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Loop mixing of the opposite parity fermion fields and its manifestation in πN scattering

Vladimir Lomov*
Institute for System Dynamics and Control Theory of SB RAS, Irkutsk
E-mail: v.p.lomov@gmail.com

Kaloshin A.E.
Irkutsk State University, Irkutsk
E-mail: shin7@rambler.ru

Kobeleva E.A.
Irkutsk State University, Irkutsk
E-mail: elenyich@mail.ru

We develop a variant of K-matrix, which includes the effect of opposite parity fermions (OPF) mixing, and apply it for description of πN partial waves S_{11} and P_{11}. OPF-mixing leads to appearance of negative energy poles in K-matrix and restoration of MacDowell symmetry, relating two partial waves. Joint analysis of PWA results for S_{11} and P_{11} confirms significance of this effect.

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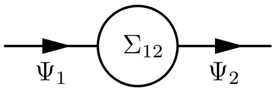
*Speaker.

1. Introduction

For fermions there exists a non-standard mixing, when fermion fields with opposite parities are mixing at loop level while parity is conserved in vertex (shortly OPF-mixing):



It is possible because fermion and antifermion have different parities. This effect was investigated in detail in Ref. [1] and was applied to πN scattering, where it leads to relation between two partial waves. In Ref. [1], the simplest physical example of manifestation of this effect was found: the partial waves P_{13} and D_{13}, where baryons $J = 3/2^{\pm}$ are produced. The OPF-mixing effect is identified in the partial wave P_{13} as rather specific interference of resonance with background generated by resonance state in D_{13} wave. The above-mentioned relation between partial waves mainly influences on a wave with lower orbital momentum and it is used as additional source of information about structure of wave with higher l.

Another physical example, where OPF-mixing may be essential, is related to the partial waves S_{11} and P_{11}, where resonances $J^P = 1/2^{\pm}, I = 1/2$ are produced. Most interesting object here is the Roper resonance $N(1440)$, which has some unusual properties and problems with quark-models identification, see, e.g. Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10]. However, in presence of several resonance states the approach of Ref. [1], that uses a matrix propagator, becomes too cumbersome. Alternatively, for description of OPF-mixing, one can use the K-matrix approach, which works for any number of states and channels.

Here we present the K-matrix approach for πN partial amplitudes with accounting of the OPF-mixing effect and apply it for description of S_{11} and P_{11} partial waves. Most serious changing as compared with its standard form is the appearance of negative energy poles in K-matrix. If, besides, we use QFT to calculate tree amplitudes (i.e. K-matrix), starting from effective Lagrangians, we obtain the partial amplitudes $\pi N \to \pi N$ satisfying the MacDowell symmetry condition:

$$f_{l,+}(W) = -f_{l+1,-}(-W), \tag{1.1}$$

which was obtained in Ref. [11] from general analytic properties of amplitudes.

We use the obtained K-matrix to describe results of partial wave analysis for S_{11} and P_{11} amplitudes. The main purpose is to see the manifestation of OPF-mixing and it naturally leads to joint fitting of these two waves.

2. OPF-mixing and K-matrix

We need to discuss the effect of OPF-mixing in amplitudes of πN scattering and its implementation in framework of K-matrix description. For a first step one may restrict oneself by a simplified case: two resonance states and two channels.

Effective Lagrangians $\pi NN'$ without derivatives and conserving the parity:

$$\mathscr{L}_{\text{int}} = g_1 \bar{N}_1(x) N(x) \phi(x) + \text{h.c.}, \qquad \text{for } J^P(N_1) = 1/2^-, \tag{2.1}$$

$$\mathscr{L}_{\text{int}} = \imath g_2 \bar{N}_2(x) \gamma^5 N(x) \phi(x) + \text{h.c.}, \quad \text{for } J^P(N_2) = 1/2^+. \tag{2.2}$$

