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The effect of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ in decays of the Ψ- and Υ-meson families

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The effect of isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ is considered in the analysis of data (from the Argus, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, and BES II Collaborations) on decays of the charmonium – $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $\psi(2S) \to J/\psi\pi\pi$ – and of the bottomonium – $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$, $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$. The analysis, which is aimed at studying the scalar mesons, is performed jointly considering the multi-channel pion-pion scattering, which is described in our model-independent approach based on analyticity and unitarity and using an uniformizing variable method, and the decays under reasonable assumptions. Results of the analysis confirm all our earlier conclusions on the scalar mesons. It is also shown that in the final states of the ψ and Υ-meson family decays (except for the $\pi\pi$ scattering) the contribution of the coupled processes, e.g., $K\overline{K} \to \pi\pi$, is important even if these processes are energetically forbidden. This is in accordance with our previous conclusions on the wide resonances. E.g., a new and natural mechanism of destructive interference in the decay $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ is indicated on the basis of that consideration, which provides a characteristic two-humped shape of the di-pion mass spectrum.

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1. Introduction

In the analysis of data on decays of the Υ-meson family –$\Upsilon(2S) \to \Upsilon(1S)\pi\pi$, $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ – the contribution of multi-channel $\pi\pi$ scattering in the final-state interactions is considered. The analysis, which is aimed at studying the scalar mesons, is performed jointly considering the isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, which are described in our model-independent approach based on analyticity and unitarity and using an uniformization procedure, and the charmonium decay processes $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\pi\pi$.

Importance of studying properties of scalar mesons is related to the obvious fact that a comprehension of these states is necessary in principle for the most profound topics concerning the QCD vacuum, because these sectors affect each other especially strongly due to possible "direct" transitions between them. However, the problem of interpretation of the scalar mesons is faraway to be solved completely [1]. E.g., applying our model-independent method in the 3-channel analyses of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ [2, 3] we have obtained parameters of the $f_0(500)$ and $f_0(1500)$ which differ considerably from results of analyses which utilize other methods (mainly those based on dispersion relations and Breit–Wigner approaches).

To make our approach more convincing, to confirm obtained results and to diminish inherent arbitrariness, we have utilized the derived model-independent amplitudes for multi-channel $\pi\pi$ scattering calculating the contribution of final-state interactions in decays $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\pi\pi$ and $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ [4, 5].

Here we add to the analysis the data on decays $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ from CLEO Collaboration. A distinction of the $\Upsilon(3S)$ decays from the above ones consists in the fact that in this case a phase space cuts off, as if, possible contributions which can interfere destructively with the $\pi\pi$-scattering contribution giving a characteristic 2-humped shape of the di-pion mass distribution in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$. These decays have been studied intensively using various approaches (see, e.g., Ref. [6] and the references therein).

After the experimental evidence for the 2-humped shape of di-pion spectrum Lipkin and Tuan [7] have suggested that the decay $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ proceeds as follows $\Upsilon(3S) \to B\overline{B} \to B^*\overline{B}\pi \to B\overline{B}\pi\pi \to \Upsilon(1S)\pi\pi$. Then in the heavy-quarkonium limit, when neglecting recoil of the final-quarkonium state, they obtained that the amplitude contains a term proportional to $\mathbf{p}_1 * \mathbf{p}_2 \propto \cos\theta_{12}$ (θ_{12} is the angle between the pion three-momenta $\mathbf{p}_1$ and $\mathbf{p}_2$) multiplied by some function of the kinematic invariants. If the latter were a constant, then the distribution $d\Gamma/d\cos\theta_{12} \propto \cos\theta_{12}^2$ (and $d\Gamma/dM_{\pi\pi}$) would have the 2-humped shape. However, this scenario was not tested numerically by fitting to data. It is possible this effect is negligible due to the small coupling of the Υ to the b-flavored sector.

Moxhay in work [8] have suggested that the 2-humped shape is a result of interference between two parts of the decay amplitude. One part, in which the $\pi\pi$ final state interaction is allowed for, is related to a mechanism which acts well in decays $\psi(2S) \to J/\psi(\pi\pi)$ and $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ and, obviously, should operate also in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$. The other part is responsible for the Lipkin – Tuan mechanism. Though there remains nothing from the latter because the author says that the term containing $\mathbf{p}_1 * \mathbf{p}_2$ does not dominate this part of amplitude and “the other tensor structures conspire to give a distribution in $M_{\pi\pi}$ that is more or less flat” – indeed, constant.

It seems, the approach of Ref. [9] resembles the above one. The authors simply supposed that a pion pair is formed in the $\Upsilon(3S)$ decay both as a result of re-scattering and direct production. One can, however, believe that the latter is not reasonable because the pions interact strongly. In present work we show that the indicated effect of destructive interference can be achieved by taking into account our previous conclusions on the wide resonances [5, 11], without any further assumptions.

2. The model-independent amplitudes for multi-channel $\pi\pi$ scattering

Considering the multi-channel $\pi\pi$ scattering, we shall deal with the 3-channel case (namely with $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$) because it was shown [10, 11] that this is a minimal number of channels needed for obtaining correct values of scalar-isoscalar resonance parameters.

