclei collision: N_{str} , multiplied by the number of charged particles per rapidity from one string $m_{nn} = 1.1$ [24].

$$N_{str}(\beta) = f N_{str}^{NN} N_c(\beta) + (1 - f) N_{AB}(\beta) \quad (2.5)$$

Here $N_c(\beta)$ is the total number of collisions, $N_{AB}(\beta)$ is the total number of wounded nucleons (nucleons-participants), $f \in [0, 1]$ is a fitting parameter dependent on the interaction energy.

$$f = 0.1731 \cdot \ln(E) - 0.4839 \quad (2.6)$$

And N_{str}^{NN} is an amount of strings in nucleon-nucleon collision. For collision energy $E \in [53; 1800] \text{AGeV}$:

$$N_{str}^{NN} = 2.56 - 0.478 \cdot \ln(E) + 0.084 \cdot (\ln(E))^2 \quad (2.7)$$

2.2 HIJING 1.38

HIJING 1.38 data was used to analyze the centrality for lead-proton collisions at 5.02 TeV [25], [26]. The version of generator is the last one which is open for access. Contrary to the direct implementation of the Glauber model HIJING 1.38 based on two-component geometrical model of minijet production includes some internal effects of nuclei interactions like soft interactions. By adjusting gluon shadowing parameter the variation of parton distribution function (PDF) is taken into account. Without gluon shadowing nuclear interactions are fully independent. Difference between PDF of nucleus and proton is found experimentally [27], [28], [29] which corresponds to the decrease of the cross-section with lesser x . The same effects were considered in different models taking into account energy conservation in elementary nuclei-nuclei interactions [30], [31], [32], [33]. Configuration of HIJING used in this work is described in more details in Appendix A.

3. Description of the method

The aim of our method is to obtain the resolution of given centrality determination method in terms of number of nucleons-participants (N_{part}). Suppose that we have some observable (for example multiplicity or data from some detector) which gives us an indication of collision centrality. Using this observable we can divide all the events into different classes and introduce a scale by the centrality percentile. In any physical analysis it is desirable to know the connection between the percents of centrality and number of nucleons-participants. In particular, for analysis of fluctuations and correlations it is important to know not only the average of number of nucleons-participants but also the variance N_{part} for given centrality class which depends both on the central position of the class and on its width.

It's clear that if, starting from some point, the decrement of the class width does not provide the decrease of the N_{part} variance anymore (it stops decreasing) then further narrowing of the class is not necessary. N_{part} is exactly known when the collision is modeled by Monte-Carlo simulations. It allows us to calculate directly all the optimal centrality classes widths.

Therefore to choose the centrality classes widths optimally we calculate the distributions of the number of events in multiplicity, then relate multiplicity and centrality percentage, choose the division of the events into centrality classes and then obtain the N_{part} variance dependence on the class on its width. Optimal width is determined by the plateau condition.

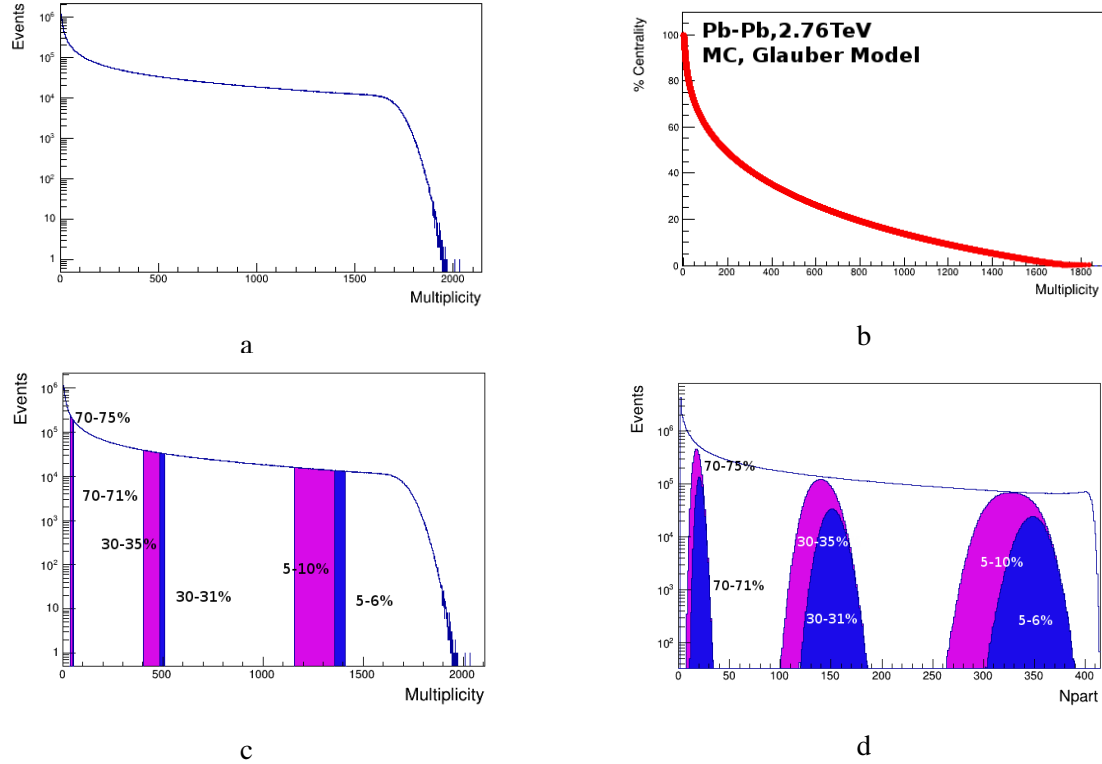


Figure 1: Procedure of centrality classes determination by multiplicity of charge particle for Pb-Pb collision at 2.76 TeV. **a:** Charge particle multiplicity distribution, **b:** Correspondence of the centrality percentile and charge particle multiplicity, **c:** Charge particle multiplicity distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%), semi-peripheral (30%) and peripheral (70%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by multiplicity.

4. Application of the method

4.1 Pb-Pb 2.76 TeV

In this section we discuss and apply the method on an example of Pb-Pb collision at the energy 2.76 TeV. The data of the generator based on Glauber model was used for the modeling of these collisions. First, it's necessarily to determine centrality and its correspondence with multiplicity. For this purpose one plots a distribution of the number of events in bins of multiplicity of charged particles (see Fig.1, a). Let's denote the distribution function as $F(x)$ and the maximum value of multiplicity as M_{max} . Then determine centrality of collision as the value of the function $\frac{F(M_{max}-x)}{N}$, where N is the total number of events. Thus one can obtain a plot of correspondence of collision centrality and multiplicity of charged particles (Fig.1, b). Let's choose the values of centrality (just for demonstration) for central (5%), semi-peripheral (30%) and peripheral (70%) collisions and different intervals with widths of 1% and 5% with the same lower bound (see Fig.1, c). We constructed distributions of events found in these intervals in the number of wounded nucleons and displayed them on the same diagram (see Fig.1, d). This plot demonstrates that the average of the number of wounded nucleons and its variance is different for different centrality classes widths. Also it's clear that the difference varies irregularly with the change of centrality.

