Femtoscopic correlations of two identical particles with nonzero spin in the model of one-particle multipole sources

Valery V. Lyuboshitz
Joint Institute for Nuclear Research (Dubna, Russia)
E-mail: Valery.Lyuboshitz@jinr.ru

Vladimir L. Lyuboshitz
Joint Institute for Nuclear Research (Dubna, Russia)

The process of emission of two identical particles with nonzero spin and different helicities is theoretically investigated within the model of one-particle multipole sources. Taking into account the unitarity of the finite rotation matrix and symmetry relations for $d$-functions, the general expression for probability of emission of two identical particles by two multipole sources with angular momentum $J$, averaged over the projections of angular momentum and over the space-time dimensions of the generation region, has been obtained. For the case of unpolarized particles, the formula for two-particle correlation function at sufficiently large 4-momentum difference $q$ is derived by performing the additional averaging over helicities. For particles with nonzero mass, this formula is simplified at the zero angle $\beta$ between the particle momenta, and also at $J = S$. The special cases of emission of two unpolarized photons by dipole and quadrupole sources, and emission of two "left" neutrinos ("right" antineutrinos) by sources with arbitrary $J$ have been also considered, and the respective explicit expressions for the correlation function are obtained.
1. Probability of emission of two identical particles with nonzero spin by two multipole sources

In the framework of the model of independent sources [1] with the angular momentum $J$ and the projections of angular momentum onto the coordinate axis $z$, equaling $M$ and $M'$, the amplitude of emission of two identical particles with the momentum $\mathbf{p}_1$, helicity $\lambda_1$ and momentum $\mathbf{p}_2$, helicity $\lambda_2$ has the following structure:

$$A_{MM'}(\mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2) =$$

$$= D_{\lambda_1 M}(\mathbf{n}_1) D_{\lambda_2 M'}(\mathbf{n}_2) e^{i P_{1z} x_1} e^{i P_{2z} x_2} + D_{\lambda_2 M}(\mathbf{n}_2) D_{\lambda_1 M'}(\mathbf{n}_1) e^{i P_{1z} x_2} e^{i P_{2z} x_1}, \quad (1.1)$$

where $x_1$ and $x_2$ are the space-time coordinates of two multipole sources, $p_{1z} x_1 = E_1 t_1 - \mathbf{p}_1 \cdot \mathbf{x}_1$, $p_{2z} x_2 = E_2 t_2 - \mathbf{p}_2 \cdot \mathbf{x}_2$,

$$D_{\lambda_1 M}(\mathbf{n}_1) = D_{\lambda_1 M}(0, \theta_1, \phi_1) = \left( d_J(0, \theta_1, \phi_1) e^{i M \phi_1} \right)_{\lambda_1 M},$$

$$D_{\lambda_2 M'}(\mathbf{n}_2) = D_{\lambda_2 M'}(0, \theta_2, \phi_2) = \left( d_J(0, \theta_2, \phi_2) e^{i M' \phi_2} \right)_{\lambda_2 M'}, \quad (1.2)$$

are elements of the finite rotation matrix corresponding to the angular momentum $J$, $\mathbf{n}_1 = \mathbf{p}_1 / |\mathbf{p}_1|$, $\mathbf{n}_2 = \mathbf{p}_2 / |\mathbf{p}_2|$, $\theta_1, \theta_2$ and $\phi_1, \phi_2$ — polar and azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively.

Thus, in accordance with Eq. (1.1), the probability of emission of two identical particles with spin $S$, respective 4-momenta $p_1, p_2$ and helicities $\lambda_1, \lambda_2$ by two multipole sources with the angular momentum $J$ and projections $M, M'$ of angular momentum onto the axis $z$ amounts to:

$$W_{MM'}(p_1, \lambda_1; p_2, \lambda_2) = \left| D_{\lambda_1 M}(\mathbf{n}_1) \right|^2 \left| D_{\lambda_2 M'}(\mathbf{n}_2) \right|^2 + \left| D_{\lambda_2 M}(\mathbf{n}_2) \right|^2 \left| D_{\lambda_1 M'}(\mathbf{n}_1) \right|^2 +$$

