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Meson mass spectrum and the Fermi coupling in the covariant confined quark model

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A new insight into the problem of hadron mass spectrum is provided in the framework of the covariant confined quark model. Along the *compositeness condition* enabling the elimination of the renormalization constant of the elementary hadron wave function in the Yukawa-type theory, we employ another equation which relates the meson mass function to the Fermi coupling of the relevant four-fermion interaction. Both equations guarantee that the Yukawa-type theory is equivalent to the Fermi-type theory, thereby providing an interpretation of the meson field as the bound state of constituent fermions (quarks). We evaluate the Fermi coupling G as a function of the meson mass M and vary the values of the mass so that to obtain a smooth behavior of $G(M)$. The conventional (pseudoscalar and vector) meson mass spectrum estimated in this manner is found to be in good agreement with the latest experimental data in a wide range of mass scale. We also compare the behavior of the obtained $G(M)$ with the strong QCD coupling α_s calculated in a QCD-inspired approach.

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1. Introduction

One of the puzzles of hadron physics is the origin of the hadron masses. The Standard Model and, in particular, QCD operate only with fundamental particles (quarks, leptons, neutrinos), gauge and the Higgs bosons. It is not yet clear how to explain the appearance of the multitude of observed hadrons and elucidate the generation of their masses. Therefore, the calculation of the hadron mass spectrum in a quality comparable to the precision of experimental data still remains one of the major problems in QCD.

Actually, even before QCD was set up as the fundamental theory of strong interactions, it was understood that it is a difficult problem to describe a composite particle within QFT as based on the relativistic S-matrix. The original Lagrangian describes free fields and their interactions while the consideration of physical processes requires the renormalization, i.e. the transition from bare or unrenormalized quantities to the physical or renormalized ones. In particular, the bare field is related to the dressed one as $\phi_0 = Z^{1/2}\phi_r$ through the wave function renormalization constant Z. The bare field ϕ_0 may be eliminated from the Lagrangian by putting $Z = 0$, that is the compositeness condition (CC) suggested first by B. Jouvet [1]. He showed that the four-fermion theory is equivalent to a Yukawa-type theory if the renormalization constant of the boson field is set to zero. The crucial point in comparison of the two theories is the renormalization of the Yukawa-type theory, i.e. the transition from the bare quantities (boson mass, boson wave function, Yukawa coupling) to the renormalized ones. Then, the physical boson mass and renormalized Yukawa coupling may be expressed through the Fermi constant via the CC. Further developments and applications of the CC may be found in [2, 3, 4, 5, 6, 7].

Relativistic models with specific forms of analytically confined propagators have been developed to study some aspects of low-energy hadron physics in a series of papers [8, 9, 10, 11]. The role of analytic confinement in the formation of two-particle bound states has been analyzed within a Yukawa-type model with specific forms of analytically confined propagators of quarks and gluons [8]. By using a ladder Bethe-Salpeter-type master equation, the masses of conventional mesons have been estimated with relative errors less than 3.5 per cent in a wide energy range. The calculated weak decay constants of light mesons were also in good accordance with the experimental data. Additionally, the lowest-state glueball mass has been predicted which is in reasonable agreement with other theoretical approaches. A phenomenological model with infrared-confined propagators has been developed to take into account the dependence of the QCD effective coupling $\alpha_s = g^2/4\pi$ on the mass [10]. By fitting the experimental masses of mesons we predicted the behavior of α_s at large distances. A new, specific and finite behavior of $\alpha_s(M)$ at the origin $M = 0$ has been derived analytically. Note, $\alpha_s(0)$ depends on the confinement scale value Λ. We fixed $\alpha_s(0) = 0.757$ for $\Lambda = 345$ MeV [9] and $\alpha_s(0) = 0.8498$ for $\Lambda = 220$ MeV [11].