Let us consider two baryon states of opposite parities with masses m_1 ($J^P = 1/2^-$), m_2 ($J^P = 1/2^+$) and two intermediate states πN, ηN. Using the effective Lagrangians we can calculate contributions of states N_1, N_2 to partial waves at tree level:
s-wave amplitudes:

$$\begin{aligned}
f^{\text{tree}}_{s,+}(\pi N\to\pi N) &= -\frac{(E_N^{(\pi)}+m_N)}{8\pi W}\left(\frac{g^2_{1,\pi}}{W-m_1}+\frac{g^2_{2,\pi}}{W+m_2}\right),\\
f^{\text{tree}}_{s,+}(\pi N\to\eta N) &= -\frac{\sqrt{\left(E_N^{(\pi)}+m_N\right)\left(E_N^{(\eta)}+m_N\right)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{W-m_1}+\frac{g_{2,\pi}g_{2,\eta}}{W+m_2}\right),\\
f^{\text{tree}}_{s,+}(\eta N\to\eta N) &= -\frac{(E_N^{(\eta)}+m_N)}{8\pi W}\left(\frac{g^2_{1,\eta}}{W-m_1}+\frac{g^2_{2,\eta}}{W+m_2}\right)
\end{aligned} \tag{2.3}$$

and *p*-wave amplitudes:

$$\begin{aligned}
f^{\text{tree}}_{p,-}(\pi N\to\pi N) &= \frac{(E_N^{(\pi)}-m_N)}{8\pi W}\left(\frac{g^2_{1,\pi}}{-W-m_1}+\frac{g^2_{2,\pi}}{-W+m_2}\right),\\
f^{\text{tree}}_{p,-}(\pi N\to\eta N) &= \frac{\sqrt{\left(E_N^{(\pi)}-m_N\right)\left(E_N^{(\eta)}-m_N\right)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{-W-m_1}+\frac{g_{2,\pi}g_{2,\eta}}{-W+m_2}\right),\\
f^{\text{tree}}_{p,-}(\eta N\to\eta N) &= \frac{(E_N^{(\eta)}-m_N)}{8\pi W}\left(\frac{g^2_{1,\eta}}{-W-m_1}+\frac{g^2_{2,\eta}}{-W+m_2}\right).
\end{aligned} \tag{2.4}$$

Here $W=\sqrt{s}$ is the total CMS energy and $E_N^{(\pi)}$ ($E_N^{(\eta)}$) is nucleon CMS energy of system πN (ηN)

$$E_N^{(\pi)} = \frac{W^2+m_N^2-m_\pi^2}{2W}. \tag{2.5}$$

Short notations for coupling constants, e.g. $g_{1,\pi} = g_{N_1\pi N}$.

The tree amplitudes (2.3)–(2.4) contain poles with both positive and negative energy, originated from propagators of N_1 and N_2 fields of opposite parities. Accounting the loop transitions results in dressing of states and also in mixing of these two fields.

Note that $W\to -W$ replacement gives

$$E_N^{(\pi)}+m_N \to -\left(E_N^{(\pi)}-m_N\right), \tag{2.6}$$

so tree amplitudes (2.3)–(2.4) exhibit the MacDowell symmetry property [11]

$$f_{p,-}(W) = -f_{s,+}(-W). \tag{2.7}$$

In *K*-matrix representation for partial amplitudes

$$f = K\left(1-\imath PK\right)^{-1}, \tag{2.8}$$

diagonal matrix $\imath P$, constructed from CMS momenta, originates from imaginary part of a loop. Therefore, *K*-matrix here is simply a matrix of tree amplitudes that should be identified with amplitudes (2.3), (2.4).