- Resonance representations on the 8-sheeted Riemann surface

The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. The matrix elements S_{ij}, where $i, j = 1,2,3$ denote channels, have the right-hand cuts along the real axis of the s complex plane (s is the invariant total energy squared), starting with the channel thresholds s_i ($i = 1,2,3$), and the left-hand cuts related to the crossed channels. The Riemann-surface sheets are numbered according to the signs of analytic continuations of the square roots $\sqrt{s-s_i}$ ($i = 1,2,3$) as follows:

	I	II	III	IV	V	VI	VII	VIII
$\mathrm{Im}\sqrt{s-s_1}$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
$\mathrm{Im}\sqrt{s-s_2}$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
$\mathrm{Im}\sqrt{s-s_3}$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$

An adequate allowance for the Riemann surface structure is performed taking the following uniformizing variable [3] where we have neglected the $\pi\pi$-threshold branch-point and taken into account the $K\overline{K}$- and $\eta\eta$-threshold branch-points and the left-hand branch-point at $s = 0$ related to the crossed channels:

$$w = \frac{\sqrt{(s-s_2)s_3} + \sqrt{(s-s_3)s_2}}{\sqrt{s(s_3-s_2)}} \qquad (s_2 = 4m_K^2 \text{ and } s_3 = 4m_\eta^2). \tag{2.1}$$

Resonance representations on the Riemann surface are obtained using formulas from [3, 12], expressing analytic continuations of the S-matrix elements to all sheets in terms of those on the physical (I) sheet that have only the resonances zeros (beyond the real axis), at least, around the physical region. In the 3-channel case, there are *7 types* of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (**a**); S_{22} – (**b**); S_{33} – (**c**); S_{11} and S_{22} – (**d**); S_{22} and S_{33} – (**e**); S_{11} and S_{33} – (**f**); S_{11}, S_{22} and S_{33} – (**g**). The resonance of every type is represented by the pair of complex-conjugate clusters (of poles and zeros on the Riemann surface). In Fig. 1 we show on the w-plane the representation of resonances of types (**a**), (**b**), (**c**) and (**g**), met in the analysis, in the 3-channel $\pi\pi$-scattering S-matrix element. The Roman numerals indicate images of the corresponding semi-sheets of the Riemann surface. The physical region extends from the point $\pi\pi$ on the imaginary axis (the first $\pi\pi$ threshold corresponding to s_1) along this axis down to the point i on the unit circle (the second threshold corresponding to s_2). Then it extends further along the unit circle clockwise in the 1st quadrant to point 1 on the real axis (the third threshold corresponding to s_3) and then along the real axis to the point $b = (\sqrt{s_2} + \sqrt{s_3})/\sqrt{s_3 - s_2}$ into which $s = \infty$ is mapped on the w-plane. The intervals $(-\infty, -b]$, $[-b^{-1}, b^{-1}]$, $[b, \infty)$ on the real axis (the shaded lines) are the images of the corresponding edges of the left-hand cut of the $\pi\pi$-scattering amplitude.

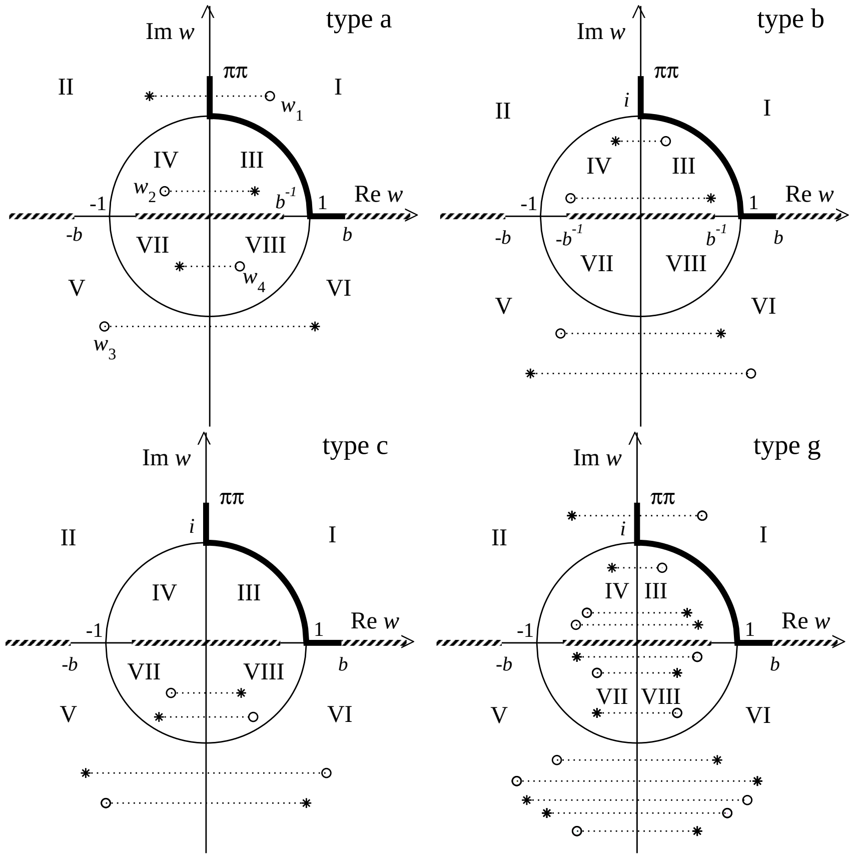

Figure 1: Uniformization w-plane: Representation of resonances of types (**a**), (**b**), (**c**) and (**g**) in the 3-channel $\pi\pi$-scattering S-matrix element.