The compositeness condition $Z_H = 0$ is also one of the key ingredients in the relativistic constituent quark model [12, 13]). The model has found numerous applications both in the meson sector [14] and in baryon physics [15]. The next step in the development of the model has been done in [16, 17], where infrared confinement was introduced to guarantee the absence of all possible threshold singularities corresponding to quark production. The implementation of quark confinement allowed to use the same values for the constituent quark masses for different quark systems (mesons, baryons, tetraquarks, etc.). In the covariant confined quark model (CCQM), the free parameters (constituent quark masses, the infrared cutoff parameter λ and the size parameters Λ_H) have been determined by a fit to available experimental data. The parameter λ is taken universal for all processes. This approach was successfully applied in the calculation of transition form factors B-mesons and Λ_b-baryons [18] as well as for strong and radiative decays of X(3872) meson treated as a tetraquark [19].

Below we apply the CCQM to the meson mass problem. We show explicitly that the four-fermion theory with the Fermi coupling G is equivalent to the Yukawa-type theory if, first, the wave function renormalization constant in the Yukawa theory is equal to zero and, second, the Fermi coupling G is inversely proportional to the meson mass function calculated at the physical meson mass.

Note, the CC is used by us to determine the renormalized Yukawa coupling g_r as a function of model parameters, hereby, the experimental values of hadron masses m_H are used. Our second constraint equation allows us to calculate the Fermi coupling G as a function of the physical mass in a quite large region ranging from the π up to B_c meson. Then, we suggest *a smoothness criterion* to generate a continuous behavior of the Fermi coupling G, update slightly the model parameters and estimate the model meson masses. The mass spectrum obtained in this manner is found to be in good agreement with the experimental data. We also compare the behavior of the 'smoothed' G with the strong QCD coupling α_s calculated in the QCD-inspired approach.

2. The compositeness condition $Z_H = 0$ and meson mass equation

1. Historically, the CC first appeared when looking for the bound state in a four-fermion theory with the Lagrangian

$$\mathscr{L}_F = \bar{q}(i\,\not{\partial} - m_q)q + \frac{G}{2}\left(\bar{q}\Gamma q\right)^2 . \tag{2.1}$$

Here, for simplicity, we drop all color and flavor indices. For the general Dirac matrix we use $\Gamma = I, i\gamma^5$, i.e. we restrict to bound states with zero spin. We will consider the bound state problem by using the one-loop (chain) approximation but the result is general and can be proved to all orders of perturbation theory. In the following, for simplicity, we will not consider the renormalization of the fermion ("quark") fields.

Let us consider the generating functional for the Fermi theory

$$Z_F = \int \mathscr{D}\bar{q} \int \mathscr{D}q\, e^{i\int dx \mathscr{L}_F(x)} . \tag{2.2}$$

By using the Gaussian functional representation for the exponential of the four-fermion interaction

$$e^{i\frac{G}{2}\langle(\bar{q}\Gamma q)^2\rangle} = N_+^{-1} \int \mathscr{D}\phi \exp\{-\frac{i}{2}\frac{1}{G}\langle\phi^2\rangle + i\langle\phi\cdot(\bar{q}\Gamma q)\rangle\}, \qquad \langle(...)\rangle = \int dx\,(...) \tag{2.3}$$

we rewrite Z_F and integrate out the obtained Gaussian path integral over quark fields.

Further we introduce the renormalized mass function of the boson with spin $S = 0$ as

$$\Pi_{S=0}(x_1 - x_2) = i\,\langle T\left\{\left[\bar{q}\Gamma q\right]_x \left[\bar{q}\Gamma q\right]_y\right\}\rangle_0 = -i\,\mathrm{tr}[\Gamma S_q(x_1 - x_2)\Gamma S_q(x_2 - x_1)] . \tag{2.4}$$

and expand its Fourier transform at the physical boson mass up to the second order:

$$\tilde{\Pi}_{S=0}(p^2) = \int dx\, e^{-ipx}\Pi_{S=0}(x) = \tilde{\Pi}_{S=0}(m^2) + (p^2 - m^2)\tilde{\Pi}'_{S=0}(m^2) + \tilde{\Pi}^{\rm ren}_{S=0}(p^2) . \tag{2.5}$$

Then, we collect the terms bi-linear in the boson fields

$$L_F^{(2)} = \frac{1}{2}\int dx\,\phi(x)\Big(-\frac{1}{G}+\tilde{\Pi}_{S=0}(m^2)+(\Box-m^2)\tilde{\Pi}'_{S=0}(m^2)\Big)\phi(x) + \frac{1}{2}\int dx_1\int dx_2\,\phi(x_1)\Pi^{\rm ren}_{S=0}(x_1-x_2)\phi(x_2)\,. \tag{2.6}$$

If we require the condition

$$G\tilde{\Pi}_{S=0}(m^2)=1 \tag{2.7}$$

and rescale the boson field as $\phi\to\phi/\sqrt{\tilde{\Pi}'_{S=0}(m^2)}$ one obtains the free Lagrangian of the boson field with the mass m and the correct residue of the Green function.