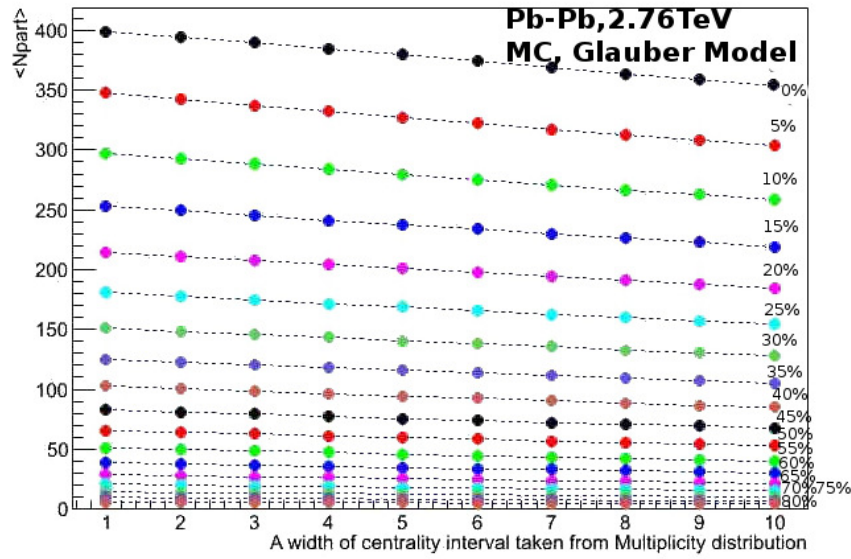


Figure 2: Pb-Pb 2.76TeV, Dependence of an average wounded nucleons from the centrality classes interval determined by multiplicity with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

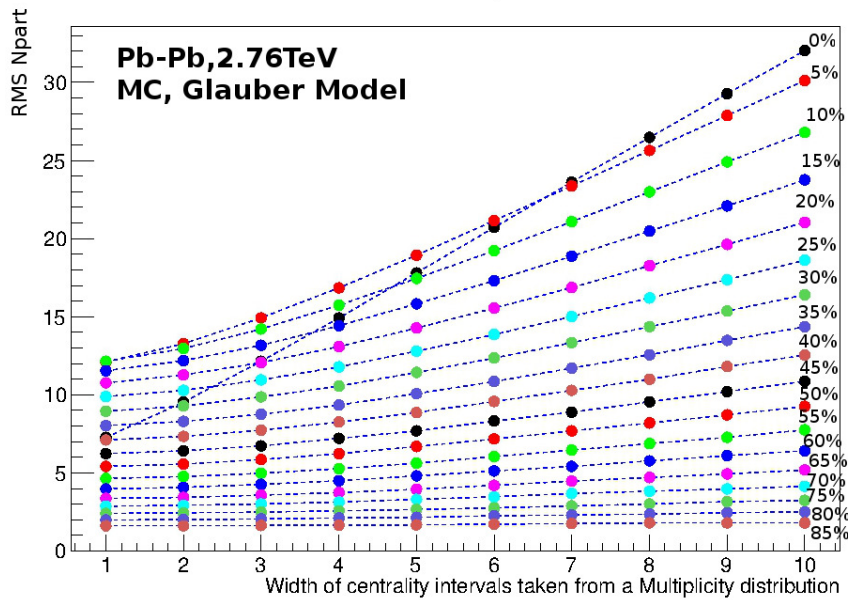


Figure 3: Pb-Pb 2.76TeV, RMS of wounded nucleons in a different centrality classes determined by multiplicity with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

The more detailed analysis is sufficient to understand this dependence. For this purpose the centrality values were chosen with periodicity of 5% (0%, 5%, ... , 80%, 85%) and intervals constructed from these points were considered of widths from 1% to 10%. Then the average number of wounded nucleons and its variance was found for such intervals (see Fig.2 and Fig.3 respectively). The points on the plots are combined into several groups: each series of points starts from the given point in centrality as its lower bound and goes through another points by changing the interval widths of centrality (for example 0%-1%, 0%-2%, ... , 0%-9%, 0%-10%). Each series is represented on the plot by its unique color.

It can be seen that for central collisions the average number of wounded nucleons (N_{part}) varies greatly when intervals are narrowing. For the peripheral collisions this variation is significantly less. For central collisions the smaller is centrality interval, the smaller is RMS (root mean square). For peripheral collisions the value of RMS is independent of the interval width. Dependence of the variance on the centrality interval width changes from the most central collisions to the most peripheral as follows: when the interval is narrowing the function goes to plateau, and this effect is more noticeable the more peripheral is collision.

For analysis of collisions it is useful to combine those events which are not qualitatively different into classes and then work with these classes (centrality classes). On our opinion, to determine the way of dividing all events into classes it is necessary to be guided by the accuracy of quantities measurement (in our case, the number of wounded nucleons, N_{part}). The analysis shows that for central collisions it is worth dividing into as narrow intervals as possible. But the wider intervals can be chosen for more peripheral collisions. The interval width is determined by the value beyond which the RMS is constant (plateau starting point). Further narrowing of the interval does not change the accuracy of the measurements.

Geometrical parameter of the determination of centrality is the impact parameter which is not possible to be measured in experiment but can be obtained by modeling of the collisions by event generators. Similar analysis can be carried out by dividing the events into the centrality classes defined by the impact parameter but not by the multiplicity. First of all it is necessarily to determine the centrality and its correspondence with the impact parameter. For this purpose it is useful to make a plot of number of events distribution in the impact parameter (see Fig.4, a). Let's denote the distribution function as $F(x)$ and the maximum value of the impact parameter as B_{max} . Unlike the previous method B_{max} is related to the most peripheral collisions. So, the centrality of collisions can be determined as a value of the function $\frac{F(x)}{N}$, where N is the total number of events. Thus one can obtain the diagram of correspondence of the collision centrality and the impact parameter (see Fig.4, b). The next steps of the procedure are the same as in the former example.