- The S-matrix parametrization

The S-matrix elements S_{ij} are parameterized using the Le Couteur-Newton relations [13]. On the w-plane, we have derived for them:

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)}, \tag{2.2}$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \quad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}, \quad S_{22}S_{33} - S_{23}^2 = \frac{d(-w)}{d(w)}. \tag{2.3}$$

The $d(w)$ is the Jost matrix determinant. The 3-channel unitarity requires the following relations to hold for physical w-values:

$$|d(-w^*)| \leq |d(w)|, \quad |d(-w^{-1})| \leq |d(w)|, \quad |d(w^{-1})| \leq |d(w)|,$$

$$|d(w^{*-1})| = |d(-w^{*-1})| = |d(-w)| = |d(w)|.$$

The S-matrix elements in Le Couteur–Newton relations are taken as the products $S = S_B S_{res}$; the main (model-independent) contribution of resonances, given by the pole clusters, is included in the resonance part S_{res}; possible remaining small (model-dependent) contributions of resonances and influence of channels which are not taken explicitly into account in the uniformizing variable are included in the background part S_B. The d-function is: for the resonance part

$$d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*) \quad (M \text{ is the number of resonance zeros}), \tag{2.4}$$

for the background part

$$d_B = \exp[-i \sum_{n=1}^{3} \frac{\sqrt{s - s_n}}{2m_n} (\alpha_n + i\beta_n)], \tag{2.5}$$

where

$$\alpha_n = a_{n1} + a_{n\sigma} \frac{s - s_\sigma}{s_\sigma} \theta(s - s_\sigma) + a_{nv} \frac{s - s_v}{s_v} \theta(s - s_v),$$
$$\beta_n = b_{n1} + b_{n\sigma} \frac{s - s_\sigma}{s_\sigma} \theta(s - s_\sigma) + b_{nv} \frac{s - s_v}{s_v} \theta(s - s_v)$$

with s_σ the $\sigma\sigma$ threshold, s_v the combined threshold of the $\eta\eta'$, $\rho\rho$, $\omega\omega$ channels. The resonance zeros w_r and the background parameters were fixed by fitting to the data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the charmonium decay processes $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $\psi(2S) \to J/\psi\pi\pi$.

- Results of the analysis of data on $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$

For the data on multi-channel $\pi\pi$ scattering we used the results of phase analyses which are given for phase shifts of the amplitudes $\delta_{\alpha\beta}$ and for the modules of the S-matrix elements $\eta_{\alpha\beta} = |S_{\alpha\beta}|$ $(\alpha, \beta = 1, 2, 3)$:

$$S_{\alpha\alpha} = \eta_{\alpha\alpha} e^{2i\delta_{\alpha\alpha}}, \quad S_{\alpha\beta} = i\eta_{\alpha\beta} e^{i\phi_{\alpha\beta}}. \tag{2.6}$$

If below the third threshold there is the 2-channel unitarity then the relations

$$\eta_{11} = \eta_{22}, \;\; \eta_{12} = (1 - {\eta_{11}}^2)^{1/2}, \;\; \phi_{12} = \delta_{11} + \delta_{22} \tag{2.7}$$

are fulfilled in this energy region.

For the $\pi\pi$ scattering, the data are taken from the threshold to 1.89 GeV are taken from many works [14]; for $\pi\pi \to K\overline{K}$, from [15]. For $\pi\pi \to \eta\eta$, we used data for $|S_{13}|^2$ from the threshold to 1.72 GeV [16].

We have found the more preferable scenarios when the $f_0(500)$ is described by the cluster of type (**a**); the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, type (**c**); and $f_0'(1500)$, type (**g**); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III.

Analyzing these data, we have obtained two solutions which are distinguished mainly in the width of $f_0(500)$. Further we show only the solution which has survived after adding to the analysis the data on decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ from the Mark III, DM2 and BES II Collaborations. In Table 1, the obtained pole-clusters for resonances are shown on the $\sqrt{s}$-plane. The poles, corresponding to the $f_0'(1500)$, on sheets III, V and VII are of the 2nd order and that on the sheet VI of the 3rd order in our approximation.

Table 1: The pole clusters for resonances on the $\sqrt{s}$-plane. $\sqrt{s_r}=\mathrm{E}_r - i\Gamma_r/2$.

Sheet		$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
II	E_r	521.6±12.4	1008.4±3.1			1512.4±4.9	
	$\Gamma_r/2$	467.3±5.9	33.5±1.5			287.2±12.9	
III	E_r	552.5±17.7	976.7±5.8	1387.2±24.4		1506.1±9.0	
	$\Gamma_r/2$	467.3±5.9	53.2±2.6	167.2±41.8		127.8±10.6	
IV	E_r			1387.2±24.4		1512.4±4.9	
	$\Gamma_r/2$			178.2±37.2		215.0±17.6	
V	E_r			1387.2±24.4	1493.9±3.1	1498.8±7.2	1732.8±43.2
	$\Gamma_r/2$			261.0±73.7	72.8±3.9	142.3±6.0	114.8±61.5
VI	E_r	573.4±29.1		1387.2±24.4	1493.9±5.6	1511.5±4.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9		250.0±83.1	58.4±2.8	179.3±4.0	111.2±8.8
VII	E_r	542.5±25.5			1493.9±5.0	1500.4±9.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9			47.8±9.3	99.9±18.0	55.2±38.0
VIII	E_r				1493.9±3.2	1512.4±4.9	1732.8±43.2
	$\Gamma_r/2$				62.2±9.2	298.4±14.5	58.8±16.4

The obtained background parameters are: $\underline{a_{11} = 0.0,\ a_{1\sigma} = 0.0199,\ a_{1v} = 0.0,}$ $\underline{b_{11} = b_{1\sigma} = 0.0,\ b_{1v} = 0.0338}$, $a_{21} = -2.4649$, $a_{2\sigma} = -2.3222$, $a_{2v} = -6.611$, $b_{21} = b_{2\sigma} = 0.0$, $b_{2v} = 7.073$, $b_{31} = 0.6421$, $b_{3\sigma} = 0.4851$, $b_{3v} = 0$; $s_\sigma = 1.6338$ GeV^2, $s_v = 2.0857$ GeV^2.

