PROCEEDINGS OF SCIENCE

Relativistic corrections to B_c mesons pair production in proton–proton collisions

A.P. Martynenko
Samara State University, 443011 Samara, Russia
Samara State Aerospace University, 443086 Samara, Russia
E-mail: a.p.martynenko@samsu.ru

A.M. Trunin*
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia
Samara State Aerospace University, 443086 Samara, Russia
E-mail: amtrnn@gmail.com

On the basis of perturbative QCD and relativistic quark model we calculate cross sections of pair B_c mesons production in proton–proton interaction. Relativistic factors in the production amplitude connected with the relative motion of heavy quarks and the transformation law of the bound state wave function are taken into account. The gluon and quark propagators entering the production amplitude are expanded in the ratio of the relative quark momenta to the meson mass up to the second order. Relativistic corrections to the quark–antiquark bound state wave functions in the rest frame are considered by means of the Breit-like potential. It turns out that the examined effects significantly decrease nonrelativistic predictions for the cross section.

XXII International Baldin Seminar on High Energy Physics Problems,
15-20 September 2014
JINR, Dubna, Russia

*Speaker.

PoS(Baldin ISHEPP XXII)118

1. Introduction

The description of heavy quarkonium represents a well-known and actively developing problem of the QCD [1]. Consistent progress in experimental studies in this area requires an adequate theoretical improvements and speculations in attempt to explain the arising puzzles in observed data. For example, the additional color-octet mechanism [2, 3] was used to achieve the agreement between theoretical predictions and the Tevatron results for $p\bar{p} \to \psi' + X$ cross section [4]. Analogously, the recent measurements of pair J/ψ production in pp–interaction [5, 6] suggest the necessity of additional contributions, required to correctly describe cross section spectrum over the transverse momentum. Due to the overwhelming gluonic density at the LHC, the source of these contributions can be, at least partially, attributed to the processes of double parton scattering [7, 8].

Another potentially important way of cross section improvement is connected with relativistic effects. Such sort of corrections was found to be crucial for plausible theoretical description of pair J/ψ and η_c mesons production in e^+e^- annihilation measured by Belle and BaBar collaborations [9]. Particularly, relativistic corrections were showed to be equal about 50% of initial nonrelativistic result in the framework of nonrelativistic quantum chromodynamics (NRQCD) [10], representing effecitve field theory for heavy quarkonium [3]. Their significant contribution to the cross section was also confirmed by calculations in light-cone approach [11] and relativistic quark model [12]. Following this precedent, the role of relativism has been investigated for the numerous processes of charmonium production in electron–positron annihilation and proton–proton collisions [13]. Relativistic effects were found to be large not only for exclusive or semi–exclusive reactions, but also in the case of several inclusive production processes [14].

In this paper we continue our study of double heavy quarkonium production in proton–proton interaction [15] by considering the pairs of pseudoscalar and vector B_c mesons on the basis of a relativistic quark model. The description of this process in nonrelativistic approximation can be found in Ref. [16]. We calculate several types of relativistic corrections to $\sigma(p+p \to B_c + \bar{B}_c + X)$ and then show how they consequently change the nonrelativistic cross section.

2. General formalism

In collinear approximation the cross section of pair B_c mesons production in high energy proton–proton collisions has the form of the convolution of partonic cross section $d\sigma[gg \to B_c + \bar{B}_c]$ with the partonic distribution functions of the initial protons [17, 18, 19]:

$$d\sigma[p+p \to B_c + \bar{B}_c + X] = \int dx_1 dx_2 f_{g/p}(x_1,\mu) f_{g/p}(x_2,\mu)\, d\sigma[gg \to B_c + \bar{B}_c], \tag{2.1}$$

where $f_{g/p}(x,\mu)$ is the distribution function for gluon in proton, $x_{1,2}$ are the longitudinal momentum fractions of gluons, μ is the factorization scale. Neglecting the proton mass and taking c.m. reference frame of the initial protons with the beam along the z-axis we can present the gluon on mass–shell momenta as $k_{1,2} = x_{1,2}\frac{\sqrt{S}}{2}(1,0,0,\pm 1)$. In (2.1) we assume that the main contribution to the cross section comes from the gluon fusion process $gg \to B_c + \bar{B}_c$, as it can be expected at the high c.m. collision energies $\sqrt{S} = 7 - 14$ TeV at the LHC [17, 20].