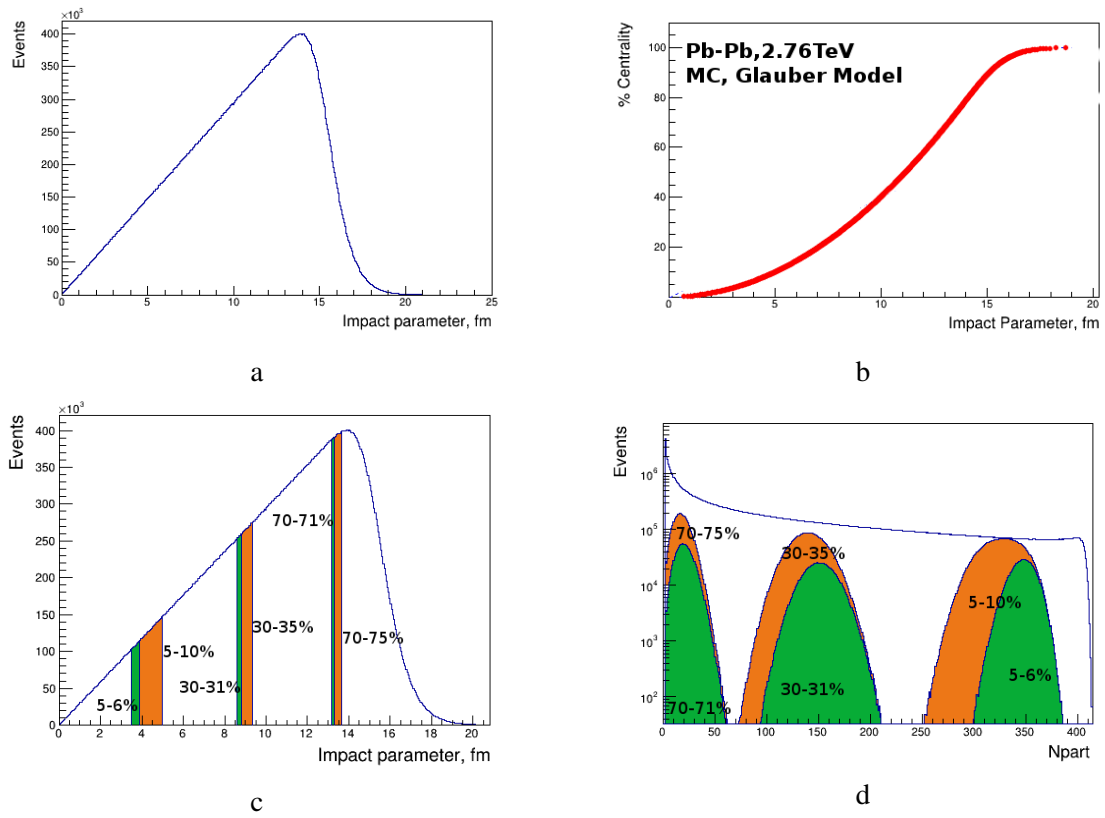


Figure 4: Procedure of centrality classes determination by impact parameter for Pb-Pb collision at 2.76 TeV. **a:** Impact parameter distribution, **b:** Correspondence of the centrality percentile and impact parameter, **c:** Impact parameter distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%), semi-peripheral (30%) and peripheral (70%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by impact parameter.

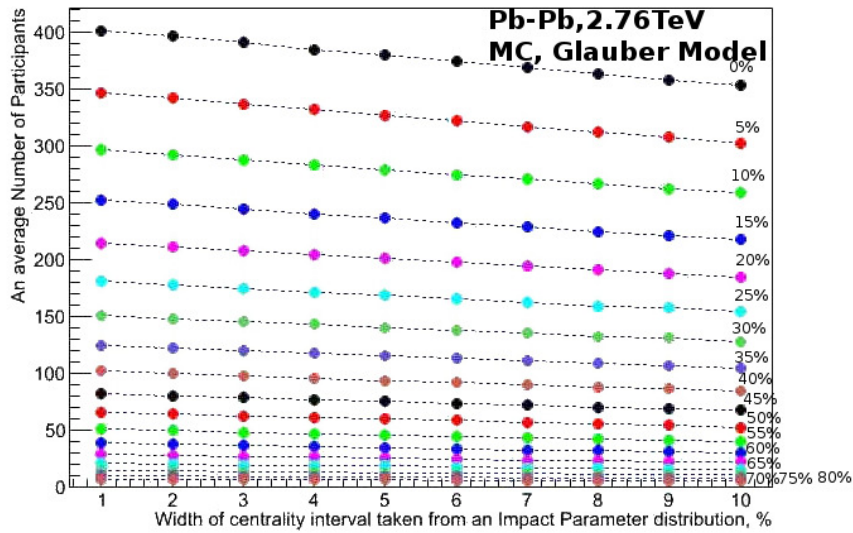


Figure 5: Pb-Pb, 2.76TeV, Dependence of an average wounded nucleons from the centrality classes interval determined by impact parameter with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

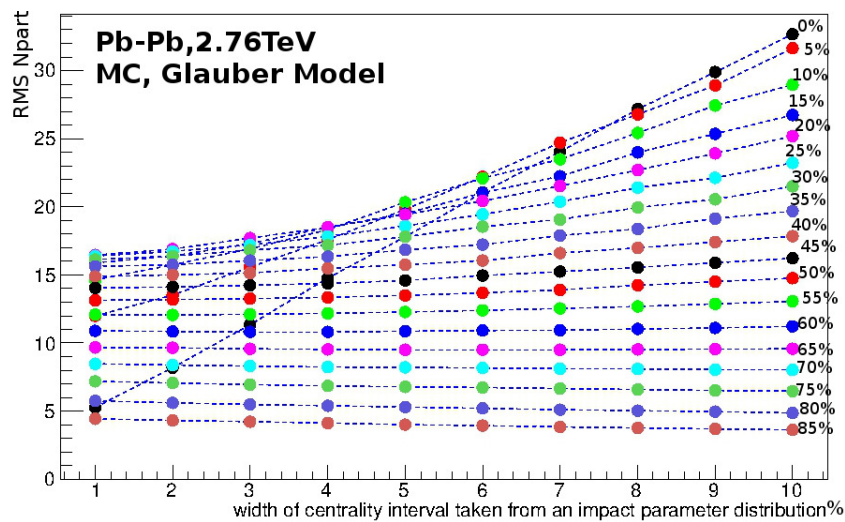


Figure 6: Pb-Pb, 2.76TeV, RMS of wounded nucleons in a different centrality classes determined by impact parameter with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

It is clear (see Fig.5 and 6) that the optimal choice of the centrality classes in case of definition of the centrality by the impact parameter is dictated by the same properties of the diagrams as in the previous case, namely the plateau starting point.

The diagrams (Fig.4 and 5, 6) show that in case of centrality definition by the impact parameter (that seemingly is more natural because the impact parameter is geometrical characteristic of a collision) the picture qualitatively is no different than the previous one.