The very simple description of the $\pi\pi$-scattering background (the underlined numbers) confirms well our assumption $\underline{S = S_B S_{res}}$ and also that representation of multi-channel resonances by the pole clusters on the uniformization plane is good and quite sufficient.

It is important that we have obtained practically zero background of the $\pi\pi$ scattering in the scalar-isoscalar channel because a reasonable and simple description of the background should be a criterion for the correctness of the approach. Furthermore, this shows that the consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable solves partly a problem of some approaches (see, e.g., [17]) that the wide-resonance parameters are strongly controlled by the non-resonant background.

Note also one more the important conclusion related to the practically zero background of the $\pi\pi$ scattering in our approach: Contribution in the $\pi\pi$ scattering amplitude from the crossed channels is given by allowing for the left-hand branch-point at $s = 0$ in the uniformizing variable and the meson-exchange contributions on the left-hand cuts. The fact that the zero background of the $\pi\pi$ scattering in the elastic-scattering region is obtained only at taking into account the left-hand branch-point in the proper uniformizing variable both in the 2-channel analysis of the processes $\pi\pi \to \pi\pi, K\overline{K}$ [18] and in the 3-channel analysis of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ indicates that the ρ-meson exchange contribution on the left-hand cut is obliterated practically by the scalar meson (the σ-meson) exchange one that has the opposite sign due to gauge invariance.

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This means that the $f_0(500)$, observed as the pole cluster of type **a**, is indeed the very large particle, not some dynamical formation. In this connection it is reasonable to interpret the effective threshold at $s_\sigma = 1.6338$ GeV^2 in the background phase-shift of the $\pi\pi$ scattering amplitude to be related to the $\sigma\sigma$ channel. Only in this channel we have obtained the non-zero background in the $\pi\pi$ scattering ($a_{1\sigma} = 0.0199$). Finally, the above-discussed situation with the contributions on the left-hand cut might prompt, in our opinion, a correct choice of potential, for example, in the so-called "hidden gauge formalism" [19].

In Figure 2 we show the results of fitting to the analyzed experimental data on $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$.

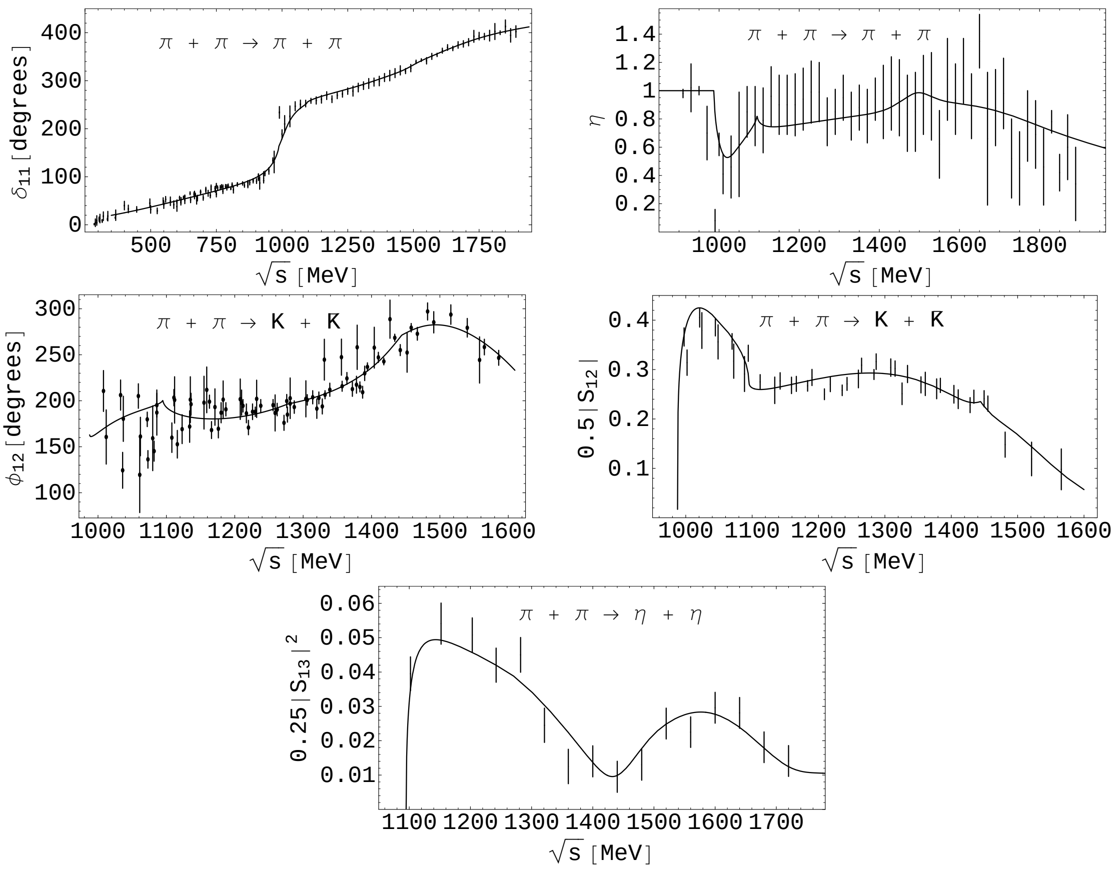

Figure 2: The phase shifts and modules of the S-matrix element in the S-wave $\pi\pi$-scattering (upper panel), in $\pi\pi \to K\overline{K}$ (middle panel), and the squared module of the $\pi\pi \to \eta\eta$ S-matrix element (lower figure).