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As a result we come to representation of partial amplitudes for s- and p-waves

$$f_s(W) = K_s(W)\big(1 - \imath P K_s(W)\big)^{-1}, \quad f_p(W) = K_p(W)\big(1 - \imath P K_p(W)\big)^{-1}, \tag{2.9}$$

where the matrices K_s, K_p (i.e. tree amplitudes (2.3), (2.4)), may be written in factorized form

$$K_s = -\frac{1}{8\pi}\rho_s \hat{K}_s \rho_s, \quad K_p = \frac{1}{8\pi}\rho_p \hat{K}_p \rho_p. \tag{2.10}$$

Here ρ_s, ρ_p are

$$\rho_s(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} + m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} + m_N}{W}} \end{pmatrix}, \tag{2.11}$$

$$\rho_p(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} - m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} - m_N}{W}} \end{pmatrix}, \tag{2.12}$$

and matrix P consists of CMS momenta as analytic functions of W. In this case "primitive" K-matrices contain poles with both positive and negative energies

$$\hat{K}_s(W) = \begin{pmatrix} \dfrac{g_{1,\pi}^2}{W-m_1} + \dfrac{g_{2,\pi}^2}{W+m_2}, & \dfrac{g_{1,\pi}g_{2,\eta}}{W-m_1} + \dfrac{g_{2,\pi}g_{2,\eta}}{W+m_2} \\ \dfrac{g_{1,\pi}g_{2,\eta}}{W-m_1} + \dfrac{g_{2,\pi}g_{2,\eta}}{W+m_2}, & \dfrac{g_{1,\eta}^2}{W-m_1} + \dfrac{g_{2,\eta}^2}{W+m_2} \end{pmatrix}, \tag{2.13}$$

$$\hat{K}_p(W) = \hat{K}_s(-W) = \begin{pmatrix} \dfrac{g_{1,\pi}^2}{-W-m_1} + \dfrac{g_{2,\pi}^2}{-W+m_2}, & \dfrac{g_{1,\pi}g_{2,\eta}}{-W-m_1} + \dfrac{g_{2,\pi}g_{2,\eta}}{-W+m_2} \\ \dfrac{g_{1,\pi}g_{2,\eta}}{-W-m_1} + \dfrac{g_{2,\pi}g_{2,\eta}}{-W+m_2}, & \dfrac{g_{1,\eta}^2}{-W-m_1} + \dfrac{g_{2,\eta}^2}{-W+m_2} \end{pmatrix}. \tag{2.14}$$

Recall that m_1 is mass of $J^P = 1/2^-$ state and m_2 is mass of $J^P = 1/2^+$ one. Generalization of this construction for the case of more channels and states is obvious.

Since CMS momenta have the property $P(-W) = -P(W)$, the MacDowell symmetry property (2.7) is extended from tree amplitudes to unitarized K-matrix ones (2.9).

From a common sense one can expect that negative energy pole should give a negligible effect in physical energy region. However, this is not the case if corresponding coupling constant is large $|g_{2,\pi}| \gg |g_{1,\pi}|$. One can compare decay width of s- and p-states

$$\Gamma(N_1 \to \pi N) = g_{N_1\pi N}^2 \Phi_s, \quad \Gamma(N_2 \to \pi N) = g_{N_2\pi N}^2 \Phi_p, \tag{2.15}$$

where Φ_s, Φ_p are corresponding phase volumes. For resonance states not far from threshold, with masses, e.g. 1.5–1.7 GeV, phase volumes differ greatly, $\Phi_s \gg \Phi_p$. If both resonances have typical hadronic width $\Gamma \sim 100$ MeV, then coupling constants differ dramatically too, $|g_{N_2\pi N}| \gg |g_{N_1\pi N}|$.

Above we use the simplest effective Lagrangians (2.1)–(2.2) to derive tree amplitudes. However, it is well-known, that spontaneous breaking of chiral symmetry requires pion field to appear in Lagrangian only through derivatives

$$\mathscr{L}_{\text{int}} = f_2 \bar{N}_2(x)\gamma^5\gamma^\mu N(x)\partial_\mu\phi(x) + \text{h.c.},\ J^P = 1/2^+,\ f_2 = \frac{g_2}{m_2+m_N}. \tag{2.16}$$