The fully renormalized generating functional of the Fermi-theory is written as

$$Z_F^{\rm ren} = \int\mathscr{D}\phi\exp\Big\{\frac{i}{2}\int dx\phi(x)(\Box-m^2)\phi(x)+\frac{i}{2}\frac{1}{\tilde{\Pi}'_{S=0}(m^2)}\int dx_1\int dx_2\,\phi(x_1)\Pi^{\rm ren}_{S=0}(x_1-x_2)\phi(x_2) -\sum_{n=3}^{\infty}\frac{i^n}{n}\Bigg[\frac{1}{\sqrt{\tilde{\Pi}'_{S=0}(m^2)}}\Bigg]^n\int dx_1\ldots\int dx_n\phi(x_1)\ldots\phi(x_n)\;{\rm tr}[\Gamma S_q(x_1-x_2)\ldots\Gamma S_q(x_n-x_1)]\Big\}\,. \tag{2.8}$$

2. The bare Lagrangian of the Yukawa interaction of the boson field ϕ_0 with the fermions reads

$$\mathscr{L}_Y = \bar{q}(i\,\not\partial-m_q)q+\frac{1}{2}\phi_0(\Box-m_0^2)\phi_0+g_0\phi_0\left(\bar{q}\Gamma q\right), \qquad \Box=-\partial^\mu\partial_\mu\,. \tag{2.9}$$

The vacuum generating functional for the Yukawa theory is

$$Z_Y=\int\mathscr{D}\phi_0\int\mathscr{D}\bar{q}\int\mathscr{D}q\,e^{i\int dx\mathscr{L}_Y(x)}\,. \tag{2.10}$$

Hereafter, we will drop all irrelevant normalization constants. Integrate out the quark fields and collect the terms bi-linear in the boson fields by using the expansion (2.5). One finds

$$L_Y^{(2)} = \frac{1}{2}\int dx\,\phi_0(x)(\Box-m_0^2+g_0^2\tilde{\Pi}_{S=0}(m^2)+(\Box-m^2)\tilde{\Pi}'_{S=0}(m^2))\phi_0(x) + \frac{1}{2}g_0^2\int dx_1\int dx_2\,\phi_0(x_1)\Pi^{\rm ren}_{S=0}(x_1-x_2)\phi_0(x_2)\,. \tag{2.11}$$

Note, the renormalization of boson mass, wave function and Yukawa coupling proceeds as follows:

$$m^2=m_0^2-g_0^2\tilde{\Pi}_{S=0}(m^2), \quad \phi_r=Z^{-1/2}\phi_0, \quad g_r=Z^{1/2}g_0, \quad Z=\frac{1}{1+g_0^2\tilde{\Pi}'_{S=0}(m^2)}\,. \tag{2.12}$$

The renormalization constant Z may be expressed via the renormalized coupling constant

$$Z=1-g_r^2\tilde{\Pi}'_{S=0}(m^2)\,. \tag{2.13}$$

Finally, the renormalized generating functional for the Yukawa theory is rewritten as follows:

$$Z_Y^{\rm ren} = \int\mathscr{D}\phi_r\exp\Big\{\frac{i}{2}\int dx\phi_r(x)(\Box-m^2)\phi_r(x)+\frac{i}{2}g_r^2\int dx_1\int dx_2\,\phi_r(x_1)\Pi^{\rm ren}_{S=0}(x_1-x_2)\phi_r(x_2) - \sum_{n=3}^{\infty}\frac{i^n}{n}g_r^n\int dx_1\ldots\int dx_n\phi_r(x_1)\ldots\phi_r(x_n){\rm tr}[\Gamma S_q(x_1-x_2)\ldots\Gamma S_q(x_n-x_1)]\Big\}\,. \tag{2.14}$$

We drop the linear boson term because it is absent for pseudoscalar mesons and it can be removed in the scalar case by a shift of the field.