Generally, *wide multi-channel states are most adequately represented by pole clusters*, because the pole clusters give the main model-independent effect of resonances. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances [18]. However, mass values are needed in some cases, e.g., in mass relations for multiplets. Therefore, we stress that such parameters of the wide multi-channel states, as *masses, total widths and coupling constants with channels, should be calculated using the poles on sheets II, IV and VIII*, because only on these sheets the analytic continuations have the forms:

$$\propto 1/S_{11}^{\mathrm{I}}, \quad \propto 1/S_{22}^{\mathrm{I}} \text{ and } \propto 1/S_{33}^{\mathrm{I}},$$

respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels. E.g., if the resonance part of amplitude is taken as

$$T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot}), \tag{2.8}$$

one obtains for the mass and total width

$$m_{res} = \sqrt{\mathrm{E}_r^2 + (\Gamma_r/2)^2} \quad \text{and} \quad \Gamma_{tot} = \Gamma_r \tag{2.9}$$

where the pole position $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ must be taken on sheets II, IV, VIII, depending on the resonance classification. In Table 2 there are given the obtained masses and total widths of states, calculated from the pole positions on sheets II, IV and VIII for resonances of types (**a**), (**b**) and (**c**), respectively.

Table 2: The masses and total widths of the f_0 resonances.

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
m_{res}[MeV]	693.9±10.0	1008.1±3.1	1399.0±24.7	1495.2±3.2	1539.5±5.4	1733.8±43.2
Γ_{tot}[MeV]	931.2±11.8	64.0±3.0	1357.0±74.4	124.4±18.4	571.6±25.8	117.6±32.8

3. The contribution of multi-channel $\pi\pi$ scattering in the final states of decays of ψ- and Υ-meson families

For decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ we have taken data from Mark III [20], from DM2 [21] and from BES II [22]; for $\psi(2S) \to J/\psi(\pi^+\pi^-)$ from Mark II [23]; for $\psi(2S) \to J/\psi(\pi^0\pi^0)$ from Crystal Ball(80) [24]; for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ from Argus [25], CLEO [26, 27], CUSB [28], and Crystal Ball(85) Collaborations [29]; finally for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ from CLEO(94) Collaboration [27, 30].

Formalism for calculating di-meson mass distributions of decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $V' \to V\pi\pi$ ($V = \psi, \Upsilon$) can be found in Ref. [31]. There is assumed that pairs of pseudo-scalar mesons of final states have $I = J = 0$ and only they undergo strong interactions, whereas a final vector meson (ϕ, ψ, Υ) acts as a spectator. The amplitudes for decays are related with the scattering amplitudes T_{ij} ($i, j = 1 - \pi\pi, 2 - K\overline{K}$) as follows

$$F(J/\psi \to \phi\pi\pi) = \sqrt{2/3}\,[c_1(s)T_{11} + c_2(s)T_{21}], \tag{3.1}$$

$$F(J/\psi \to \phi K\overline{K}) = \sqrt{1/2}\,[c_1(s)T_{12} + c_2(s)T_{22}], \tag{3.2}$$

$$F(V(2S) \to V(1S)\pi\pi\ (V = \psi, \Upsilon)) = [(d_1, e_1)T_{11} + (d_2, e_2)T_{21}], \tag{3.3}$$

$$F(\Upsilon(3S) \to \Upsilon(1S, 2S)\pi\pi) = [(f_1, g_1)T_{11} + (f_2, g_2)T_{21}] \tag{3.4}$$

where $c_1 = \gamma_{10} + \gamma_{11}s$, $c_2 = \alpha_2/(s - \beta_2) + \gamma_{20} + \gamma_{21}s$, $(d_i, e_i) = (\delta_{i0}, \rho_{i0}) + (\delta_{i1}, \rho_{i1})s$ and $(f_i, g_i) = (\omega_{i0}, \tau_{i0}) + (\omega_{i1}, \tau_{i1})s$ are functions of couplings of the J/ψ, $\psi(2S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ to channel i;

α_2, β_2, γ_{i0}, γ_{i1}, δ_{i0}, ρ_{i0}, δ_{i1}, ρ_{i1}, ω_{i0}, ω_{i1}, τ_{i0} and τ_{i1} are free parameters. The pole term in c_2 is an approximation of possible ϕK states, not forbidden by OZI rules when considering quark diagrams of these processes. Obviously this pole should be situated on the real s-axis below the $\pi\pi$ threshold.