It is not difficult to understand how inclusion of derivative changes tree amplitudes and, hence K-matrix. Pole contribution $\pi(k_1)N(p_1) \to N_2(p) \to \pi(k_2)N(p_2)$ in that case takes the form:

$$T = f_2^2 \bar{u}(p_2)\gamma^5\hat{k}_2\frac{1}{\hat{p}-M}\gamma^5\hat{k}_1 u(p_1). \tag{2.17}$$

With the use of equations of motion, we see that inclusion of derivative at vertex leads to the following modification of resonance contribution

$$g_2^2\frac{1}{\hat{p}-M} \to f_2^2(\hat{p}+m_N)\frac{1}{\hat{p}-M}(\hat{p}+m_N). \tag{2.18}$$

Separation of the positive and negative energy poles is performed with the off-shell projector operators $\Lambda^\pm = 1/2\big(1\pm\hat{p}/W\big)$

$$f_2^2(\hat{p}+m_N)\frac{1}{\hat{p}-m_N}(\hat{p}+m_N) = \Lambda^+\frac{f_2^2(W+m_N)^2}{W-M} + \Lambda^-\frac{f_2^2(W-m_N)^2}{-W-M}, \tag{2.19}$$

where the first term gives contribution to p-wave and second one to s-wave. Modification of the pole contributions in “primitive” K-matrices (2.13)–(2.14) is evident

$$g_2^2 \to f_2^2(W-m_N)^2, \quad \text{for } s\text{-wave}, \tag{2.20}$$
$$g_2^2 \to f_2^2(W+m_N)^2, \quad \text{for } p\text{-wave}. \tag{2.21}$$

One can expect that the inclusion of derivatives most strongly affects on threshold properties of s-wave due to dumping factor $(W-m_N)^2$.

3. Partial amplitudes P_{11} and S_{11} of πN scattering

First of all, let us try to describe S_{11} and P_{11} waves separately. p-wave is described rather well by our formulas with derivative in vertex (2.20)– (2.21), see Fig. 1 where solid lines represent our amplitudes (2.9)–(2.14) in the presence of derivative in vertex (2.20)– (2.21). In this case the s-wave states are missing in amplitudes, the p-wave K-matrix has two positive energy poles.

Quality of description is defined by:

$$\chi^2/\text{DOF} = 273/95. \tag{3.1}$$

The use of vertices without derivative leads to impairment of quality of description: $\chi^2 > 350$, again we need two poles with close masses.

Both variants give a negative background contribution to S_{11} wave, comparable in magnitude with other contributions, as it seen on Fig. 2. This figure shows background contribution to s-wave generated by p-wave states, i.e. in this case K-matrix for s-wave (2.13) has only negative

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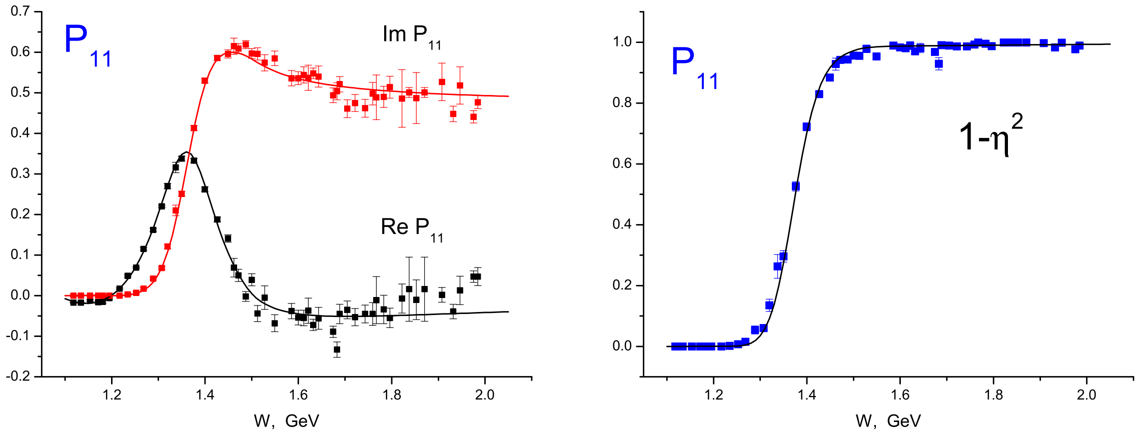