Comparing both renormalized generating functionals of Eqs. (2.14) and (2.8) we conclude that the condition for their equality is

$$g_r = \frac{1}{\sqrt{\tilde{\Pi}'_{S=0}(m^2)}} \tag{2.15}$$

or, according to Eq. (2.13),

$$Z = 1 - g_r^2\,\tilde{\Pi}'_{S=0}(m^2) = 0\,. \tag{2.16}$$

Thus the vanishing of the wave function renormalization constant in the Yukawa theory may be interpreted as the elimination of the bare field $\phi_0 = Z^{1/2}\phi_r$ for a composite boson.

3. Mass function in the covariant quark model

The interaction of the ground-state pseudoscalar and vector mesons with their constituent quarks is described in the covariant quark model by a Lagrangian which reads

$$\mathscr{L}_{\rm int} = g_H H(x)\,J_H(x)\,; \qquad J_H(x) = \int dx_1 \int dx_2 F_H(x;x_1,x_2)\bar{q}_2(x_2)\Gamma_H q_1(x_1)\,. \tag{3.1}$$

Here, $\Gamma_P = i\gamma^5$ and $\Gamma_V^\mu = \gamma^\mu$ are chosen for the pseudoscalar and vector mesons, respectively. The vector meson field ϕ^μ has the Lorentz index μ and satisfies the transversality condition

$$\partial_\mu \phi^\mu = 0\,. \tag{3.2}$$

For the vertex function F_H we use the translational invariant form

$$F_H(x,x_1,x_2) = \delta(x - w_1x_1 - w_2x_2)\Phi_H((x_1-x_2)^2) \tag{3.3}$$

where $w_i = m_{q_i}/(m_{q_1}+m_{q_2})$ so that $w_1 + w_2 = 1$. The Fourier transform of the vertex function is chosen in a Gaussian form

$$\tilde{\Phi}_H(-p^2) = \int dx\, e^{ipx}\Phi_H(x^2) = e^{p^2/\Lambda_H^2} \tag{3.4}$$

for both the pseudoscalar and vector mesons. The size parameter Λ_H is an adjustable quantity. Since the calculation of the Feynman diagrams proceeds in the Euclidean region where $p^2 = -p_E^2$, the vertex function decreases very rapidly for $p_E^2 \to \infty$ and thereby provides ultraviolet convergence in the evaluation of any diagram.

The mass functions for the pseudoscalar (spin $S=0$) and vector mesons (spin $S=1$) are:

$$\Pi_{PP}(x-y) \;=\; +i\,\langle T\left\{J_P(x)J_P(y)\right\}\rangle_0, \tag{3.5}$$

$$\Pi_{VV}^{\mu\nu}(x-y) \;=\; -i\,\langle T\left\{J_V^\mu(x)J_V^\nu(y)\right\}\rangle_0\,. \tag{3.6}$$

Using the Fourier transforms of the vertex functions in (3.4) and the quark propagator in the Schwinger representation

$$S_q(x-y) = \int \frac{d^4k}{(2\pi)^4 i}\frac{e^{-ik(x-y)}}{m_q - \not{k}}\,. \tag{3.7}$$

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one can easily find the Fourier transforms of the mass functions

$$\tilde{\Pi}_{PP}(p^2) = N_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P^2(-k^2) \mathrm{tr}\Big(\gamma^5 S_1(k+w_1 p)\gamma^5 S_2(k-w_2 p)\Big), \tag{3.8}$$

$$\begin{aligned}\tilde{\Pi}_{VV}^{\mu\nu}(p) &= N_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_V^2(-k^2) \mathrm{tr}\Big(\gamma^\mu S_1(k+w_1 p)\gamma^\nu S_2(k-w_2 p)\Big) \\ &= g^{\mu\nu}\tilde{\Pi}_{VVg}(p^2) + p^\mu p^\nu \tilde{\Pi}_{VVpp}(p^2)\end{aligned} \tag{3.9}$$

where $N_c = 3$ is a number of color degrees of freedom. Due to the transversality of the vector field the second term in Eq. (3.9) is irrelevant in our considerations. The first remaining term in Eq. (3.9) can be expressed as

$$\tilde{\Pi}_{VVg}(p^2) = \frac{1}{3}\big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\big)\tilde{\Pi}_{VV}^{\mu\nu}(p)\,. \tag{3.10}$$