4.2 Pb-Pb 17.3 GeV

Similar analysis was made for lower range of energy, namely SPS-energies. Qualitatively picture is the same, therefore we think that it is relevant to show only diagrams (see Fig.7, 8, 9, 10, 11, 12).

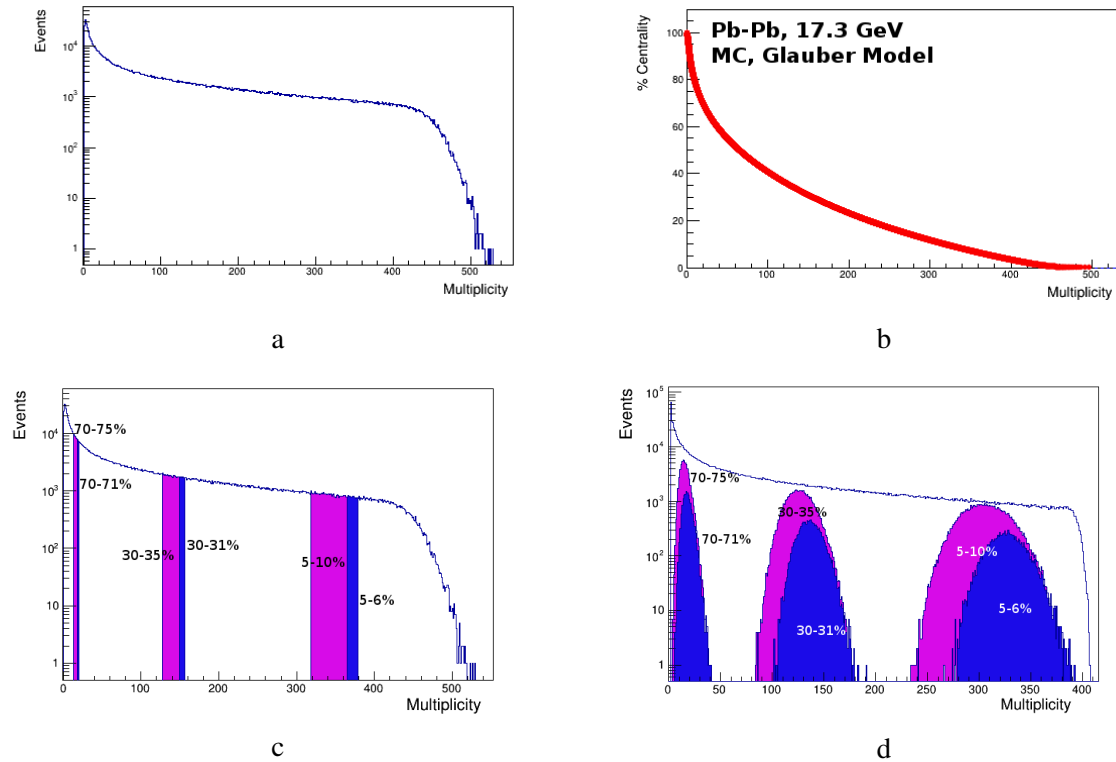


Figure 7: Procedure of centrality classes determination by multiplicity of charge particle for Pb-Pb collision at 17.3 GeV. **a:** Charge particle multiplicity distribution, **b:** Correspondence of the centrality percentile and charge particle multiplicity, **c:** Charge particle multiplicity distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%), semi-peripheral (30%) and peripheral (70%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by multiplicity.

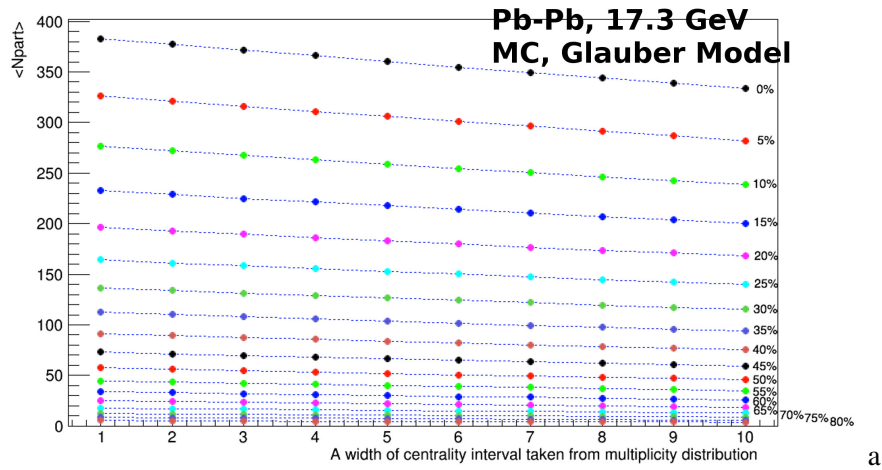


Figure 8: Pb-Pb 17.3GeV, Dependence of an average wounded nucleons from the centrality classes interval determined by multiplicity with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

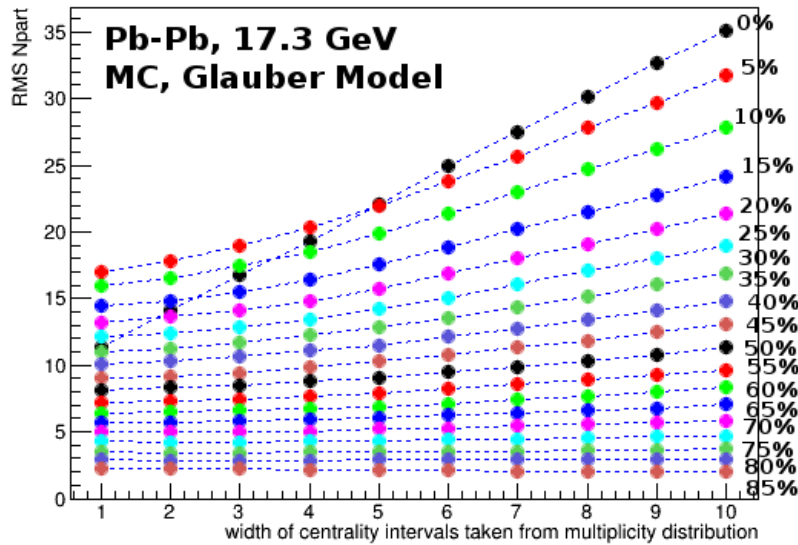


Figure 9: Pb-Pb 17.3GeV, RMS of wounded nucleons in a different centrality classes determined by multiplicity with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

