PROCEEDINGS OF SCIENCE

From pion formfactor to exclusive Drell-Yan processes

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The factorized amplitude for pion formfactor at high Q^2 was the first case of QCD application for hard exclusive processes. It may be naturally extended to describe the exclusive Drell-Yan process by substituting of one or two pion Distribution Amplitudes by Generalized Parton Distributions. Two respective mechanisms for the lepton pair production in exclusive proton-meson collisions are considered and compared. Amplitudes and differential cross sections are calculated.

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1. Introduction

The strength of the interaction of quarks and gluons depends on the distance between them. If the distance increases, the interaction strength rises, and vice versa. These effects are called “confinement” and “asymptotic freedom”, respectively. At high energies and momentum transfers the coupling constant is small, which results in the possibility of using perturbation theory.

The QCD factorization, leading to separation of small and large distances contributions, was first proved for inclusive processes, the first of which was the well-known Deep-Inelastic Scattering. These processes appear to be more suitable for theoretical analysis, relying on the Operator Products Expansion. At the same time, there are also exclusive processes in which final states are detected. Exclusive processes are rare and occupy the small amount of phase space. However, they are clean processes due to the absence of background appearing in hadrons-initiated inclusive reactions.

In the analysis of exclusive processes the method laid down by A.V. Efremov (A.V.E.) and I.F. Ginzburg. [1] appears to be indispensable. The analysis of Feynman diagrams asymptotics paved the way to QCD factorization [2]. It was successfully applied [3] by A.V.E. in collaboration with his student A.V. Radyushkin to describe in QCD [**?**] the pion formfactor being one of the most simple exclusive amplitudes.

The expression appeared there contained the square of the inverse moment of pion distribution amplitude (DA)

$$\int_0^1 \frac{dx}{x}\phi(x).$$

The similar feature happened to be common to some other exclusive processes containing two non-perturbative ingredients.

The first case comes to the consideration when one of the DA’s is substituted by the Generalized Parton Distribution (GPD) [5]. In the case of space-like photon in the initial state it corresponds to exclusive electroproduction [6] (see also [7, 8]). Passing to the time-like photon in the final state brings into consideration the first, "classical", mechanism of exclusive Drell-Yan process.

Another mechanism comes into consideration when second DA is also substituted by GPD [9]. One cannot directly apply collinear factorization here and analytic continuation was propsed instead. Later the analytical properties of probability amplitudes were considered in [10, 11].

In what follows we present the study [12] the hard exclusive collision $p\pi^- \to p\pi^-\gamma^*$, being the exclusive analog of the Drell-Yan process. For simplicity, the (unpolarized) virtual photon is considered instead of the pair of leptons, which corresponds to averaging over their direction angles in the center-of-mass frame.

This process is considered in two approaches generalizing the pion formfactor case.. The first one corresponds to the inversion of the "classical" electroproduction reaction [6], when the proton is described by the GPD and the pion by the Distribution Amplitude (DA). The second corresponds to the description of both pion and proton by their GPDs.

2. Classical mechanism

The diagrams are shown in Fig. 1. In order to have the pion in the final state, one should deal here with the $p \to p\pi^-$ GPD [7] (Transitional Distribution Amplitudes). It is natural to normalize them, in the spirit of chiral invariance, to the inverse pion coupling constant, canceled with the same coupling in the definition of pion distribution amplitude. The momenta of the hadrons are assumed to be lightlike.

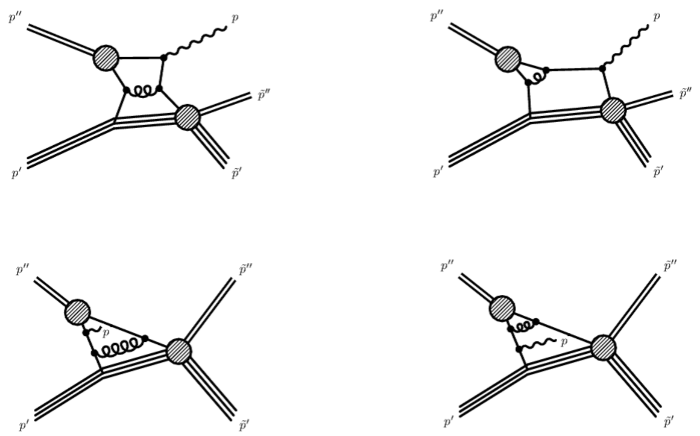

Figure 1: Classical mechanism

Fermion lines correspond to either quarks or antiquarks. Eight diagrams should be taken into account.