By using the calculational technique from Ref. [16] one finds

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int\limits_0^{1/\lambda^2} \frac{dt\,t}{a_H^2} \int\limits_0^1 d\alpha\, e^{-t z_0 + z_H} \left\{ \frac{n_H}{a_H} + m_{q_1} m_{q_2} + \big(w_1 - \frac{b}{a_H}\big)\big(w_2 + \frac{b}{a_H}\big)p^2 \right\}, \tag{3.11}$$

where

$$\begin{aligned} z_0 &= \alpha m_{q_1}^2 + (1-\alpha) m_{q_2}^2 - \alpha(1-\alpha)p^2\,, \qquad z_H = \frac{2s_H t}{2s_H + t}(\alpha - w_2)^2 p^2\,, \\ a_H &= 2s_H + t\,, \qquad b = (\alpha - w_2)t\,. \end{aligned} \tag{3.12}$$

Here $n_P = 2$ and $n_V = 1$. We use the result of the fit [17] for the value of infrared cutoff with $\lambda = 181$ MeV. The parameter s_H is related to the size parameter Λ_H as $s_H = 1/\Lambda_H^2$. Note that in the case $\lambda \to 0$ the branching point appears at $p^2 = (m_{q_1} + m_{q_2})^2$. At this point the integral over t becomes divergent as $t \to \infty$ because of $z_0 = 0$ at $\alpha = m_{q_2}/(m_{q_1} + m_{q_2})$. By introducing an infrared cutoff on the upper limit of the scale integration one can avoid the appearance of the threshold singularity.

The compositeness condition

$$Z_H = 1 - g_H^2\, \tilde{\Pi}'_H(m_H^2) = 0\,, \tag{3.13}$$

where g_H is the renormalized Yukawa coupling constant, now has a clear mathematical meaning because the mass function Π_H in Eq. (3.11) is well defined.

As discussed in the previous section, the Yukawa theory defined by the interaction Lagrangian of Eq. (3.1) is equivalent to the Fermi theory defined by the interaction Lagrangian

$$\mathscr{L}^F_{\rm int} = \frac{G}{2} J_H^2(x) \tag{3.14}$$

if the wave function renormalization constant Z_H is equal to zero and the Fermi coupling G satisfies the equation

$$G\tilde{\Pi}_H(m_H^2) = 1\,. \tag{3.15}$$

Now we are able to investigate the dependence of the Fermi coupling G on the hadron masses.

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4. Numerical results

A first fit of the model parameters has originally been performed in Ref. [16], where the above described method for implementing infrared quark confinement was used for the first time. The leptonic decay constants which are known either from experiment or, from lattice simulations have been chosen as input quantities to adjust the model parameters. A given meson H in the interaction Lagrangian Eq. (3.1) is characterized by the coupling constant g_H, the size parameter Λ_H and two of the four constituent quark masses, m_q ($m_u = m_d$, m_s, m_c, m_b). Moreover, there is the infrared confinement parameter λ which is universal for all hadrons. Note, the physical values for the hadron masses have been used in the fit. In the beginning we have $2n_H + 5$ adjustable parameters for n_H number of mesons. The compositeness condition provides n_H constraints and allows one to express all coupling constants g_H through other model parameters. The remaining $n_H + 5$ parameters are determined by a fit to experimental data. As input data the values of the leptonic decay constants and some electromagnetic decay widths are chosen.

Later on, several updated fits were indicated in Ref. [17]. In this paper we will use one of them which is slightly different from the published version. The reason is that in the published version [17] the value of the charm quark mass was found to be $m_c = 2.16$ GeV which is a somewhat higher than the value needed to describe some observables of in the charm sector. The results of the (overconstrained) least–squares fit used in the present study can be found in Tables 1 and 2.