The expression for decays $J/\psi \to \phi(\pi\pi, K\overline{K})$

$$N|F|^2\sqrt{(s-s_1)\lambda(m^2_{J/\psi}, s, m^2_\phi)}, \tag{3.5}$$

where $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2xz$ is the Källen function, and the analogues relations for $V(2S) \to V(1S)\pi\pi$ ($V = \psi, \Upsilon$) and $\Upsilon(3S) \to \Upsilon(1S, 2S)\pi\pi$ give the di-meson mass distributions. N (normalization to experiment) is: for $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$, 4.3439 for ARGUS, 2.1776 for CLEO(84), 1.2011 for CUSB; for $\Upsilon(2S) \to \Upsilon(1S)\pi^0\pi^0$, 0.0788 for Crystal Ball(85); for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-$ and $\pi^0\pi^0)$, 0.5096 and 0.2235 for CLEO(07), and for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-$ and $\pi^0\pi^0)$, 7.7397 and 3.8587 for CLEO(94), respectively. Parameters of the coupling functions of the decay particles (J/ψ, $\psi(2S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$) to channel i, obtained in the analysis, are: $(\alpha_2, \beta_2) = (0.0843,\ 0.0385)$, $(\gamma_{10}, \gamma_{11}, \gamma_{20}, \gamma_{21}) = (1.1826, 1.2798, -1.9393, -0.9808)$, $(\delta_{10},\ \delta_{11},\ \delta_{20},\ \delta_{21}) = (-0.1270,\ 16.621,\ 5.983,\ -57.653)$, $(\rho_{10},\ \rho_{11},\ \rho_{20},\ \rho_{21}) = (0.4050,\ 47.0963,\ 1.3352, -21.4343)$, $(\omega_{10},\ \omega_{11},\ \omega_{20},\ \omega_{21}) = (1.0827, -2.7546,\ 0.8615,\ 0.6600)$, $(\tau_{10},\ \tau_{11},\ \tau_{20},\ \tau_{21}) = (7.3875, -2.5598,\ 0.0,\ 0.0)$.

A satisfactory combined description of all considered processes is obtained with the total $\chi^2/\text{ndf} = 640.302/(564-70) \approx 1.30$; for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.15$; for $\pi\pi \to K\overline{K}$, $\chi^2/\text{ndf} \approx 1.65$; for $\pi\pi \to \eta\eta$, $\chi^2/\text{ndp} \approx 0.87$; for decays $J/\psi \to \phi(\pi^+\pi^-, K^+K^-)$, $\chi^2/\text{ndp} \approx 1.21$; for $\psi(2S) \to J/\psi(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 2.43$; for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 1.01$; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 0.97$, for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 0.54$.

In Figures 3–7 we show our fitting to the experimental data on the above indicated decays of quarkonia in the combined analysis with the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. Note the important role of the BES II data: Namely the di-pion mass distribution in Figure 4 rejects the solution with the narrower $f_0(500)$ – the corresponding curve lies considerably below the data from the threshold to about 850 MeV. The dips in the energy dependence of di-pion spectra (Figure 7, upper panel) are the result of a destructive interference between the $\pi\pi$ scattering and $K\overline{K} \to \pi\pi$ contributions to the final states of the decays $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$.

The description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and charmonia decays and the resulting resonance parameters practically did not change when compared to the case without bottomonia decays.

4. Conclusions

- The combined analysis was performed for the data on the isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the decays of heavy quarkonia $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\pi\pi$, $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$, $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, and BES II Collaborations.

- It is shown that in the final states of ψ and Υ-meson family decays (except for the $\pi\pi$ scattering) the contribution of coupled processes, e.g., $K\overline{K} \to \pi\pi$, is important even if these

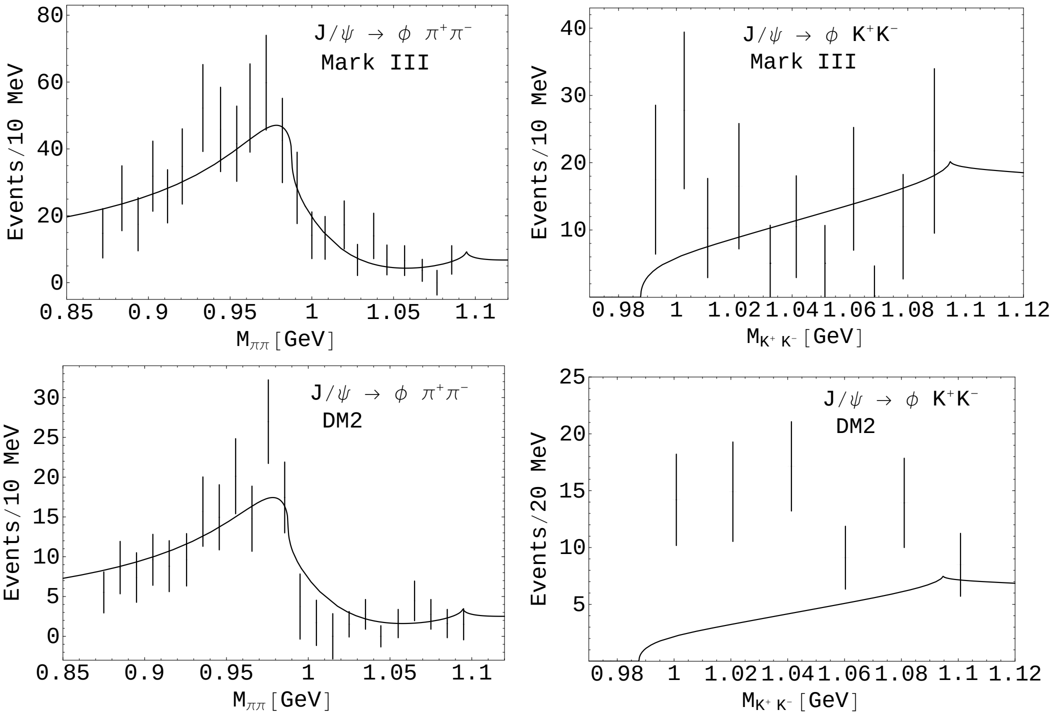

Figure 3: The $J/\psi \to \phi\pi\pi$ and $J/\psi \to \phi K\overline{K}$ decays.