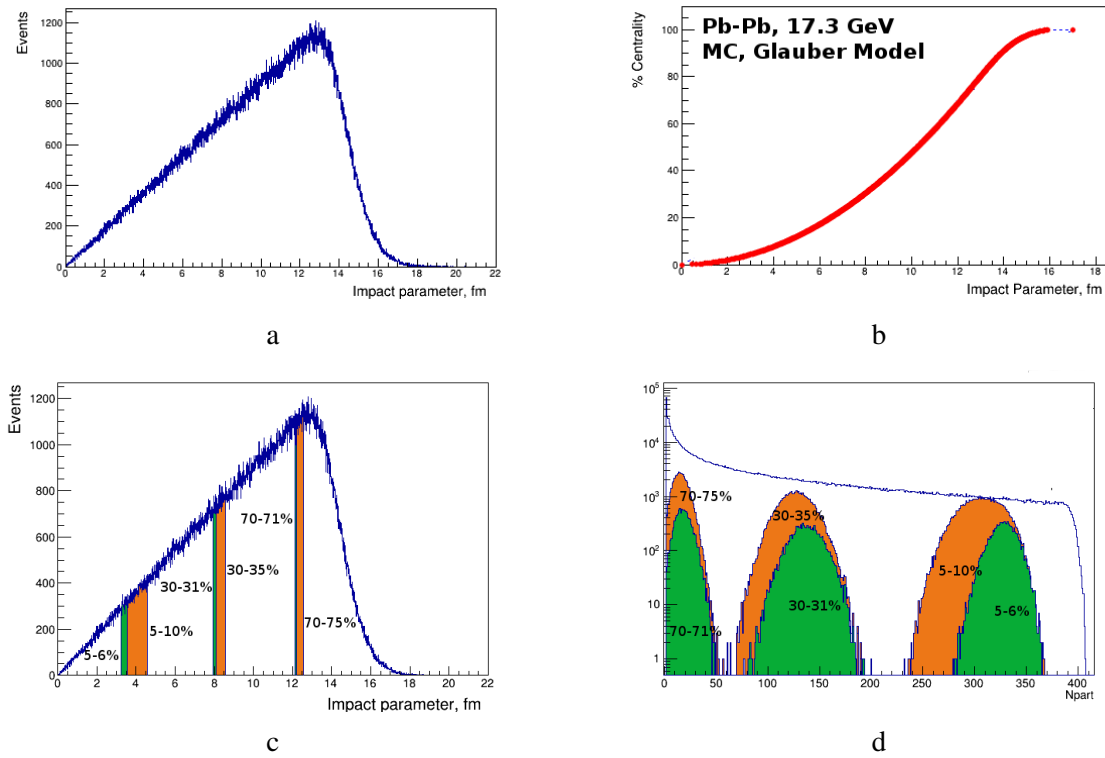


Figure 10: Procedure of centrality classes determination by impact parameter for Pb-Pb collision at 17.3 GeV. **a:** Impact parameter distribution, **b:** Correspondence of the centrality percentile and impact parameter, **c:** Impact parameter distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%), semi-peripheral (30%) and peripheral (70%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by impact parameter.

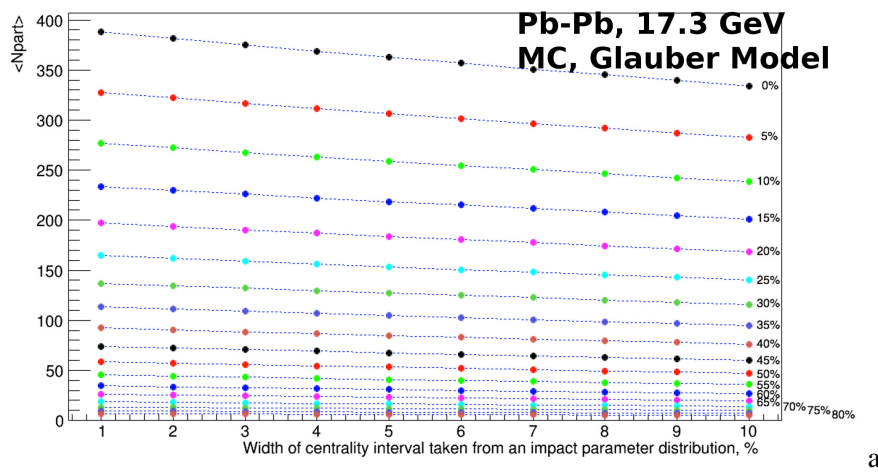


Figure 11: Pb-Pb, 17.3 GeV, Dependence of an average wounded nucleons from the centrality classes interval determined by impact parameter with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0% to peripheral collisions 85%.

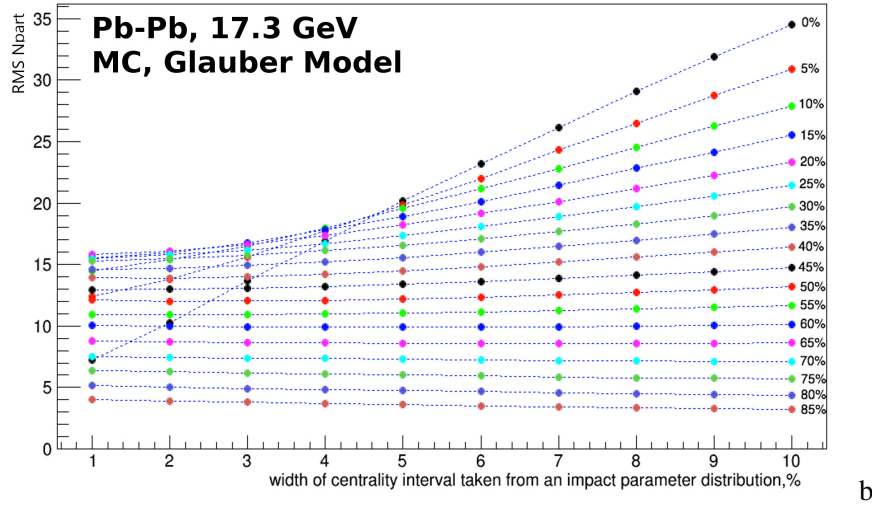


Figure 12: Pb-Pb, 17.3 GeV, RMS of wounded nucleons in a different centrality classes determined by impact parameter with the centrality interval width 1%, 2%, 3%...10% and lower bound from central 0 % to peripheral collisions 85 %.

4.3 $Be^7 - Be^9$ 16.8 GeV

Analysis was carried out not only for heavy-ion collisions but also for light-ion collisions. As the light ions we chose $Be^7 - Be^9$ because the analysis of these collisions can be implemented for the NA61/SHINE experiment. Data obtained by the generator based on Glauber model was used for modeling such collisions. Procedure of our method was repeated (see Section 3). The diagram of the events number distribution in the number of wounded nucleons is of a great interest (see Fig.13, d). As can be seen (see Fig.13) independently of the choice of the centrality intervals (their bounds and widths) the variance of the number of wounded nucleons is so great that the very concept of the centrality class can not be applied effectively.