$$+ 2 (-1)^{S} \text{Re} \left( D_{\lambda_1 M}(\mathbf{n}_1) D_{\lambda_2 M'}(\mathbf{n}_2) D_{\lambda_2 M}(\mathbf{n}_2) D_{\lambda_1 M'}(\mathbf{n}_1) \right) \cos(q x), \quad (1.3)$$

where $q = p_1 - p_2$ is the difference of 4-momenta of two identical particles and $x = x_1 - x_2$ is the difference of 4-coordinates of two one-particle multipole sources.

Now let us average this expression over the angular momentum projections $M, M'$ and over the space-time dimensions of the emission region. In doing so, we take into account that, due to the unitarity of the finite rotation matrix, the following relations hold:
Let us emphasize that the quantity \( r = (d_{\lambda_1 \lambda_2}^{(J)}(\beta))^2 \) has the meaning of the degree of non-orthogonality (non-distinguishability) of particle states with different helicities with respect to the momenta, the angle between which equals \( \beta = \theta_1 - \theta_2 : \langle \lambda_1 | \lambda_2 \rangle \neq 0 \).

2. **Correlation function for two unpolarized particles in the model of one-particle multipole sources**

If the emitted identical particles with the momenta \( \mathbf{p}_1, \mathbf{p}_2 \) are unpolarized, then – after averaging over all the \( (2S + 1) \) values of helicity allowed at spin \( S \) – we obtain:

\[
\begin{align*}
\sum_{M=-J}^{J} |D_{\lambda_1 M}(\mathbf{n}_1)|^2 &= \sum_{M'=-J}^{J} |D_{\lambda_2 M'}(\mathbf{n}_2)|^2 = \\
&= \sum_{M=-J}^{J} |D_{\lambda_2 M}(\mathbf{n}_2)|^2 = \sum_{M'=-J}^{J} |D_{\lambda_1 M'}(\mathbf{n}_1)|^2 = 1. \quad (1.4)
\end{align*}
\]

Using the well-known symmetry relation \( (d_{\lambda}^{(J)}(\beta))_{\lambda_1 \lambda_2} = (d_{\lambda}^{(J)}(-\beta))_{\lambda_2 \lambda_1} \) [2], we come to the result:

\[
\overline{W}_{MM}(p_1, \lambda_1; p_2, \lambda_2) = \frac{1}{(2J+1)^2} \left( 2 + 2 \left( d_{\lambda_1 \lambda_2}^{(J)}(\beta) \right)^2 (-1)^{2S} \langle \cos(qx) \rangle \right). \quad (1.7)
\]
Femtoscopic correlations of two identical particles with nonzero spin...

Valery V. Lyuboshitz

\[ \overline{W}(q) = \left( 2(2S + 1)^2 + (-1)^{2S} \sum_{\lambda_1 = -S}^{S} \sum_{\lambda_2 = -S}^{S} |d^{(J)}_{\lambda_1 \lambda_2}(\beta)|^2 \langle \cos(qx) \rangle \right) \frac{1}{(2J + 1)^2} \frac{1}{(2S + 1)^2}. \]  

(2.1)

At sufficiently large momentum differences \( q \) the correlation function, normalized by unity, will take the form:

\[ R(q) = 1 + \frac{(-1)^{2S}}{(2S + 1)^2} \sum_{\lambda_1 = -S}^{S} \sum_{\lambda_2 = -S}^{S} |d^{(J)}_{\lambda_1 \lambda_2}(\beta)|^2 \langle \cos(qx) \rangle. \]  

(2.2)

In particular, if \( \beta = 0 \), then we have \( d^{(J)}_{\lambda_1 \lambda_2}(0) = \delta_{\lambda_1 \lambda_2} \), and formula (2.2) is simplified:

\[ R(q) = 1 + \frac{(-1)^{2S}}{(2S + 1)} \langle \cos(qx) \rangle. \]  

(2.3)

Besides, taking into account the unitarity of the matrix \( d^{(J)}_{\lambda_1 \lambda_2}(\beta) \), it is easy to see from Eq. (2.2) that at \( J = S \) formula (2.3) is valid at any angles between the momenta \( p_1 \) and \( p_2 \). Let us stress that Eq. (2.3) is related to particles with nonzero mass.