Table 1: Input values for the leptonic decay constants f_H (in MeV) and our least-squares fit values.

	Fit Values	Data	Ref.		Fit Values	Data	Ref.
f_π	128.4	130.4 ± 0.2	[20, 21]	f_ρ	221.2	221 ± 1	[20]
f_K	156.0	156.1 ± 0.8	[20, 21]	f_ω	204.2	198 ± 2	[20]
f_D	206.7	206.7 ± 8.9	[20, 21]	f_ϕ	228.2	227 ± 2	[20]
f_{D_s}	257.5	257.5 ± 6.1	[20, 21]	$f_{J/Psi}$	415.0	415 ± 7	[20]
f_B	189.7	192.8 ± 9.9	[22]	f_{K^*}	215.0	217 ± 7	[20]
f_{B_s}	235.3	238.8 ± 9.5	[22]	f_{D^*}	223.0	245 ± 20	[24]
f_{η_c}	386.6	438 ± 8	[23]	$f_{D_s^*}$	272.0	272 ± 26	[24]
f_{B_c}	445.6	489 ± 5	[23]	f_{B^*}	196.0	196 ± 44	[24]
f_{η_b}	609.1	801 ± 9	[23]	$f_{B_s^*}$	229.0	229 ± 46	[24]
				f_Υ	661.3	715 ± 5	[20]

The agreement between the fit and input values is quite satisfactory. We do not include decay results for the $\eta(\eta')$-mesons because the primary goal of our present study is to understand the origin of the meson masses in the framework of the CCQM. The $\eta(\eta')$-mesons have the additional features like the mixing angle and an possibly important gluon admixture to the conventional $q\bar{q}$-structure of the η'.

Some aspects of the nonleptonic B_s-meson decays with $\eta(\eta')$ in the final states were recently

Table 2: Input values for some basic electromagnetic decay widths and our least-squares fit values (in keV).

Process	Fit Values	Data [20]
$\pi^0 \to \gamma\gamma$	5.07×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	3.47	5.0 ± 0.4
$\rho^\pm \to \pi^\pm\gamma$	76.3	67 ± 7
$\omega \to \pi^0\gamma$	687	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	57.7	50 ± 5
$K^{*0} \to K^0\gamma$	129	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	0.59	1.5 ± 0.5
$J/\Psi \to \eta_c\gamma$	1.90	1.58 ± 0.37

discussed in Ref. [25]. The results of the fit for the values of the quark masses m_{q_i}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} are given in (4.1) and in Table 3, respectively.

The constituent quark masses and the values for the size parameters fall into the expected range. The size parameters show the expected general pattern: the geometrical size of a meson, which is inversely proportional to Λ_{H_i}, decreases when the mass increases.

$m_{u/d}$	m_s	m_c	m_b	λ	
0.235	0.442	1.61	5.07	0.181	GeV

(4.1)

Table 3: The fitted values of the size parameters Λ_H in GeV.

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	ϕ	J/Ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

The present numerical least-squares fit and the values for the model parameters supersede the results of a similar analysis given in [16], where a different set of electromagnetic decays has been used. In the present fit we have also updated some of the theoretical/experimental input values.

Our prime goal is to study the behavior of the Fermi coupling G in Eq. (3.15) as a function of the hadron masses by keeping other parameters (infrared cutoff parameter λ, size parameters Λ_H and constituent quark masses m_q) fixed. The original dependence of G on the hadron mass is obtained by directly taking the physical values, resulting in a sawtooth-like behavior. Therefore, we suggest to change the values of the input hadron masses in such a way to get a relatively smooth

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dependence of G on the masses. *A smoothness criterion* might be considered as a possibility, when values for the meson masses are computed through Eq. (3.15) as a function of the other model parame y $G\lambda^2$ on these masses is show es.