The amplitude in the Born approximation is

$$S_{p\pi^{-}1} = -i\frac{(2\pi)^4}{2N_c^2}\frac{eg^2_{(s)}}{(2V)^{\frac{5}{2}}\sqrt{\varepsilon'\varepsilon''\tilde{\varepsilon}'\tilde{\varepsilon}''\varepsilon}}\cdot e^*_\lambda\cdot\frac{1}{(\bar{P},p'')}\cdot\left(2\bar{P}^\lambda-\frac{p''^\lambda}{\xi}\right)\cdot I_1\cdot\delta(p'+p''-\tilde{p}'-\tilde{p}''-q)\,, \tag{2.1}$$

where

$$I_1 = \int_{-1}^{1}\mathrm{d}x\int_0^1\frac{e_d H_{p\pi^-}(x)\Phi(y)}{(x+\xi)y+i\varepsilon_g}\mathrm{d}y+\int_{-1}^{1}\mathrm{d}x\int_0^1\frac{e_u H_{p\pi^-}(x)\Phi(y)}{(x-\xi)y-i\varepsilon_g}\mathrm{d}y; \tag{2.2}$$

p', p'' are the initial momenta; $\tilde{p}', \tilde{p}''$ are the final momenta; $\bar{P}'$ is the average momentum; $\xi_1\in[0,1]$; $e_u=\frac{2}{3}$; $e_d=-\frac{1}{3}$; $H_{p\pi^-}(x)$ is Transitional Distribution Amplitudes, determined as

$$H_{p\pi^-}(x) = \begin{cases} H^{(ud)}_{p\pi^-}(x) & x>0; \\ -H^{(\bar{d}\bar{u})}_{p\pi^-}(-x) & x<0. \end{cases} \tag{2.3}$$

and $\Phi(y)$ is the distribution amplitude of quarks and antiquarks in the pion. In this expression the symmetry of $\Phi(y)$ was taken into consideration.

3. GPD×GPD mechanism

The relevant contribution can be represented as the sum of some subprocesses, shown in Fig. 2.

Fermion lines also correspond to quarks or antiquarks. Thirty-two diagrams should be taken into account (we take into consideration two types of quarks). The amplitude in the Born approximation is

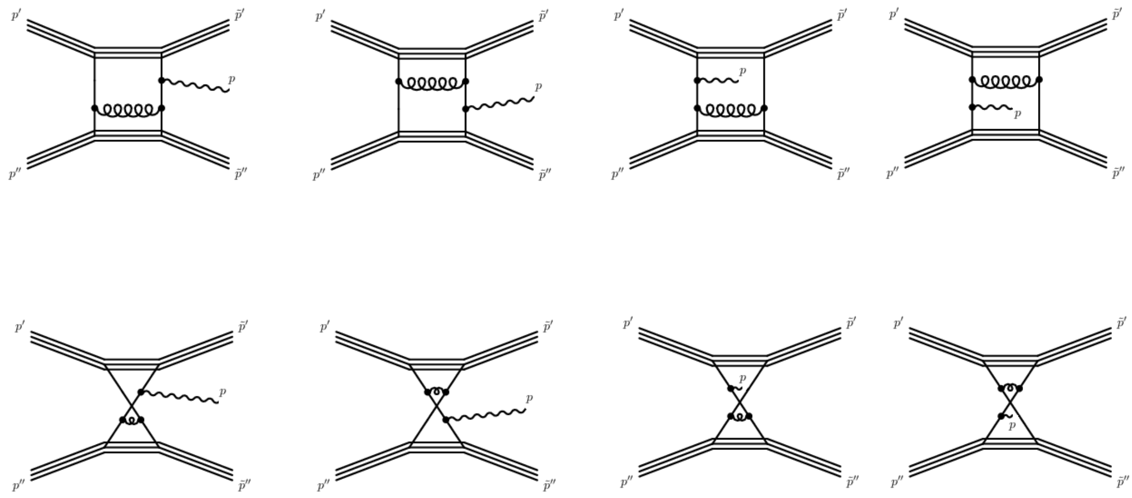

Figure 2: GPD×GPD mechanism

$$S_{p\pi^-2} = -i\frac{(2\pi)^4}{2N_c^2}\frac{eg_{(s)}^2}{(2V)^{\frac{5}{2}}\sqrt{\varepsilon'\varepsilon''\tilde{\varepsilon}'\tilde{\varepsilon}''\varepsilon}} \cdot e_\lambda^* \cdot \frac{1}{(\bar{P}',\bar{P}'')} \cdot \left(\frac{\bar{P}'^\lambda}{\xi_2} - \frac{\bar{P}''^\lambda}{\xi_1}\right) \cdot I_2 \cdot \delta(p'+p''-\tilde{p}'-\tilde{p}''-q)\ , \tag{3.1}$$