3. Special cases of pair correlations of two unpolarized photons and two neutrinos

In the case of emission of two unpolarized photons, when the mass equals zero, spin \( S = 1 \) and each of the helicities \( \lambda_1, \lambda_2 \) takes only two \( (2S) \) values: \(-1\) and \( 1 \), irrespective of the momentum direction, the correlation function for dipole sources has the form [3]:

\[ R(q) = 1 + \frac{1}{4} \left[ (d^{(1)}_{11}(\beta))^2 + (d^{(1)}_{1,-1}(\beta))^2 + (d^{(1)}_{-1,1}(\beta))^2 + (d^{(1)}_{-1,-1}(\beta))^2 \right] \langle \cos(qx) \rangle. \]  

(3.1)

Taking into account the equalities:

\[ d^{(1)}_{11}(\beta) = d^{(1)}_{1,-1}(\beta) = \frac{1 + \cos \beta}{2}, \quad d^{(1)}_{-1,1}(\beta) = d^{(1)}_{-1,-1}(\beta) = \frac{1 - \cos \beta}{2}, \]  

(3.2)

we find:

\[ R(q) = 1 + \frac{1}{4} (1 + \cos^2 \beta) \langle \cos(qx) \rangle. \]  

(3.3)

At very small angles between the photon momenta \( \beta \ll 1 \) we obtain:
Femtoscopic correlations of two identical particles with nonzero spin...

Valery V. Lyuboshitz

\[ R(q) = 1 + \frac{1}{2} \langle \cos(qx) \rangle. \] (3.4)

For the case of quadrupole sources, the correlation function is as follows:

\[ R(q) = 1 + \frac{1}{4} \left[ (d^{(2)}_{11}(\beta))^2 + (d^{(2)}_{-1,1}(\beta))^2 + (d^{(2)}_{-1,-1}(\beta))^2 + (d^{(2)}_{1,-1,1}(\beta))^2 \right] \langle \cos(qx) \rangle. \] (3.5)

Using the equalities:

\[ d^{(2)}_{11}(\beta) = d^{(2)}_{-1,1}(\beta) = \frac{1 + \cos\beta}{2} (2\cos\beta - 1), \] (3.6)

\[ d^{(2)}_{1,-1,1}(\beta) = d^{(2)}_{-1,1}(\beta) = \frac{1 - \cos\beta}{2} (2\cos\beta + 1), \] (3.7)

we find the correlation function of two unpolarized photons emitted by the quadrupole sources:

\[ R(q) = 1 + \frac{1}{4} \left( 4\cos^2\beta - 3\cos^2\beta + 1 \right) \langle \cos(qx) \rangle. \] (3.8)

At \( \beta \approx 0 \) we have: \( R(q) = 1 + \frac{1}{2} \langle \cos(qx) \rangle \), i.e. here we also obtain the standard formula (see Eq. (3.4)), corresponding to two directions of polarization for each of the photons [3].

Let us consider also the case of emission of two "left" neutrinos (two "right" antineutrinos), with helicity taking only one value \( \lambda_1 = \lambda_2 = \pm \frac{1}{2} \). For this case, the correlation function in the model of multipole sources is as follows:

\[ R(q) = 1 - \left( d^{(J)}_{\pm \frac{1}{2}}(\beta) \right)^2 \langle \cos(qx) \rangle. \] (3.9)

In particular, at \( J = S = \frac{1}{2} \) we obtain:

\[ R(q) = 1 - \cos\beta \frac{\beta}{2} \langle \cos(qx) \rangle. \] (3.10)

In the limit \( \beta \to 0 \) Eq. (3.10) gives:

\[ R(q) = 1 - \langle \cos(qx) \rangle. \] (3.11)

References