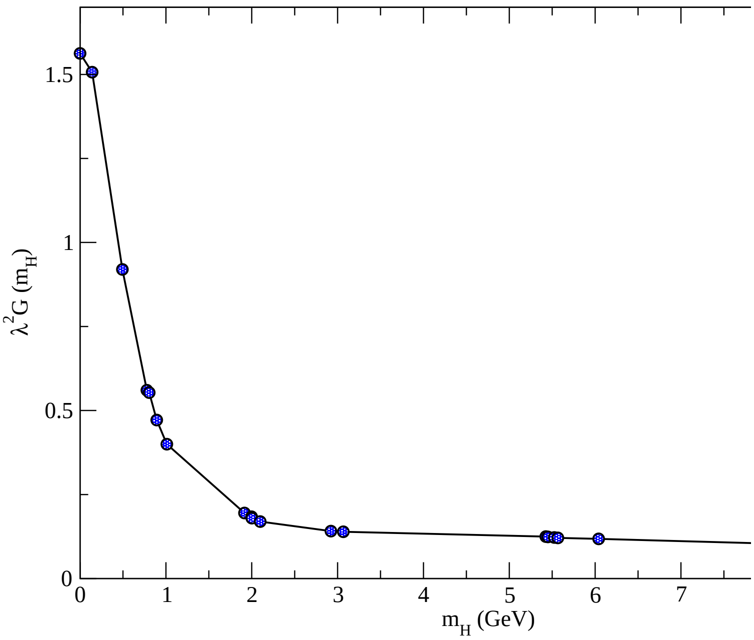

Figure 1: The dependence of the Fermi coupling on the fitted meson masses.

The estimated values for the meson masses found in this manner are shown in Table 4.

One can see that they are in quite good agreement with the experimental data. For completeness in Table 5 we also present our results for the effective couplings $G\lambda^2$ in the case of exact fit (when the values of meson masses are taken from data) and in the case of the smooth fit.

It might be interesting to compare the behavior of G with the effective QCD coupling constant α_s obtained in the relativistic models with specific forms of analytically confined quark and gluon propagators [8, 9, 10]. In these models the nonlocal four-quark interaction is induced by one-gluon exchange between biquark currents. Since the quark currents are connected via the confined gluon propagator having the dimension of an inverse mass squared in momentum space, the resulting coupling α_s is dimensionless. In Fig. 2 we compare the mass dependence of the rescaled dimensionless Fermi coupling $\alpha_s^{\mathrm{model}} \equiv 1.74\, G\lambda^2$ [solid line] estimated for the model parameters given by Eq. (4.1) with the effective QCD coupling α_s [dashed line] obtained in [9, 10].

The idea of such a comparison by rescaling of one coupling to another, where both of them are having the plato behavior, looks quite reasonable. Note that this region is almost the same in both approaches that is actually non-trivial information. After rescaling we are able to compare the behavior of two curves in the region of the small masses. They are different due to different dynamics (confinement, quark propagators, vertex functions, etc.) implemented in these approaches. Note, the particular choice of the model parameters used in Ref. [9] are $m_{u/d} = 0.193, m_s = 0.293, m_c = 1.848, m_b = 4.693$ GeV for the constituent quark masses and $\Lambda = 0.345$ GeV for the confinement

Table 4: The fitted values for the meson masses in MeV

	Model	Data [20]		Model	Data [20]
m_π	141.0	139.57018 ± 0.0003	m_{η_c}	2922.0	2983.7 ± 0.7
m_K	493.0	493.677 ± 0.016	$m_{J/\Psi}$	3067.0	3096.916 ± 0.011
m_ρ	778.0	775.26 ± 0.25	m_B	5425.0	5279.26 ± 0.17
m_ω	806.0	782.65 ± 0.12	m_{B^*}	5450.0	5325.2 ± 0.4
m_{K^*}	893.0	891.66 ± 0.26	m_{B_s}	5524.0	5366.77 ± 0.24
m_ϕ	1011.0	1019.45 ± 0.02	$m_{B_s^*}$	5566.0	5415.8 ± 1.5
m_D	1915.0	1869.62 ± 0.15	m_{B_c}	6041.0	6274.5 ± 1.8
m_{D_s}	1998.0	1968.50 ± 0.32	m_{η_b}	8806.0	9398.0 ± 3.2
m_{D^*}	2001.0	2010.29 ± 0.13	m_Υ	8880.0	9460.30 ± 0.26
$m_{D_s^*}$	2099.0	2112.3 ± 0.5			