Figure 1: The results of fitting of P_{11}-wave of πN scattering. Dots show results of PWA [12], solid lines represent our amplitudes in the presence of derivative in vertex. K-matrix has only p-wave states. On the right side: p-wave inelasticity [12], the curve corresponds to lines on the left side. Partial wave normalization corresponds to Ref. [12]: $\mathrm{Im} f = |f|^2 + (1-\eta^2)/4$.

energy poles. Variant without derivative in vertex gives a larger background contribution, rapidly changing near thresholds. It seems that description of P_{11} partial wave without derivative in vertices contradicts to data on S_{11}. On Fig. 2 some typical curves are shown, there exist different variants with sharp behavior near thresholds. The presence of derivative in a vertex suppresses the threshold region in background contribution due to factor $(W - m_N)^2$, but in resonance region this is rather large contribution, see Fig. 2.

Attempt to describe S_{11} without background has no success: it doesn't allow to reach even qualitative agreement with PWA.

As a next step, let us add the background contribution, arising from p-wave states (solid lines on Fig. 1) with fixed parameters of p-wave.

One can see from Fig. 3 that quality of description is unsatisfactory in this case but double-peak behavior is arisen in partial wave for the first time. It means that to describe S_{11} wave a background contribution is necessary and its value is close to solid line curves at Fig. 1

4. Joint fit of S_{11} and P_{11}

Let's perform the joint analysis of S_{11} and P_{11} amplitudes, when resonance states in one wave generate background in other and vice versa. In this case K-matrices (2.13)–(2.14) have poles with both positive and negative energies: we use two s-wave and two p-wave poles. This leads to noticeable improvement of description, as can be seen from Fig. 4; in this case $\chi^2/\mathrm{DOF} = 850/190$.

At last, background can be generated not only by negative energy poles but by other terms. We accounted it by adding to elastic amplitudes $\pi N \to \pi N$ a smooth contributions of the form:

$$\hat{K}_s^B = A + B(W - m_N)^2, \quad \hat{K}_p^B = A + B(W + m_N)^2, \tag{4.1}$$

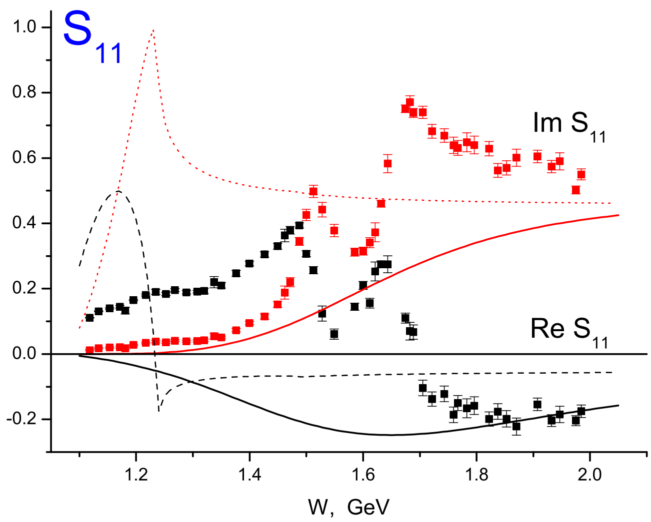

Figure 2: Background contribution to *s*-wave, generated by *p*-wave states. Solid lines represent variant with derivative in vertex (corresponding to curves on Fig. 1), dashed lines – variant without derivative in vertex.