In the quasipotential approach the production amplitude for the gluonic subprocess can be expressed as a convolution of a perturbative production amplitude of $(\bar{b}c)$ and $(b\bar{c})$ quark–antiquark

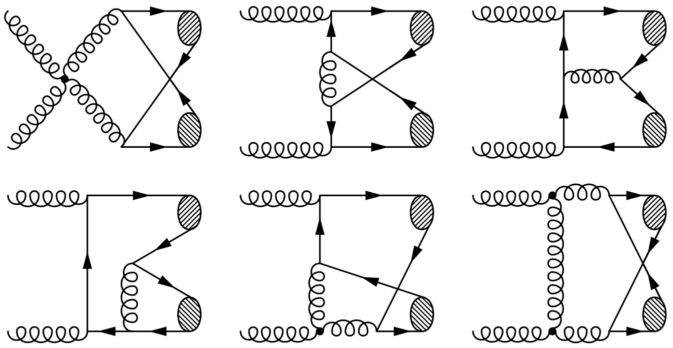

Figure 1: The leading order diagrams contributing to $gg \to B_c + \bar{B}_c$ subprocess. The others can be obtained by reversing the quark lines or interchanging the initial gluons

pairs $\mathscr{T}(p_1, p_2; q_1, q_2)$ and the quasipotential wave functions of the final B_c and $\bar{B}_c$ mesons $\Psi(p,P)$ and $\Psi(q,Q)$ [12, 15]:

$$\mathscr{M}[gg \to B_c + \bar{B}_c](k_1,k_2,P,Q) = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \bar{\Psi}(p,P)\bar{\Psi}(q,Q) \otimes \mathscr{T}(p_1,p_2;q_1,q_2), \quad (2.2)$$

where $p_{1,2}$ are four–momenta of c and $\bar{b}$ (anti)quarks forming B_c meson, and $q_{1,2}$ are the appropriate four–momenta for $\bar{c}$ and b (anti)quarks in $\bar{B}_c$ meson. They are defined in terms of total momenta $P(Q)$ and relative momenta $p(q)$ as follows:

$$p_{1,2} = \eta_{1,2}P \pm p, \quad (pP) = 0; \qquad q_{1,2} = \eta_{1,2}Q \pm q, \quad (qQ) = 0,$$
$$\eta_{1,2} = \frac{M^2 \pm m_c^2 \mp m_b^2}{2M^2}, \quad (2.3)$$

where $M = M_{B_c} = M_{\bar{B}_c}$ is the meson mass, $p = L_P(0,\mathbf{p})$ and $q = L_Q(0,\mathbf{q})$ are the relative four–momenta obtained by the Lorentz transformation of four–vectors $(0,\mathbf{p})$ and $(0,\mathbf{q})$ to the reference frames moving with the four–momenta P and Q of the final mesons B_c and $\bar{B}_c$. In Eq. (2.2) we integrate over the relative three–momenta of quarks and antiquarks in the final state.

In the leading order in the strong coupling constant α_s, there are 31 Feynman diagrams contributing to the described gluon fusion subprocess $gg \to B_c + \bar{B}_c$, which are presented in Fig. 1. Analytical evaluation of the diagrams and subsequent calculation of traces in the corresponding expressions were performed by FeynArts for Mathematica [21] and FORM [22]. Then we obtain the following result for the leading order production amplitude (2.2):

$$\mathscr{M}[gg \to B_c + \bar{B}_c](k_1,k_2,P,Q) = \frac{1}{9} M\pi^2\alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{Tr}\,\mathfrak{M}, \quad (2.4)$$

$$\mathfrak{M} = \bar{\Psi}(p,P)\gamma_\beta \Gamma_1^{\beta\omega} \bar{\Psi}(q,Q)\gamma_\omega + \bar{\Psi}(p,P)\gamma_\beta \bar{\Psi}(q,Q)\gamma_\omega \Gamma_2^{\beta\omega}$$
$$+\bar{\Psi}(p,P)\hat{\varepsilon}_1 \frac{m_c - \hat{k}_1 + \hat{p}_1}{(k_1-p_1)^2 - m_c^2} \gamma_\beta \bar{\Psi}(q,Q)\Gamma_3^\beta + \bar{\Psi}(p,P)\gamma_\beta \frac{m_c + \hat{k}_1 - \hat{q}_1}{(k_1-q_1)^2 - m_c^2} \hat{\varepsilon}_1 \bar{\Psi}(q,Q)\Gamma_4^\beta$$
$$+\bar{\Psi}(p,P)\hat{\varepsilon}_2 \frac{m_c - \hat{k}_2 + \hat{p}_1}{(k_2-p_1)^2 - m_c^2} \gamma_\beta \bar{\Psi}(q,Q)\Gamma_5^\beta + \bar{\Psi}(p,P)\gamma_\beta \frac{m_c + \hat{k}_2 - \hat{q}_1}{(k_2-q_1)^2 - m_c^2} \hat{\varepsilon}_2 \bar{\Psi}(q,Q)\Gamma_6^\beta,$$

where $\varepsilon_{1,2}$ are polarization vectors of the initial gluons, the hat symbol means contraction of the four–vector with the Dirac gamma–matrices, and a number of vertex functions Γ_i was introduced to make the entry of the amplitude (2.4) more compact. We explicitly extracted in (2.4) the normalization factors $\sqrt{2M}$ of the quasipotential wave functions.