We tried to make the same analysis for determination of the centrality by the impact parameter (see Fig.14). Results are not much different from the previous.

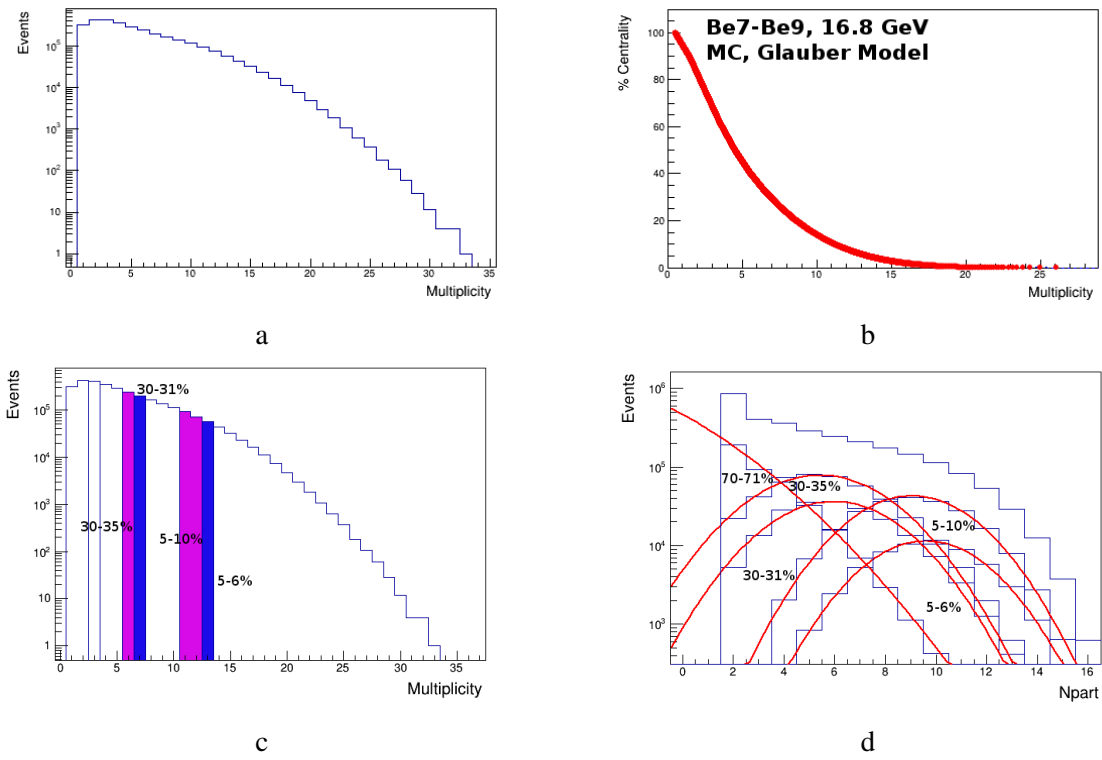


Figure 13: Procedure of centrality classes determination by multiplicity of charge particle for $Be^7 - Be^9$ 16.8 GeV. **a:** Charge particle multiplicity distribution, **b:** Correspondence of the centrality percentile and charge particle multiplicity, **c:** Charge particle multiplicity distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%) and semi-peripheral (30%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by multiplicity.

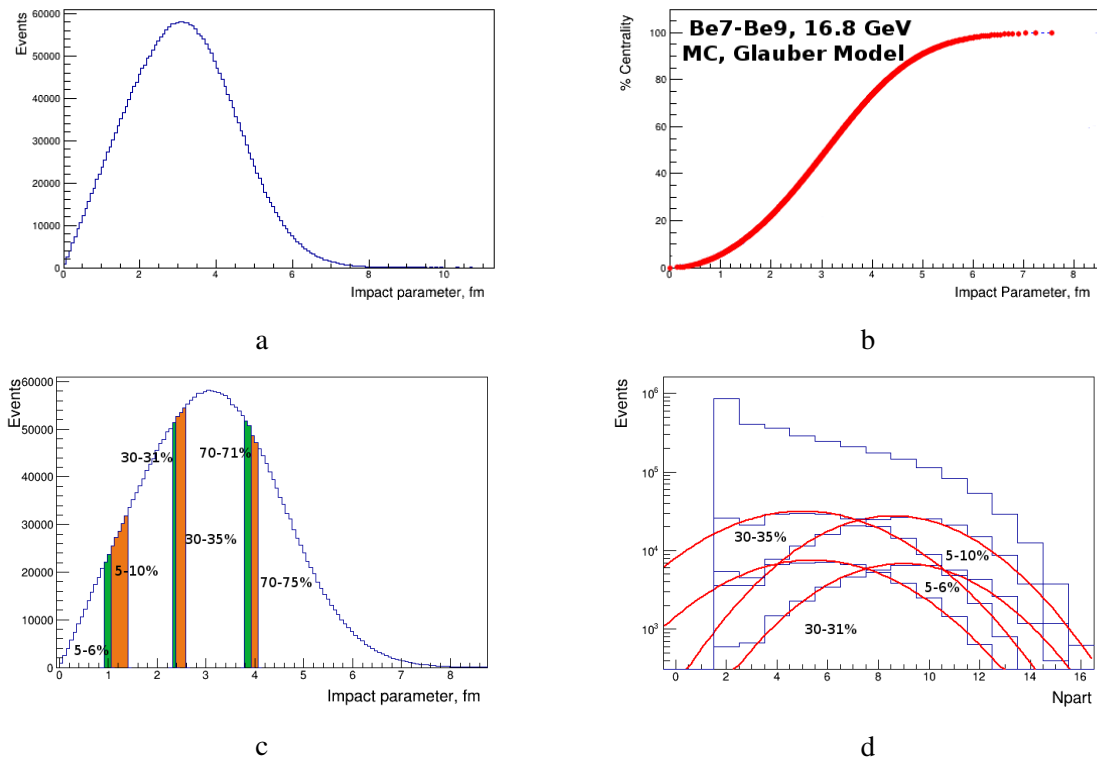


Figure 14: Procedure of centrality classes determination by impact parameter for $Be^7 - Be^9$ 16.8 GeV. **a:** Impact parameter distribution, **b:** Correspondence of the centrality percentile and impact parameter, **c:** Impact parameter distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%) and semi-peripheral (30%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by impact parameter.