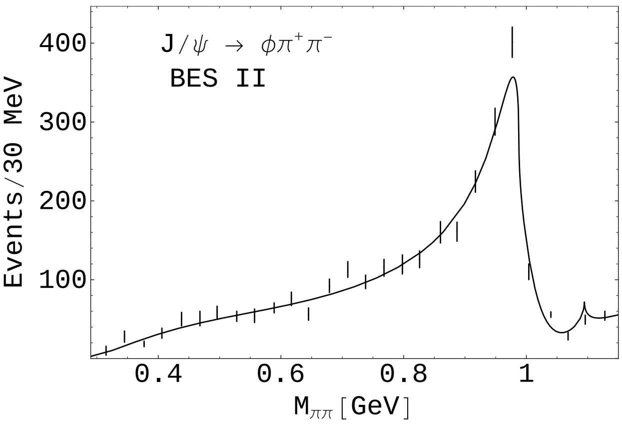

Figure 4: The $J/\psi \to \phi\pi\pi$ decay; the data of BES II Collaboration.

processes are energetically forbidden. This is in accordance with our previous conclusions on the wide resonances [5, 11]: when a wide resonance cannot decay into a channel, which opens above its pole mass and which is strongly coupled (e.g. the $f_0(500)$ and the $K\overline{K}$ channel), one should consider this resonance as a multi-channel state with allowing for the indicated channel taking into account the Riemann-surface sheets related to the threshold branch-point of this channel and performing the combined analysis of the considered and coupled channels. E.g., on the basis of that consideration the new and natural mechanism

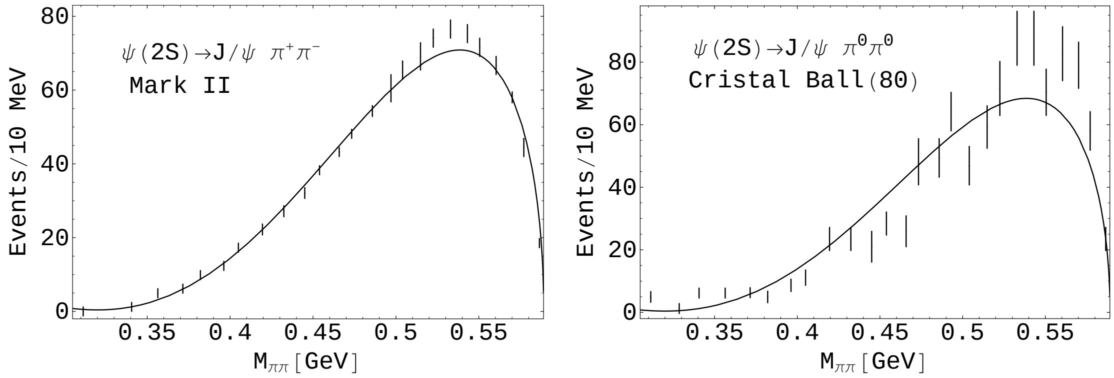

Figure 5: The $\psi(2S) \to J/\psi\pi\pi$ decay.

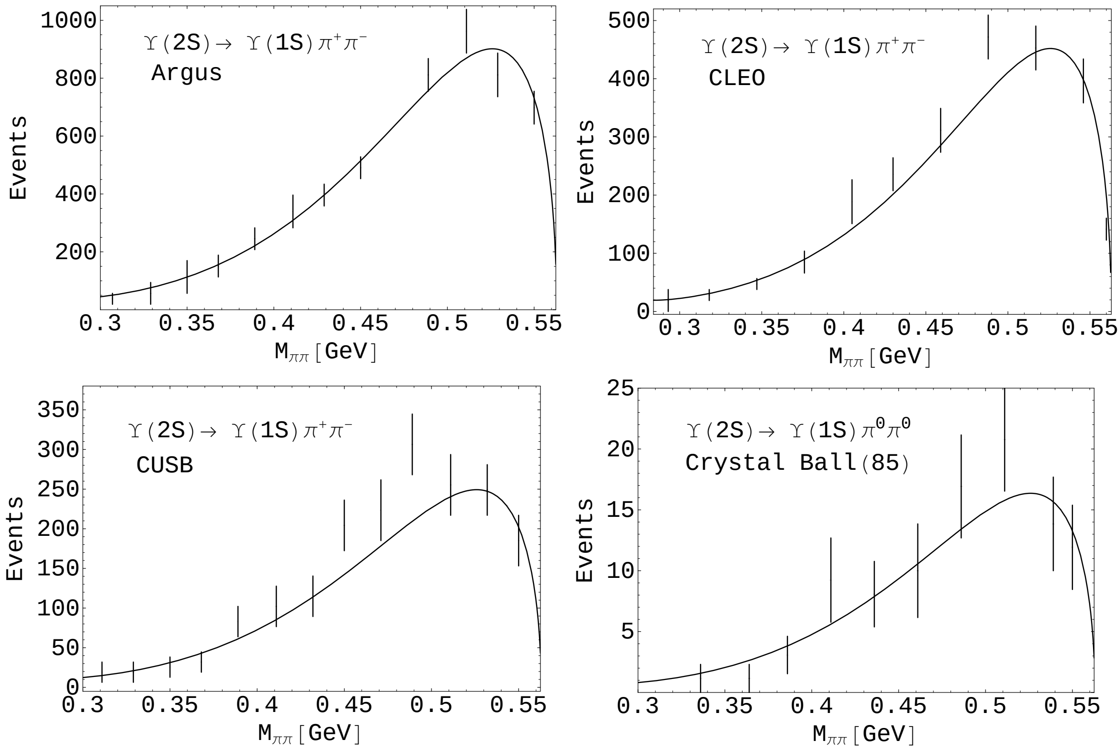

Figure 6: The $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ decay.