The formation of B_c mesons from quark–antiquark pairs is determined in the quark model by the quasipotential wave functions $\Psi(p,P)$ and $\Psi(q,Q)$. These wave functions are calculated initially in the meson rest frame and then transformed to the reference frames moving with the four–momenta P and Q. The law of such transformation was derived in the Bethe–Salpeter approach in Ref. [23] and in the quasipotential method in Ref. [24]. We use the last one and obtain the following expressions:

$$\bar\Psi(p,P)=\frac{\bar\Psi_0(\mathbf{p})}{\sqrt{\frac{e_c(p)}{m_c}\frac{e_c(p)+m_c}{2m_c}\frac{e_b(p)}{m_b}\frac{e_b(p)+m_b}{2m_b}}}\left[\frac{\hat v_1-1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m_b(e_b(p)+m_b)}-\frac{\hat p}{2m_b}\right]$$
$$\times\Sigma^P(1+\hat v_1)\left[\frac{\hat v_1+1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m_c(e_c(p)+m_c)}+\frac{\hat p}{2m_c}\right],$$
$$\bar\Psi(q,Q)=\frac{\bar\Psi_0(\mathbf{q})}{\sqrt{\frac{e_c(q)}{m_c}\frac{e_c(q)+m_c}{2m_c}\frac{e_b(q)}{m_b}\frac{e_b(q)+m_b}{2m_b}}}\left[\frac{\hat v_2-1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m_c(e_c(q)+m_c)}+\frac{\hat q}{2m_c}\right]$$
$$\times\Sigma^Q(1+\hat v_2)\left[\frac{\hat v_2+1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m_b(e_b(q)+m_b)}-\frac{\hat q}{2m_b}\right], \qquad (2.5)$$

where $m_{c,b}$ are quark masses, $e_{c,b}(p)=\sqrt{p^2+m_{c,b}^2}$, $v_1=P/M$, $v_2=Q/M$, and $\Sigma^{P,Q}$ is equal to γ_5 and $\hat\varepsilon_{P,Q}$ for pseudoscalar (B_c) and vector (B_c^*) mesons. The polarization vectors $\varepsilon_{P,Q}$ of vector mesons fulfill the relations $(\varepsilon_P P)=0$ and $(\varepsilon_Q Q)=0$.

Leading order vertex functions Γ_i in (2.4) have the following explicit form:

$$\Gamma_1^{\beta\omega}=18D_\mu{}^\beta(p_1+q_1)D_\nu{}^\omega(p_2+q_2)\big[2\varepsilon_1\varepsilon_2 g^{\mu\nu}-\varepsilon_1^\mu\varepsilon_2^\nu-\varepsilon_1^\nu\varepsilon_2^\mu$$
$$+iD_{\kappa\lambda}(k_1-p_1-q_1)\mathfrak{E}_1^{\kappa\mu}(p_1+q_1)\mathfrak{E}_2^{\lambda\nu}(p_2+q_2)+iD_{\kappa\lambda}(k_1-p_2-q_2)\mathfrak{E}_1^{\lambda\nu}(p_2+q_2)\mathfrak{E}_2^{\kappa\mu}(p_1+q_1)\big]$$
$$-i\varepsilon_1^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat k_1+\hat p_1}{(k_1-p_1)^2-m_c^2}\left[\gamma^\mu\frac{m_c+\hat k_2-\hat q_1}{(k_2-q_1)^2-m_c^2}\hat\varepsilon_2-8\hat\varepsilon_2\frac{m_c-\hat p_2-\hat q_1-\hat q_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma^\mu\right.$$
$$\left.-9i\mathfrak{E}_2^{\nu\mu}(p_2+q_2)D_{\nu\rho}(k_1-p_1-q_1)\gamma^\rho\right]-i\varepsilon_2^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat k_2+\hat p_1}{(k_2-p_1)^2-m_c^2}$$
$$\times\left[\gamma^\mu\frac{m_c+\hat k_1-\hat q_1}{(k_1-q_1)^2-m_c^2}\hat\varepsilon_1-8\hat\varepsilon_1\frac{m_c-\hat p_2-\hat q_1-\hat q_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma^\mu-9i\mathfrak{E}_1^{\nu\mu}(p_2+q_2)D_{\nu\rho}(k_1-p_2-q_2)\gamma^\rho\right]$$
$$+8iD^{\beta\omega}(p_2+q_2)\frac{m_c+\hat p_1+\hat p_2+\hat q_2}{(p_1+p_2+q_2)^2-m_c^2}\left[\hat\varepsilon_1\frac{m_c+\hat k_2-\hat q_1}{(k_2-q_1)^2-m_c^2}\hat\varepsilon_2+\hat\varepsilon_2\frac{m_c+\hat k_1-\hat q_1}{(k_1-q_1)^2-m_c^2}\hat\varepsilon_1\right]$$
$$+9D_\nu{}^\omega(p_2+q_2)\left[\mathfrak{E}_2^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_1-q_1)\frac{m_c+\hat k_1-\hat q_1}{(k_1-q_1)^2-m_c^2}\hat\varepsilon_1\right.$$
$$\left.+\mathfrak{E}_1^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_2-q_2)\frac{m_c+\hat k_2-\hat q_1}{(k_2-q_1)^2-m_c^2}\hat\varepsilon_2\right],$$