Table 5: Values for effective couplings $G\lambda^2$ in cases of exact and smooth fit

	Exact fit	Smooth fit		Exact fit	Smooth fit
π	1.508	1.507	η_c	0.128	0.141
K	0.919	0.920	J/Ψ	0.129	0.139
ρ	0.571	0.560	B	0.215	0.125
ω	0.673	0.553	B^*	0.237	0.124
K^*	0.476	0.472	B_s	0.192	0.122
ϕ	0.377	0.400	B_s^*	0.232	0.121
D	0.224	0.195	B_c	0.0905	0.118
D_s	0.197	0.184	η_b	0.0612	0.0986
D^*	0.168	0.180	Υ	0.0600	0.0984
D_s^*	0.158	0.170			

scale. Despite the different model origins and input parameter values, the behaviors of two curves are very similar to each other in the intermediate and heavy mass regions above ~ 2 GeV. Their values at the origin are mostly determined by the confinement mechanisms realized in different ways in these models. This could explain why they have different behaviors in the low-energy region below 2 GeV.

In conclusion, we have represented a brief sketch of an approach to the bound state problem in quantum field theory which is based on the compositeness condition $Z_H = 0$. By using the

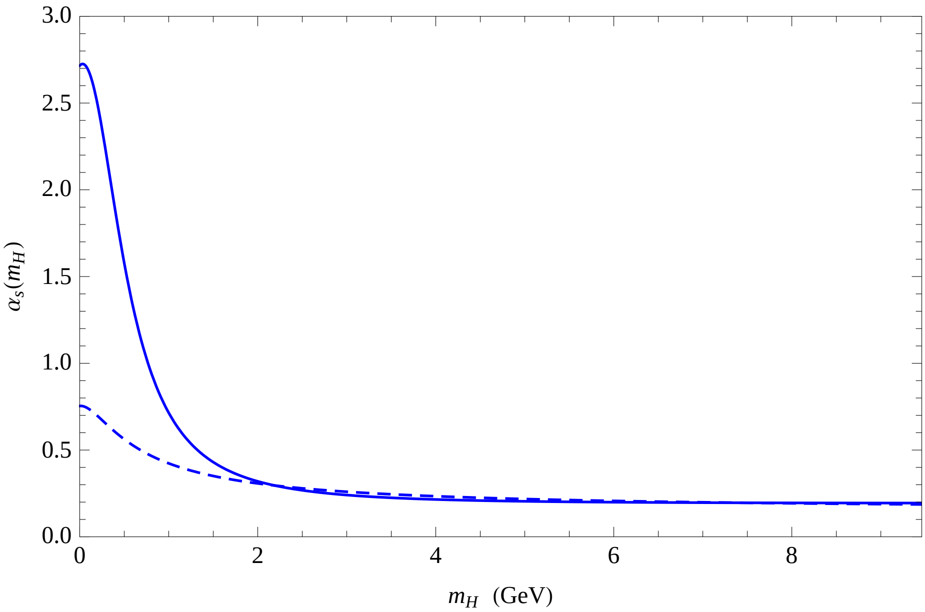

Figure 2: The comparison of the effective coupling $\alpha_s(m_H)$ obtained in [9, 10] (dashed line) with our calculated dimensionless coupling $\lambda^2 G(m_H)$ (solid line) scaled to the curve $\alpha_s(m_H)$ in the region of large masses.

functional integral we have demonstrated explicitly that the four-fermion theory with the Fermi coupling G is equivalent to the Yukawa-type theory if, first, the wave function renormalization constant in the Yukawa theory is equal to zero and, second, the Fermi coupling G is inversely proportional to the meson mass function calculated for the physical meson mass.

We have given details for the calculation of the mass function for pseudoscalar and vector mesons in the framework of the covariant quark model. We updated the fit of the model parameters and calculated the Fermi coupling G as a function of physical masses in a quite large region from the π up to B_c mesons.

We have suggested *a smoothness criterion* for the curve just varying the meson masses in such a way to obtain the smooth behavior for the Fermi coupling G. The mass spectrum obtained in this manner is found to be in good agreement with the experimental data. We have compared the behavior of G with the strong QCD coupling α_s calculated in the QCD-inspired approach.

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