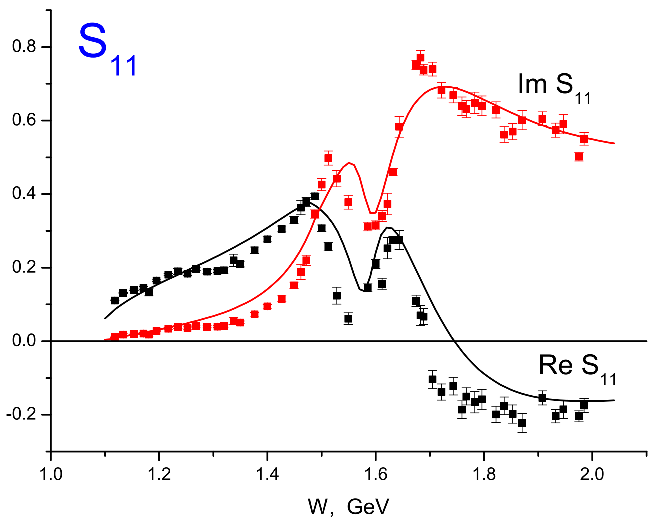

Figure 3: Results of *s*-wave fitting with fixed parameters for *p*-wave states. Parameters of *p*-wave correspond to curves on Fig. 1, *s*-wave contains two states with *K*-matrix masses 1.55 and 1.75 GeV.

which do not violate the MacDowell symmetry property. Note that we have quite good description $\chi^2/\mathrm{DOF} = 584/187$ and background contribution in S_{11} is close to simplest variant of Fig. 2.

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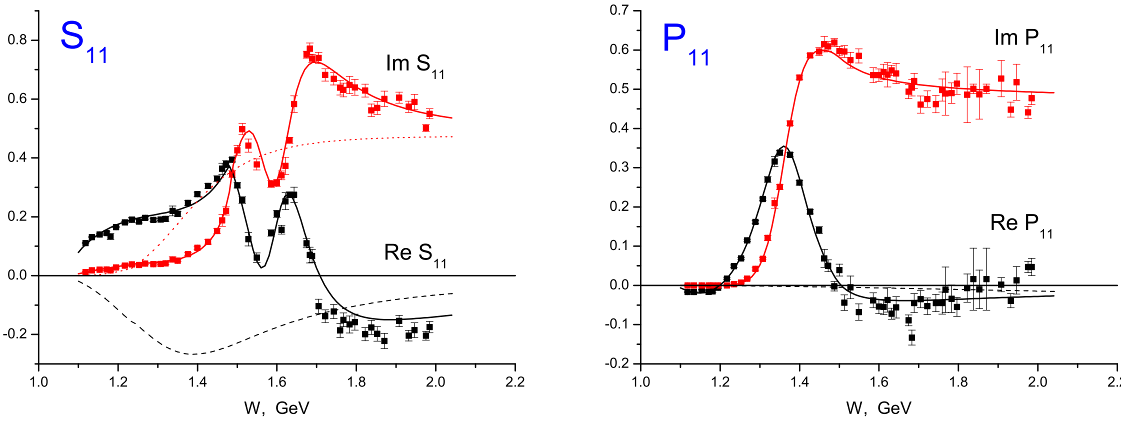

Figure 4: Result of joint fitting of S_{11} and P_{11}-waves of πN scattering. Dashed lines show real and imaginary parts of (unitarized) background contribution.