$$\Gamma_2^{\beta\omega} = -i\varepsilon_1^{\omega} D_\mu{}^\beta(p_1+q_1)\frac{m_b-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m_b^2}\Big[\gamma^\mu\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2 - 8\hat{\varepsilon}_2\frac{m_b-\hat{p}_1-\hat{p}_2-\hat{q}_1}{(p_1+p_2+q_1)^2-m_b^2}\gamma^\mu$$
$$-9i\mathfrak{E}_2^{\nu\mu}(p_1+q_1)D_{\nu\rho}(k_1-p_2-q_2)\gamma^\rho\Big] - i\varepsilon_2^{\omega} D_\mu{}^\beta(p_1+q_1)\frac{m_b-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m_b^2}$$
$$\times\Big[\gamma^\mu\frac{m_b+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m_b^2}\hat{\varepsilon}_1 - 8\hat{\varepsilon}_1\frac{m_b-\hat{p}_1-\hat{p}_2-\hat{q}_1}{(p_1+p_2+q_1)^2-m_b^2}\gamma^\mu - 9i\mathfrak{E}_1^{\nu\mu}(p_1+q_1)D_{\nu\rho}(k_1-p_1-q_1)\gamma^\rho\Big]$$
$$+8iD^{\beta\omega}(p_1+q_1)\frac{m_b+\hat{p}_1+\hat{q}_1+\hat{q}_2}{(p_1+q_1+q_2)^2-m_b^2}\Big[\hat{\varepsilon}_1\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2+\hat{\varepsilon}_2\frac{m_b+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m_b^2}\hat{\varepsilon}_1\Big]$$
$$+9D_\mu{}^\beta(p_1+q_1)\Big[\mathfrak{E}_2^{\nu\mu}(p_1+q_1)D_\nu{}^\omega(k_1-p_2-q_2)\frac{m_b+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m_b^2}\hat{\varepsilon}_1$$
$$+\mathfrak{E}_1^{\nu\mu}(p_1+q_1)D_\nu{}^\omega(k_1-p_1-q_1)\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2\Big],$$
$$\Gamma_3^\beta = iD_\mu{}^\beta(k_1-p_1-q_1)\Big[8\gamma^\mu\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2-\hat{\varepsilon}_2\frac{m_b-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m_b^2}\gamma^\mu\Big],$$
$$\Gamma_4^\beta = iD_\mu{}^\beta(k_1-p_1-q_1)\Big[8\hat{\varepsilon}_2\frac{m_b-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m_b^2}\gamma^\mu-\gamma^\mu\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2\Big],$$
$$\Gamma_5^\beta = iD_\mu{}^\beta(k_1-p_2-q_2)\Big[8\gamma^\mu\frac{m_b+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m_b^2}\hat{\varepsilon}_1-\hat{\varepsilon}_1\frac{m_b-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m_b^2}\gamma^\mu\Big],$$
$$\Gamma_6^\beta = iD_\mu{}^\beta(k_1-p_2-q_2)\Big[8\hat{\varepsilon}_1\frac{m_b-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m_b^2}\gamma^\mu-\gamma^\mu\frac{m_b+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m_b^2}\hat{\varepsilon}_1\Big], \quad (2.6)$$