of destructive interference in the decay $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ is indicated, which provides the two-humped shape of the di-pion mass distribution.

- Results of the analysis confirm all of our earlier conclusions on the scalar mesons, main of which are:
  1) Confirmation of the $f_0(500)$ with a mass of about 700 MeV and a width of 930 MeV (the pole on sheet II is $521.6 \pm 12.4 - i(467.3 \pm 5.9)$ MeV). This mass value accords with prediction by S. Weinberg [32] on the basis of mended symmetry, with the analysis using the large-N_c consistency conditions between the unitarization and resonance saturation [33] and

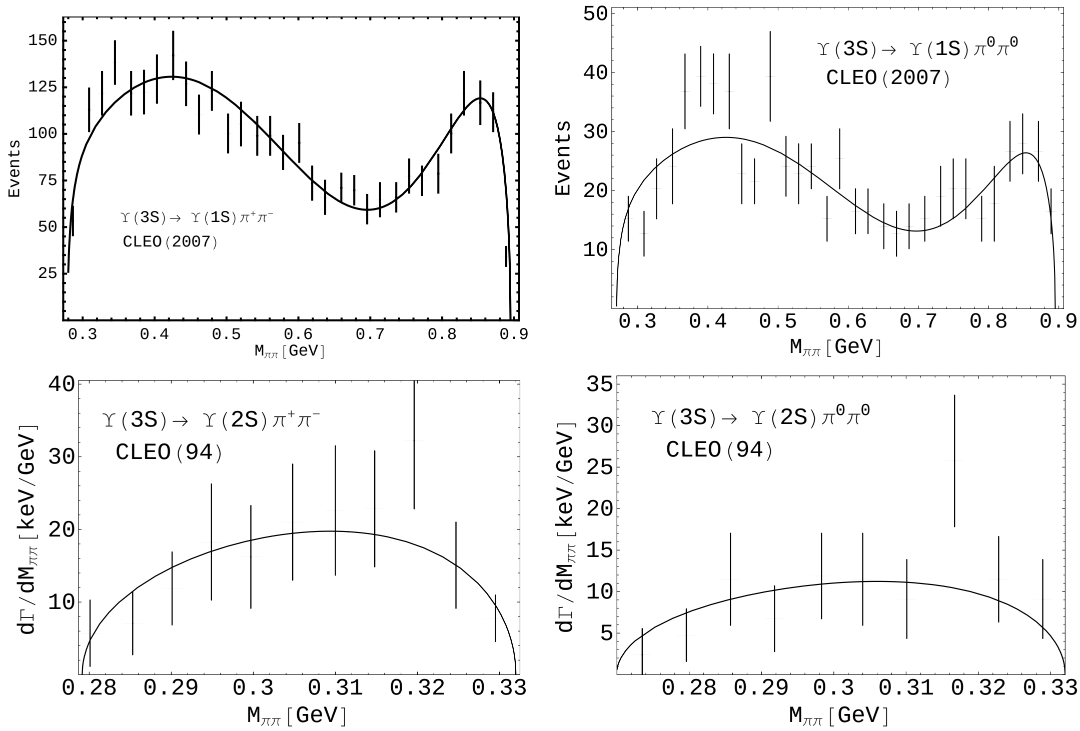

Figure 7: The decays $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$.

with the prediction of the soft-wall AdS/QCD approach [34].
2) Indication for the $f_0(980)$ (the pole on sheet II is $1008.4 \pm 3.1 - i(33.5 \pm 1.5)$ MeV) to be neither the $q\bar{q}$ state nor $K\overline{K}$ molecule, but, possibly, a bound $\eta\eta$ state.
3) Indication for the $f_0(1370)$ and $f_0(1710)$ to have a dominant $s\bar{s}$ component. This is in agreement with a number of experiments: Conclusion about the $f_0(1370)$ quite agrees with the one of the Crystal Barrel Collaboration work [35] where the $f_0(1370)$ is identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation. This explains also quite well why one did not find this state considering only the $\pi\pi$ scattering [36, 37]. Conclusion about the $f_0(1710)$ is consistent with the experimental facts that this state is observed in $\gamma\gamma \to K_S K_S$ [38] and not observed in $\gamma\gamma \to \pi^+\pi^-$ [39].
4) Indication for two states in the 1500-MeV region: the $f_0(1500)$ ($m_{res} \approx 1495$ MeV, $\Gamma_{tot} \approx 124$ MeV) and the $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV). The $f_0'(1500)$ is interpreted as a glueball taking into account its biggest width among the enclosing states [40].

- Thus, one can conclude that the considered bottomonia decay data do not offer new insights into the nature of the scalar mesons, which were not already deduced in our previous analysis of pseudoscalar meson scattering data and the charmonia decays [5].

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