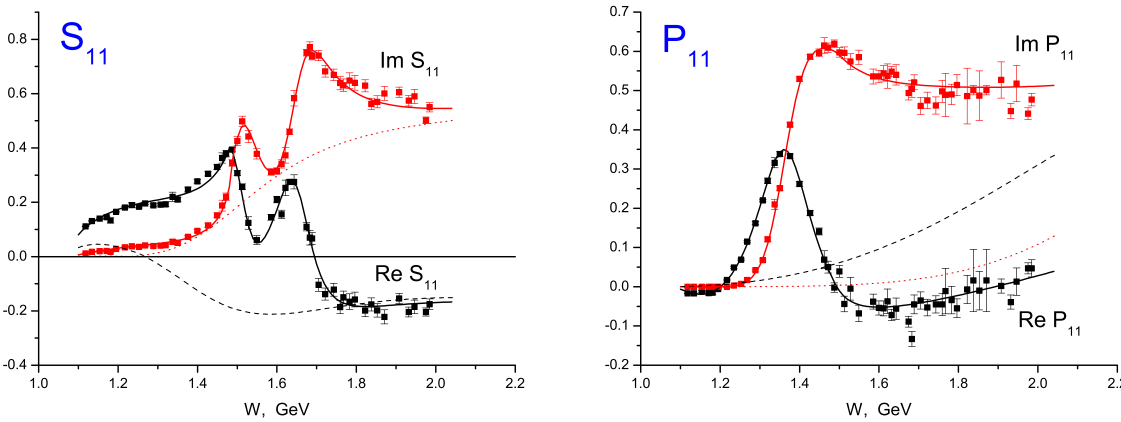

Figure 5: Result of joint fitting of S_{11} and P_{11} waves of πN scattering.

5. Poles in complex plane

In Table 1 we present the pole masses and widths obtained by continuation of our amplitudes to complex W plane. As a whole, we see that our values for m_p, Γ_p are rather close to previously obtained. The only hint for disagreement is appearance at some sheets of a stable pole $1/2^+$ with $m_p \approx 1500$ MeV instead of generally accepted mass $m_p \approx 1365$ MeV.

6. Conclusions

In this report we investigated the manifestation of OPF-mixing in πN partial waves S_{11} and P_{11}, where baryons $1/2^{\pm}$, $I = 1/2$ are produced. We found that the effect of mixing of fermion fields with opposite parity can be readily realized in the framework of K-matrix approach. It allows to

Partial wave, PDG values	This work	Some other works
S_{11}, $1/2^-$		
N(1535) (1510, 70)	(1507, 87)	(1502, 95), (1648, 80) [12]
N(1650) (1655, 165)	(1659, 149)	(1519, 129), (1669, 136) [13]
P_{11}, $1/2^+$		
N(1440) (1365, 190)	(1365, 194)	(1359, 162) [12]
	(1500, 160)	(1385, 164) [14]
		(1387, 147) [13]

Table 1: Pole masses and widths (M_R, Γ_R) extracted from poles position in the complex plane W: $W_0 = M_R - \imath\Gamma_R/2$.

have simple expressions for amplitudes in the case of any resonance states and reaction channels. Note that s- and p-wave K-matrices, (2.13)–(2.14), have poles with both positive and negative energies and are related with each other by $\hat{K}_p(W) = \hat{K}_s(-W)$.

The so-constructed partial waves possess the well-known MacDowell symmetry that connects two partial waves under substitution $W \to -W$. Up to now, this symmetry did not play any role in data analysis since it connects physical and unphysical regions. However, taking OPF-mixing into account, MacDowell symmetry leads to physical consequences: resonance in one partial wave gives rise to background contribution in another and vice versa. This connection between two waves, as in case of $3/2^{\pm}$ resonances,[1] works mainly in one direction: it generates large negative background in a wave with lower orbital momentum. So we come to idea of joint analysis of two partial waves and it allows to get an additional information about dynamics in higher l wave. Such an example can be seen at Fig. 2, where two variants of background in S_{11} are depicted.

Our main purpose here was to see the effects of OPF-mixing in the amplitudes S_{11}, P_{11} and to estimate their value. So, following Ref. [15], we have used simplified three-channel formalism in which σN is some quasi-channel, imitating different $\pi\pi N$ intermediate states. In spite of the rough approach we obtained rather good description of S_{11} and P_{11} waves, comparable well with more comprehensive analyses [16, 17, 18, 19] with number of channels up to 6. We suppose that OPF-mixing (or MacDowell symmetry) can be taken into account not only in K-matrix formalism but in framework of more detailed dynamical multi-channel approach.

Note that obtained pole positions not always coincide with the results of previous analyses. For example, for $N(1440)$ state we found on most sheets a very stable pole with Re$W \approx 1500$ MeV instead of "standard" value ≈ 1360 MeV, see Table 1. After various verifications we suppose that this is result of crudity of used approximation (effective σN channel). But it is possible that there exists some dependency on details of description and it needs more close investigation.