where we introduce the tensors

$$\mathfrak{E}_{1,2}^{\mu\nu}(x) = g^{\mu\nu}(k_{1,2}-2x)\varepsilon_{1,2}+\varepsilon_{1,2}^\mu(2k_{1,2}^\nu-x^\nu)+\varepsilon_{1,2}^\nu(k_{1,2}^\mu+x^\mu), \quad \mathfrak{E}_{1,2}^\mu(x)=\varepsilon_{2,1}^\nu\mathfrak{E}_{1,2}^{\mu\nu}(x) \quad (2.7)$$

and $D_{\mu\nu}(k)$ is the gluon propagator, which is subsequently taken in the Feynman gauge.

The production amplitude (2.4) and vertex functions (2.6) contain relative momenta p and q in exact form. In order to take into account relativistic corrections of the second order in p and q we expand all inverse denominators of the quark and gluon propagators:

$$\frac{1}{(p_{1,2}+q_{1,2})^2} = \frac{1}{Z_0}\Big[1\mp\frac{2\eta_{1,2}(pQ+qP)}{Z_0}-\frac{p^2+2pq+q^2}{Z_0}+\ldots\Big],$$
$$\frac{1}{(p_2+q_1+q_2)^2-m_c^2} = \frac{1}{Z_1}\Big[1+\frac{2pQ-p^2}{Z_1}+\frac{4(pQ)^2}{Z_1^2}+\ldots\Big], \quad (2.8)$$
$$\frac{1}{(k_2-q_1)^2-m_c^2} = \frac{1}{Z_2}\Big[1+\frac{2k_2q-q^2}{Z_2}+\frac{4(k_2q)^2}{Z_2^2}+\ldots\Big],$$

where $s=(k_1+k_2)^2=(P+Q)^2=x_1x_2S$ and $t=(P-k_1)^2=(Q-k_2)^2$ are the Mandelstam variables for the gluonic subprocess, and leading order expansion denominators are $Z_0=s\,\eta_{1,2}^2$, $Z_1=s\,\eta_1+\eta_2^2M^2-m_c^2$, and $Z_2=t\,\eta_1-\eta_1\eta_2M^2-m_c^2$. The amplitude (2.4) contains 16 different denominators to be expanded in the manner of Eq. (2.8). Temporarily neglecting the bound state corrections, we found that expansion denominators have one of the following form: $s\eta_{1,2}$, $s\eta_{1,2}^2$, $\eta_{1,2}(M^2-t)$ or $\eta_{1,2}(M^2-s-t)$. Then, taking into account kinematical restrictions for s and t, along with the nonrelativistic estimate $\eta_1 = m_c/(m_c+m_b) \approx 1/4$, we conclude that expansion parameters in (2.8) are at least as small as $4p^2/M^2$ and $4q^2/M^2$.

Preserving in the expanded amplitude terms up to the second order both in the relative momenta p and q, we perform angular integration using the following relations for S-wave mesons:

$$\int \frac{\Psi_0^S(\mathbf{p})}{\sqrt{\frac{e_c(p)}{m_c}\frac{e_c(p)+m_c}{2m_c}\frac{e_b(p)}{m_b}\frac{e_b(p)+m_b}{2m_b}}}\frac{d\mathbf{p}}{(2\pi)^3} = \frac{1}{\sqrt{2}\pi}\int_0^\infty \frac{p^2 R_S(p)}{\sqrt{\frac{e_c(p)}{m_c}\frac{e_c(p)+m_c}{2m_c}\frac{e_b(p)}{m_b}\frac{e_b(p)+m_b}{2m_b}}}dp,$$

$$\int \frac{p_\mu p_\nu \Psi_0^S(\mathbf{p})}{\sqrt{\frac{e_c(p)}{m_c}\frac{e_c(p)+m_c}{2m_c}\frac{e_b(p)}{m_b}\frac{e_b(p)+m_b}{2m_b}}}\frac{d\mathbf{p}}{(2\pi)^3} = -\frac{g_{\mu\nu}-v_{1\mu}v_{1\nu}}{3\sqrt{2}\pi}\int_0^\infty \frac{p^4 R_S(p)}{\sqrt{\frac{e_c(p)}{m_c}\frac{e_c(p)+m_c}{2m_c}\frac{e_b(p)}{m_b}\frac{e_b(p)+m_b}{2m_b}}}dp, \quad (2.9)$$

where $R_S(p)$ is the radial wave function.