4.4 p-Pb 5.02 TeV

The third case that should be considered is a collision of a heavy and a light particles. Collision of p and Pb is an actual topic of discussions. This experiment is actively carrying out in CERN at the energy 5.02 TeV. HIJING generator (to see about its configuration in more details see Appendix A) was used for modeling of this collision. Analysis was carried out with according to the method described above (see Section 3).

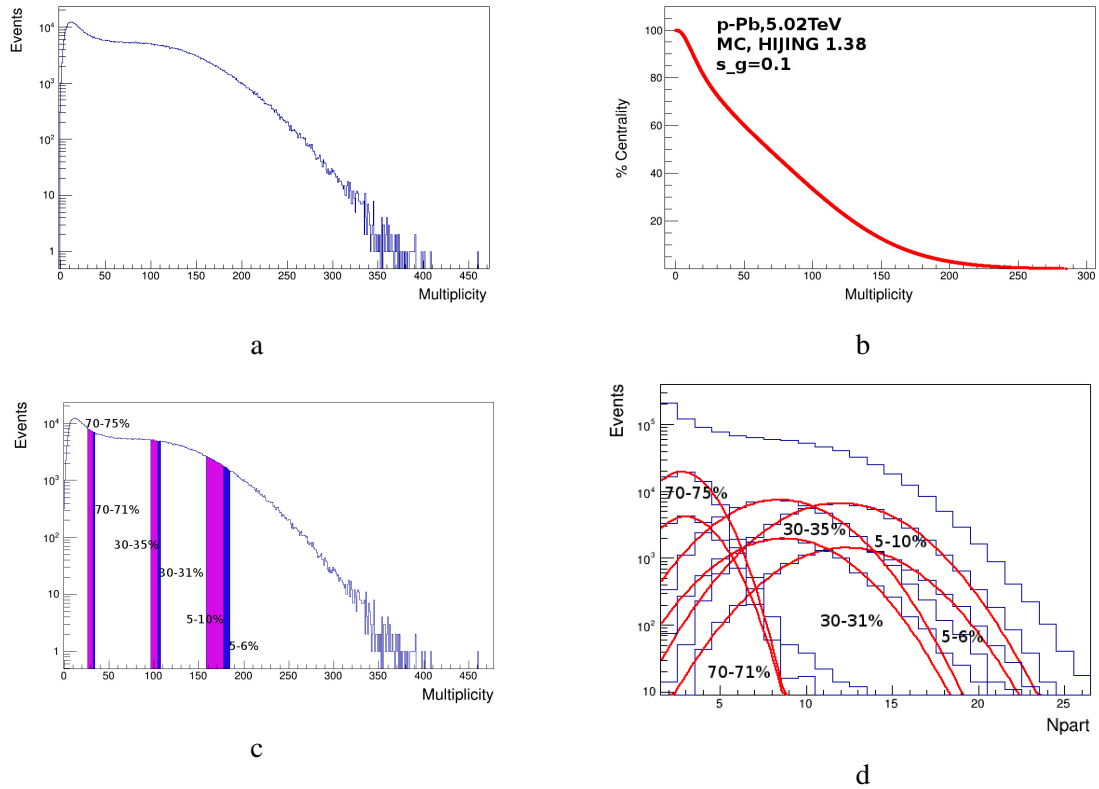


Figure 15: Procedure of centrality classes determination by multiplicity of charge particle for p-Pb collision at 5.02 TeV. **a:** Charge particle multiplicity distribution, **b:** Correspondence of the centrality percentile and charge particle multiplicity, **c:** Charge particle multiplicity distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%), semi-peripheral (30%) and peripheral (70%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by multiplicity.

As it can be seen from the plots (see Fig.15) in this case an error in the average number of wounded nucleons is so great that the division into centrality classes can not be used effectively. Analysis of the centrality dependence on the impact parameter (see Fig.16) showed the same results.

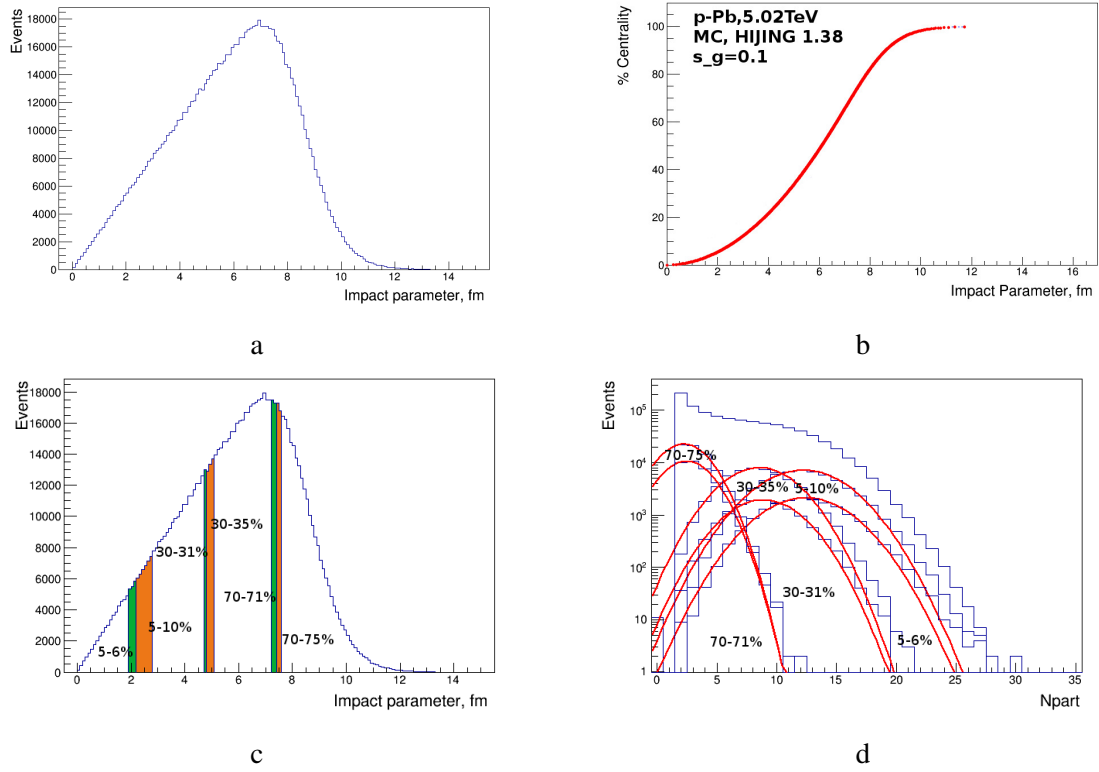


Figure 16: Procedure of centrality classes determination by impact parameter for p-Pb 5.02 TeV. **a:** Impact parameter distribution, **b:** Correspondence of the centrality percentile and impact parameter, **c:** Impact parameter distribution with values of centrality (just for demonstration) with different intervals widths of 1% and 5% with the same lower bound for central (5%) and semi-peripheral (30%) collisions, **d:** Number of nucleons-participants distribution in different centrality classes determined by impact parameter.

