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How is neutrino energy reconstruction affected by nuclear effects?

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The existence of three nonvanishing angles in neutrino mixing matrix is now fully established, paving the way for studies of charge-parity symmetry violation in the lepton sector, which may shed light on the matter-antimatter asymmetry in the universe. As neutrino oscillations are energy dependent, and modern experiments employ polychromatic beams, precise oscillation measurements require neutrino energy to be precisely deduced from the measured kinematics of the reaction products. This paper is focused on difficulties related to energy reconstruction in charged-current quasielastic events. I analyze in detail the standard reconstruction method, paying special attention to the parameter of nucleon separation energy in the reconstruction formula. The role of backgrounds is covered by reviewing results available in the literature. I also discuss the effects of final-state interactions and present a possible improvement of the energy reconstruction method.

16th International Workshop on Neutrino Factories and Future Neutrino Beam Facilities—NUFACT2014, 25–30 August, 2014
University of Glasgow, United Kingdom

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1. Introduction

The past few years have brought significant progress in the determination of neutrino oscillation parameters [1]. In addition to the mixing angles θ_{12} and θ_{23}, governing oscillations of solar [2] and atmospheric [3] neutrinos, respectively, the mixing angle θ_{13} is now well established to have nonvanishing value, mainly by the results reported from reactor experiments [4].

Thanks to the large enough value of θ_{13}, the Dirac phase δ_{CP}, appearing in the oscillation matrix, has the potential to give an important—or event dominant—contribution to the matter-antimatter asymmetry in the universe [5]. It is noteworthy that preference for the maximal charge-parity (CP) symmetry violation, $\delta_{CP} = -\pi/2$, has been recently observed combining results from the T2K and reactor experiments [6].

While ongoing experimental programs, such as the T2K and NOνA [7], are expected to provide further constrains on the δ_{CP} value and the neutrino-mass hierarchy, their decisive determination will not be possible without the next generation of experiments [8], planned to study neutrino oscillations with unprecedented precision.

As the oscillation probability is a function of neutrino energy, precise determination of the oscillation parameters requires precise determination of the energy. In modern experiments, neutrino beams are tertiary, originating from decaying pions produced by the primary proton beams. As a consequence, they are nonmonochromatic, and for oscillation analysis, the neutrino energy in an event must be inferred from the measured kinematics of the *detected* particles. It has to be kept in mind, however, that propagating through nuclear medium, the particles produced in the primary vertex of neutrino interaction may undergo final-state interactions, changing their energy or leading to their absorption or production of secondary particles. Therefore, an accurate estimate of nuclear effects in the Monte Carlo simulations involved in data analysis is a prerequisite for a precise measurement of neutrino oscillation parameters.

An accurate reconstruction of hadron energy is a formidable experimental task, and therefore various experiments collect the data in the region where charged-current (CC) quasielastic (QE) scattering,

$$\nu_\ell + n \to \ell^- + p \qquad \text{and} \qquad \bar{\nu}_\ell + p \to \ell^+ + n,$$

for nucleons bound in nuclei, is the dominant reaction mechanism [9]. Should CC QE interaction involve a free nucleon at rest, the (anti)neutrino energy could be exactly determined from the measured kinematics—the energy E_ℓ and the production angle θ—of the charged lepton only. The energy and momentum conservation then give

$$E_\nu^{(N\text{ at rest})} = \frac{2ME_\ell + M'^2 - M^2 - m_\ell^2}{2(M - E_\ell + |\mathbf{k}_\ell|\cos\theta)}, \tag{1.1}$$

with the initial (final) nucleon's mass M (M'), the charged lepton's mass m and its momentum $|\mathbf{k}_\ell| = \sqrt{E_\ell^2 - m_\ell^2}$.

However, owing to the low cross sections, neutrino experiments have to employ nuclear targets, in which nucleons undergo Fermi motion. In this case, the exact expression for the neutrino energy [10],

$$E_\nu = \frac{2E_{\mathbf{p}}E_\ell - 2|\mathbf{k}_\ell||\mathbf{p}|\cos\vartheta_\ell + M'^2 - (E_{\mathbf{p}}^2 - \mathbf{p}^2) - m_\ell^2}{2(E_{\mathbf{p}} - |\mathbf{p}|\cos\vartheta_\nu - E_\ell + |\mathbf{k}_\ell|\cos\theta)}, \tag{1.2}$$

involves not only the norm of the struck-nucleon's momentum $\mathbf{p}$, but also the angles ϑ_ν and ϑ_ℓ its vector forms with the neutrino-beam direction and with the charged-lepton's momentum, respectively. The struck nucleon is off the mass shell, and its energy can be cast in the form $E_\mathbf{p} = M_A - E_{A-1}$, with the mass of the target nucleus M_A and the energy of the residual $(A-1)$-nucleon system E_{A-1}.