In order to calculate the cross section we average the squared modulus of the amplitude over polarizations of the initial gluons and also sum it over final particle polarizations in the case of pair vector mesons production:

$$\sum_\lambda \varepsilon_{1,2}^\mu \varepsilon_{1,2}^{*\,\nu} = \frac{k_1^\mu k_2^\nu + k_1^\nu k_2^\mu}{(k_1 k_2)} - g^{\mu\nu}, \qquad \sum_\lambda \varepsilon_{P,Q}^\mu \varepsilon_{P,Q}^{*\ \nu} = v_{1,2}^\mu v_{1,2}^\nu - g^{\mu\nu}. \quad (2.10)$$

After averaging over 8×8 possible initial gluons color states, we obtain the following expression for the differential cross section of pair B_c mesons production:

$$\begin{gathered} d\sigma[gg\to B_c+\bar{B}_c](s,t) = \frac{\pi M^2\alpha_s^4}{65\,536\,s^2}|\tilde{R}(0)|^4\times \\ \big[F^{(1)}(s,t) - 4(\omega_{01}+\omega_{10}-\omega_{11})F^{(1)}(s,t) - 4m_c^{-1}m_b^{-1}(m_c^2\omega_{\frac{1}{2}\frac{3}{2}} + m_b^2\omega_{\frac{3}{2}\frac{1}{2}})F^{(1)}(s,t) \\ +6(\omega_{01}+\omega_{10})^2F^{(1)}(s,t) + \omega_{\frac{1}{2}\frac{1}{2}}(1-3\omega_{01}-3\omega_{10})F^{(2)}(s,t) + \omega_{\frac{1}{2}\frac{1}{2}}^2F^{(3)}(s,t)\big]. \end{gathered} \quad (2.11)$$

The parameter $\tilde{R}(0)$ in (2.11) has the following definition

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sqrt{\frac{(e_c(p)+m_c)(e_b(p)+m_b)}{2e_c(p)\,2e_b(p)}}R(p)p^2\,dp \quad (2.12)$$

and represents the relativistic generalization for the value of coordinate wave function at the origin $R(0)$. The relativistic parameters ω_{nk} are expressed through momentum integrals of the radial wave function $R(p)$:

$$\begin{gathered} I_{nk} = \int_0^{m_c} p^2R(p)\sqrt{\frac{(e_c(p)+m_c)(e_b(p)+m_b)}{2e_c(p)\,2e_b(p)}}\left(\frac{e_c(p)-m_c}{e_c(p)+m_c}\right)^n\left(\frac{e_b(p)-m_b}{e_b(p)+m_b}\right)^k dp, \\ \omega_{nk} = \sqrt{\frac{2}{\pi}}\frac{I_{nk}}{\tilde{R}(0)}. \end{gathered} \quad (2.13)$$

The functions $F^{(i)}(s,t)$ in (2.11) represent leading order term in heavy quark velocity expansion of the cross section and relativistic corrections to it. Their analytical expressions are extremely lengthy, so we do not present them here.

Table 1: Numerical values of the parameters describing pseudoscalar and vector B_c mesons

Meson state	$n^{2S+1}L_J$	M, GeV	$\tilde{R}(0)$, GeV$^{3/2}$	ω_{10}	ω_{01}	$\omega_{\frac{1}{2}\frac{1}{2}}$	ω_{11}	$\omega_{\frac{1}{2}\frac{3}{2}}$	$\omega_{\frac{3}{2}\frac{1}{2}}$
B_c	1^1S_0	6.289	0.96	0.0439	0.0054	0.0153	0.00055	0.00020	0.0016
B_c^*	1^3S_1	6.323	0.86	0.0462	0.0056	0.0161	0.00057	0.00020	0.0016

3. Numerical results and discussion

The quasipotential wave functions are obtained by numerical solving of the Schrödinger equation with effective relativistic Hamiltonian based on the QCD generalization of the Breit potential completed with the scalar and vector exchange confinement terms, as it is described in details in our previous works [15, 25]. We give the values of B_c meson masses and relativistic parameters (2.12), (2.13) in Table 1. Note that our definition (2.13) of relativistic integrals I_{nk} contains a cutoff at the value of c-quark mass $\Lambda = m_c$. Although the integrals (2.13) are convergent, there are some uncertainties in their calculation related with the determination of the wave function in the region of relativistic momenta $p \gtrsim m_c$ in our model.