5. Results of the analysis

We think that the method described in this paper is effective enough to consequentially and precisely determine the optimal centrality classes. With help of this method we obtained quantitative results that are important for analysis of fluctuations and correlations.

Analysis allows us to be certain that for the collisions with few participants division into centrality classes is inappropriate. The only events that can be differed in such case are the most central and the most peripheral collisions. For collisions with greater number of participants it is reasonable to use the centrality classes. And the choice of the classes should be carried out using the mentioned procedure and taking into account the features and details described above, namely: the greater is number of participants the more detailed division can be done, the less is the number of participants, the less reasonable specification appears to be. On the example of peripheral Pb-Pb collisions with appropriately small number of participants the effects are the same that in p-Pb and $Be^7 - Be^9$ collisions. The method introduced in the paper allows us to consider the question of events division into centrality classes consequentially using the RMS of the number of participants for different centrality (the plateau starting points, see Fig.2 and Fig.5). We plan to make this process automatic to have a mechanism that can obtain the optimal choice of centrality classes based

on modeling of the collisions. We recommend to use the method to choose the centrality classes in experiments. We think that the qualitative picture that was obtained allows to significantly decrease the error of measurements. That, in turn, is critical in experiments aimed at finding the effects related to fluctuations of those measurements.

A. Appendix: Configuration of HIJING

Dependence of the generated data on such a natural parameter as the gluon shadowing was used for configuration of the HIJING generator. The difference in parton distribution function (PDF) for a nucleus and a proton is explained by introduction of this parameter. It has been experimentally determined that PDF for nucleus and for proton is different and leads to decrease of the cross-section in case of small x . The same effects were considered in different models, taking into account energy conservation in the elementary nuclei-nuclei interactions [30], [31], [32], [33]. Without gluon shadowing nuclear interactions are meant to be independent (as it is in the Glauber model). The choice of this parameter allows us to model data that describes the experiment more precisely.

As it can be seen from the plots (Fig.18) distribution of the events number for p-Pb collision at 5.02 TeV both in multiplicity of charged particles and in the number of nucleons-participants differs greatly with the changing of this parameter. Using of HIJING generator seems to be most expedient for us but the choice of the generator does not affect the method used in this paper. To fix the parameter of gluon shadowing the dependence of the normalized charged particles multiplicity distribution in the rapidity window $(-2, 2)$ on the gluon shadowing parameter s_g (see Fig.17, left [34]) was compared to the experimental data obtained by the ALICE collaboration (see Fig.17, right). For the demonstration of the method with using of HIJING we chose the value of gluon shadowing parameter $s_g = 0.1$, that gives an appropriate agreement with the experiment.

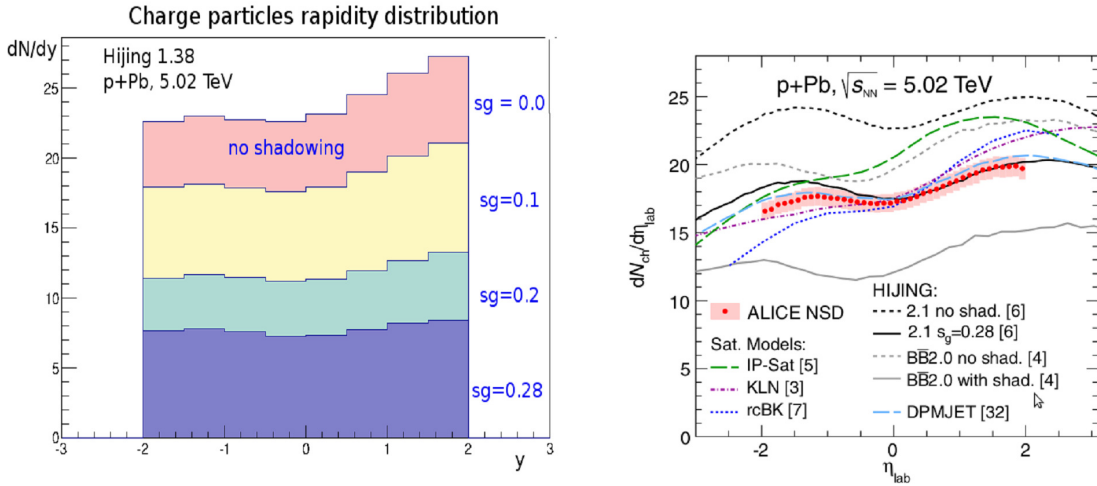


Figure 17: (left): the normalized multiplicity of charged particles in the rapidity window $(-2, 2)$ with the different gluon shadowing parameter s_g in comparison with the experimental data obtained by the ALICE Collaboration [34] (right).

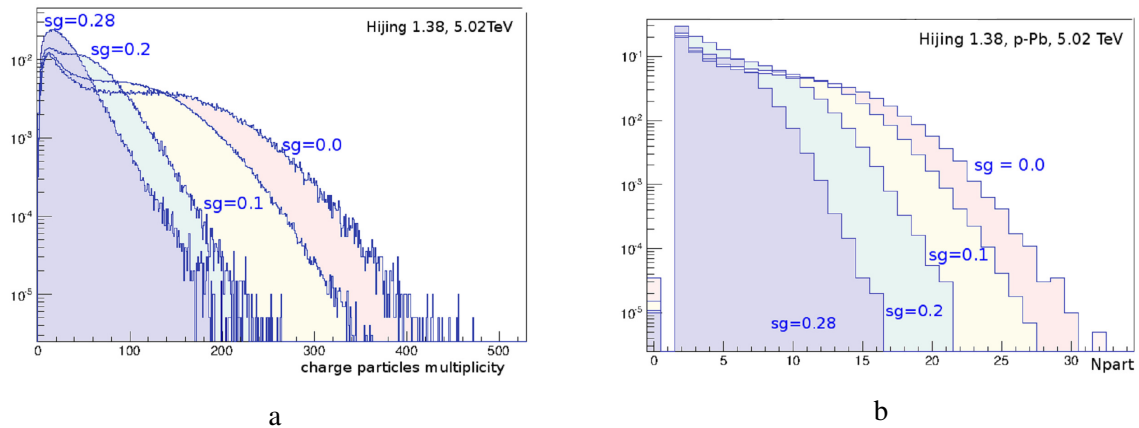


Figure 18: p-Pb, 5.02TeV, a:Charge particle multiplicity distribution with the different gluon shadowing parameter, **b** Nucleons-participants number distribution with the different gluon shadowing parameter

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