where

$$I_2 = \int_{-1}^{1} \frac{e_u H_p^{(u)}(x) H_{\pi^-}^{(u)}(y)}{(x+\xi_1)(y-\xi_2)-i\varepsilon_g}\mathrm{d}x\mathrm{d}y - \int_{-1}^{1} \frac{e_u H_p^{(u)}(x) H_{\pi^-}^{(u)}(y)}{(x-\xi_1)(y+\xi_2)-i\varepsilon_g}\mathrm{d}x\mathrm{d}y$$

$$+\int_{-1}^{1} \frac{e_d H_p^{(d)}(x) H_{\pi^-}^{(d)}(y)}{(x+\xi_1)(y-\xi_2)-i\varepsilon_g}\mathrm{d}x\mathrm{d}y - \int_{-1}^{1} \frac{e_d H_p^{(d)}(x) H_{\pi^-}^{(d)}(y)}{(x-\xi_1)(y+\xi_2)-i\varepsilon_g}\mathrm{d}x\mathrm{d}y; \tag{3.2}$$

$$H_p^{(q)}(x) = \begin{cases} H_p^{(q)}(x) & x>0; \\ -H_p^{(\bar{q})}(-x) & x<0; \end{cases} \tag{3.3}$$

$$H_{\pi^-}^{(q)}(x) = \begin{cases} H_{\pi^-}^{(q)}(x) & x>0; \\ -H_{\pi^-}^{(\bar{q})}(-x) & x<0, \end{cases} \tag{3.4}$$

where q corresponds to u- or d-quarks.

4. Differential cross section

The differential cross section of the exclusive Drell-Yan process is

$$\frac{d\sigma}{\mathrm{d}^3\tilde{p}'\mathrm{d}^3\tilde{p}''} = \frac{\alpha_{(em)}\alpha_{(s)}^2}{2^5 N_c^4 \pi^2 \tilde{\varepsilon}'\tilde{\varepsilon}''} \cdot \frac{1}{(p',p'')^2} \cdot$$

$$\cdot \left[\frac{8s^2}{s_1 s_2}|I_1|^2 + \frac{8s}{s_1+s_2-s}|I_2|^2 + 4s\left(\frac{1}{s_2} + \frac{s}{s_1(s_1+s_2-s)}\right)(I_1^* I_2 + I_2^* I_1)\right] \cdot \delta(q^2 - m_\gamma^2),$$

where $s = (p' + p'')^2$; $s_1 = (\tilde{p}' + q)^2$; $s_2 = (\tilde{p}'' + q)^2$; $q = p' + p'' - \tilde{p}' - \tilde{p}''$; m_γ is the virtual photon mass.

As it can be seen, the first two terms in this expression are symmetric with respect to the interchange of s_1 and s_2, while the interference term is not. This is because the exchange of the final hadron states in the GPD^2-process should be accompanied by the exchange of the initial states with corresponding redefinition of the variables due to the difference between proton and pion. However, the exchange of the final hadron states in the classical subprocess does not require any change in the initial state. This is the reason of the absence of the symmetry of the interference term.

The integrals in I_2 have an infrared divergence. However, the real part of these integrals can be calculated as a principal value. It is an interesting possibility to construct the variable sensitive to real part only.

One should take into account the purely electromagnetic process, when the lepton pair is produced by the collisions of two (quasi-real) photons which are emitted by the colliding hadrons and this emission is described by hadron (charge) formfactors. Note that for proton-proton collisions at LHC the respective muon pair production cross-sections are very large. The amplitude is pure real and their interference with DY amplitudes selects the real parts of the latter. Note also that the C-parity of the pair in this electromagnetic process is positive, while in the DY processes it is clearly negative. Due to the different C-parities of the interfering processes, the resulting interference term in the cross-section should integrate to zero when averaged over the lepton emission angles and will be manifested in a charge asymmetry which may be used to select this contribution.

We may also consider this real part as the whole integral in the unphysical region. There is some reason to believe [9] in the possibility of analytical continuation to the physical region. At the same time, the integrals in I_1 can be calculated without such assumptions.

5. Conclusions

In this paper, we considered two mechanisms for an exclusive process of proton-pion collision at leading order. We got the cross section of the whole process. The interference of the subprocesses was taken into account. The explanation of the symmetry properties of the interference term was given.

The integrals of the Generalized Parton Distributions in one of the mechanisms have infrared divergences. However, there are reasons to believe in the possibility of calculation of them in assumption of the absence of the imaginary part with subsequent analytical continuation to the physical region. Also, there should be the specific charge asymmetry due to interference with the purely electromagnetic process where only the infrared-stable real part of the amplitude contributes.

Both mechanisms are the straightforward generalizations of the QCD factorization approach to pion formfactor led down by A.V.E. 40 years from now.

We would like to dedicate this work to Anatoli Vasil'evich Efremov on the occasion of his 80th Birthday.

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