The numerical results for the total cross section of pair pseudoscalar and vector B_c mesons production corresponding to the energies $\sqrt{S} = 7$ and 14 TeV are presented in Table 2. The integration in (2.1) is performed with partonic distribution functions from CTEQ5L and CTEQ6L1 sets [26]. The renormalization and factorization scales are set equal to transverse mass $\mu = m_T = \sqrt{M^2 + P_T^2}$. The leading order result for strong coupling constant with initial value $\alpha_s(\mu = M_Z) = 0.118$ is used.

In nonrelativistic limit all parameters ω_{nk} are equal zero and only $F^{(1)}(s,t)$ term survives in square brackets of (2.11). Then, replacing $\tilde{R}(0)$ by nonrelativistic value of radial wave function at the origin $R(0) = \sqrt{2/\pi}\int p^2 R(p)\,dp$ and assuming that meson mass is equal to the sum of masses of constituent quarks $M_0 = m_b + m_c$, we obtain our nonrelativistic predictions for the cross section presented in third and fifth columns of Table 2. Nonrelativistic result $R(0) = 1.18$ GeV$^{3/2}$ from our model lies close to the value $R(0) = 1.23$ GeV$^{3/2}$, which is commonly used in literature [16, 27]. Note that in the described limit our expressions for the cross section (2.11) and function $F^{(1)}(s,t)$ coincide with the appropriate analytical results presented in Ref. [16] in the framework of NRQCD.

As it shown in Table 2, relativistic corrections decrease the cross section for B_c^* mesons vector pair more than twice. In the case of pseudoscalar pair production their negative contribution amounts about 25%. Total significant decrease of the cross sections results from cumulative effect of the several sources of relativistic corrections taken into account in Eq. (2.11). First of all, there are corrections to quark–antiquark interaction considered by means of the generalized Breit potential in our model [15, 25]. They lower the appropriate wave functions by $15-30\%$, but due to the presence of the forth degree of the generalized relativistic parameter $\tilde{R}(0)$ the corresponding cross sections fall around three times. The second and the following terms in square brackets of Eq. (2.11) represent perturbative corrections in the values of quark momenta p^2 and q^2 originating from the production amplitude (2.4). Such relativistic contributions to the amplitude can be found in exact form in the quasipotential wave functions transformation law (2.5) and were extracted

Table 2: Cross sections of the pair pseudoscalar and vector B_c mesons production in proton–proton collisions

Energy $\sqrt{S}$	Meson pair	CTEQ5L		CTEQ6L1	
		σ_{nonrel}, nb	σ_{rel}, nb	σ_{nonrel}, nb	σ_{rel}, nb
$\sqrt{S}=7$ TeV	$B_c^*+\bar{B}_c^*$	0.96	0.46	0.88	0.42
	$B_c+\bar{B}_c$	0.57	0.44	0.52	0.39
$\sqrt{S}=14$ TeV	$B_c^*+\bar{B}_c^*$	2.09	1.00	1.83	0.88
	$B_c+\bar{B}_c$	1.24	0.93	1.08	0.81

explicitly in propagator expansions (2.8). These terms manifest themselves in 25% growth of the cross section. Finally, we consider the effects connected with the non-zero values of the mesons bound state energy $W = M - m_c - m_b \neq 0$. For described S-wave B_c mesons bound energy is negative, which leads to 30 and 40% cross section increase in the case of vector and pseudoscalar pairs, respectively.

The total error of the numerical results for the cross section (2.11) is estimated to be 49%. In order to obtain this value we sum in quadrature the errors from several sources of uncertainty in our analysis: relativistic wave functions (40%), forth and higher orders perturbative contributions to the amplitude (25%), and partonic distribution functions accuracy (15%) [15]. In this paper we considered pair B_c production in leading order in the strong coupling constant α_s. Next-to-leading order calculations for the similar process of pair J/ψ mesons production were performed in Ref. [28] in nonrelativistic approximation, where the corresponding impact on the cross section is shown to be about 10% for the LHCb kinematical conditions, covering effectively the whole range in transverse momentum P_T. The process $gg \to 2J/\psi$ is described by the same color singlet diagrams from Fig. 1, so that all analytical results for vector $B_c^* + \bar{B}_c^*$ pair remains valid for charmonium production in the limit $m_b \to m_c$. Then, we can assume that NLO α_s corrections to the cross section values presented in Table 2 also will be at the level of 10%. Nevertheless, such corrections become important in high P_T regions realized in the kinematical cuts of the CMS experiment [6, 8, 28].

Acknowledgments

The authors are grateful to D. Ebert, R. N. Faustov, and V. O. Galkin for useful discussions. The work was supported by Dynasty Foundation program for young scientists and by the Ministry of education and science of Russia under Competitiveness Enhancement Program of SSAU for 2013–2020 years.