Summarizing, we found out that effect of a loop OPF-mixing is seen in PWA results as a connection between partial waves S_{11} and P_{11}. We assume that this connection may be of interest as possibility to obtain additional information about P_{11} wave and baryons $1/2^+$.

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References

[1] A.E. Kaloshin, E.A. Kobeleva, and V.P. Lomov, *Mixing of fermion fields of opposite parities and baryon resonances*, *Int.J.Mod.Phys.* **A26** 2307 (2011).

[2] O. Krehl, C. Hanhart, S. Krewald, and J. Speth, *What is the structure of the Roper resonance?*, *Phys.Rev.* **C62** 025207 (2000).

[3] M. Batinic, I. Slaus, A. Svarc, and B.M.K. Nefkens, *$\pi N \to \eta N$ and $\eta N \to \eta N$ partial wave T matrices in a coupled, three channel model*, *Phys.Rev.* **C51** 2310 (1995).

[4] L.Ya. Glozman and D.O. Riska, *The Spectrum of the nucleons and the strange hyperons and chiral dynamics*, *Phys.Rept.* **268** (1996).

[5] S. Capstick and W. Roberts, *Quark models of baryon masses and decays*, *Prog.Part.Nucl.Phys.* **45** 241 (2000).

[6] N. Mathur, Y. Chen, S.J. Dong, T. Draper, I. Horvath, et al, *Roper resonance and $S_{11}(1535)$ from lattice QCD*, *Phys.Lett.* **B605** 137 (2005).

[7] M. Dillig and M. Schott, *Mesonic content of the nucleon and the Roper resonance*, *Phys.Rev.* **C75** 067001 (2007).

[8] A.V. Sarantsev, M. Fuchs, M. Kotulla, U. Thoma, J. Ahrens, et al, *New results on the Roper resonance and the $P(11)$ partial wave*, *Phys.Lett.* **B659** 94 (2008).

[9] B. Julia-Diaz and D.O. Riska, *The Role of $qqqq\bar{q}$ components in the nucleon and the N(1440) resonance*, *Nucl.Phys.* **A780** 175 (2006).

[10] C.D. Roberts, *Hadron Properties and Dyson–Schwinger Equations*, *Prog.Part.Nucl.Phys.* **61** 50 (2008).

[11] S.W. MacDowell, *Analytic properties of partial amplitudes in meson-nucleon scattering*, *Phys. Rev.* **116** 774 (1959).

[12] R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, and R.L. Workman, *Extended partial-wave analysis of πN scattering data*, *Phys.Rev.* **C74** 045205 (2006).

[13] M. Doring, C. Hanhart, F. Huang, S. Krewald, and U.-G. Meissner, *Analytic properties of the scattering amplitude and resonances parameters in a meson exchange model*, *Nucl.Phys.* **A829** 170 (2009).

[14] G. Hohler, *What can be learned from elastic electron-nucleon scattering experiments?*, *πN Newslett.* 108 (1993).

[15] S. Ceci, J. Stahov, A. Svarc, S. Watson, and B. Zauner, *Resolution of the multichannel anomaly in the extraction of S-matrix resonance-pole parameters*, *Phys.Rev.* **D77** 116007 (2008).

[16] S.X. Nakamura, *Extraction of P_{11} resonance from πN data and its stability*, *AIP Conf.Proc.* **1374** 505 (2011).

[17] H. Kamano, S.X. Nakamura, T.-S.H. Lee, and T. Sato, *Extraction of P_{11} resonances from πN data*, *Phys.Rev.* **C81** 065207 (2010).

[18] B. Golli and S. Sirca, *Roper resonances in chiral quark models*, *Eur.Phys.J.* **A38** 271 (2008).

[19] M.W. Paris and R.L. Workman, *Toward a unified description of hadro- and photoproduction: S-wave π- and η-photoproduction amplitudes*, *Phys.Rev.* **C82** 035202 (2010).