References

[1] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011).

[2] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D **46**, 1914 (1992).

[3] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).

[4] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995).

[5] R. Aaij *et al.* (LHCb Collaboration), Phys. Lett. B **707**, 52 (2012).

[6] V. Khachatryan *et al.* (CMS Collaboration), J. High Energy Phys. **1409**, 094 (2014).

[7] S.P. Baranov, A.M. Snigirev, and N.P. Zotov, Phys. Lett. B **705**, 116 (2011); A. Novoselov, arXiv:1106.2184 (2011); A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, and A.A. Novoselov, Phys. Rev. D **86** 034017 (2012).

[8] J.-P. Lansberg and H.-S. Shao, arXiv:1410.8822 (2014).

[9] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **89**, 142001 (2002); Phys. Rev. D **70**, 071102 (2004); B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. D **72**, 031101 (2005).

[10] E. Braaten and J. Lee. Phys. Rev. D **67**, 054007 (2003); **72**, 099901(E) (2005); G.T. Bodwin, J. Lee, and C. Yu, Phys. Rev. D **77**, 094018 (2008).

[11] A.E. Bondar and V.L. Chernyak, Phys. Lett. B **612**, 215 (2005); V.V. Braguta, A.K. Likhoded, and A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005); G.T. Bodwin, D. Kang, and J. Lee, Phys. Rev. D **74**, 114028 (2006); A.V. Berezhnoy, Phys. At. Nucl. **71**, 1803 (2008); V.V. Braguta, Phys. Rev. D **79**, 074018 (2009).

[12] D. Ebert and A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006); D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko, Phys. Lett. B **672**, 264 (2009).

[13] V.V. Braguta, A.K. Likhoded, and A.V. Luchinsky, Phys. Lett. B **635**, 299 (2006); Y. Fan, Y.-Q. Ma, and K.-T. Chao, Phys. Rev. D **79**, 114009 (2009); E.N. Elekina and A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010); Y.-J. Li, G.-Z. Xu, K.-Y. Liu, and Y.-J. Zhang, J. High Energy Phys. **1307**, 051 (2013); arXiv:1407.3783 [hep-ph] (2014); A.P. Martynenko and A.M. Trunin, Proceedings of Science **QFTHEP2011**, 051 (2011); Phys. At. Nucl. **77**, 777 (2014).

[14] Z.-G. He, Y. Fan, and K.-T. Chao, Phys. Rev. D **81**, 054036 (2010); Y. Jia, Phys. Rev. D **82**, 034017 (2010); G.-Z. Xu, Y.-J. Li, K.-Y. Liu, and Y.-J. Zhang, Phys. Rev. D **86**, 094017 (2012).

[15] A.P. Martynenko and A.M. Trunin, Phys. Rev. D **86**, 094003 (2012); Phys. Lett. B **723**, 132 (2013).

[16] R. Li, Y.-J. Zhang, and K.-T. Chao, Phys. Rev. D **80**, 014020 (2009).

[17] V.V. Kiselev, A.K. Likhoded, S.R. Slabospitsky, and A.V. Tkabladze, Sov. J. Nucl. Phys. **49**, 682 (1989).

[18] E. Braaten, S. Fleming, and T.C. Yuan, Annu. Rev. Nucl. Part. Sci. **46**, 197 (1996).

[19] M. Krämer, Prog. Part. Nucl. Phys. **47**, 141 (2001); J.-P. Lansberg, Int. J. Mod. Phys. A **21**, 3857 (2006).

[20] B. Humpert and P. Méry, Z. Phys. C **20**, 83 (1983).

[21] J. Kublbeck, M. Böhm, and A. Denner, Comp. Phys. Comm. **60**, 165 (1990); T. Hahn, Comp. Phys. Comm. **140**, 418 (2001).

[22] J.A.M. Vermaseren, arXiv:math-ph/0010025 (2000).

[23] S.J. Brodsky and J.R. Primack, Ann. Phys. (N.Y.) **52**, 315 (1969).

[24] R.N. Faustov, Ann. Phys. (N.Y.) **78**, 176 (1973).

[25] A.P. Martynenko and A.M. Trunin, Phys. Rev. D **89**, 014004 (2014).

[26] H.L. Lai *et al.*, Eur. Phys. J. C **12**, 375 (2000); J. Pumplin *et al.*, J. High Energy Phys. **0207**, 012 (2002).

[27] E.J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).

[28] L.-P. Sun, H. Han, and K.-T Chao, arXiv:1404.4042 (2014).

PoS(Baldin ISHEPP XXII)118