As the information on $\mathbf{p}$ and E_{A-1} is unavailable, one needs to resort to the approximations $|\mathbf{p}| \approx 0$ and $E_\mathbf{p} \approx M - \varepsilon$, ε being the nucleon separation energy, defining the reconstructed energy as

$$E_\nu^{\rm rec} = \frac{2(M-\varepsilon)E_\ell + M'^2 - (M-\varepsilon)^2 - m_\ell^2}{2(M-\varepsilon-E_\ell+|\mathbf{k}_\ell|\cos\theta)}. \tag{1.3}$$

Note that—for example in backward scattering—the expression $(E_\ell - |\mathbf{k}_\ell|\cos\theta)$ may be greater than $(M-\varepsilon)$ and therefore, the reconstructed energy does not have to be positive.

To perform oscillation analysis, the true neutrino energy needs to be unfolded from $E_\nu^{\rm rec}$. This procedure may involve the probability distributions that a charged lepton of given kinematics originates from the interaction of a neutrino of energy E_ν, such as those for ν_μ scattering shown in Fig. 1 [11]. While for fixed E_ℓ and θ, Eq. (1.3) yields a single value of $E_\nu^{\rm rec}$, in realistic situation, the charged lepton of given kinematics may be produced with different probabilities by a range of true neutrino energies. It is a consequence of nucleons Fermi motion, distribution of their initial energy, and final-state interactions.

Because the procedure of unfolding is significantly affected by the description of nuclear structure and dynamics, precise oscillation measurements need to rely on theoretical models capable of accurate estimate of nuclear effects in available electron scattering data.

Owing to the relevance for the ongoing oscillation experiments, this paper focuses on difficulties of energy reconstruction for CC QE event candidates at the energy ∼1 GeV. In Sec. 2, as a pedagogical example, an ideal case of monochromatic beam of unknown energy is considered. Additional complications arising for polychromatic beams are discussed in Sec. 3. In Sec. 4, the effect of final-state interactions is discussed and a new method allowing for improved energy reconstruction is presented. Finally, Sec. 5 summarizes the paper.

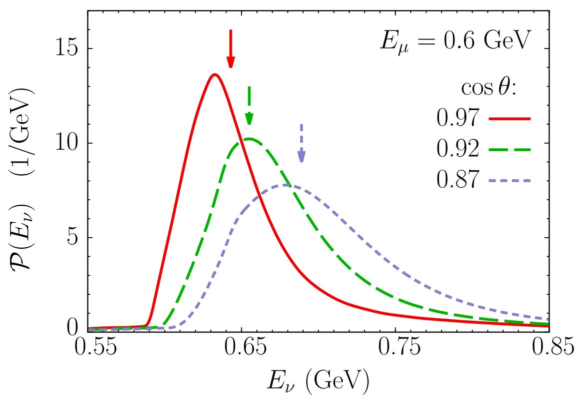

Figure 1: Comparison of the ν_μ energy reconstructed from Eq. (1.3), represented by arrows, with the realistic calculations [11] of the energy distributions (lines), obtained for the muon energy 0.6 GeV and the $\cos\theta$ values of 0.97, 0.92, and 0.87.

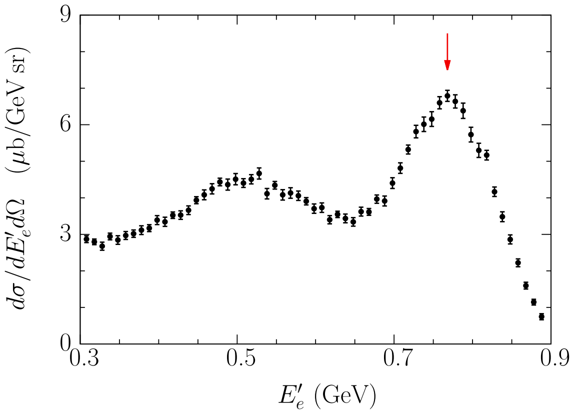

Figure 2: Typical example of the inclusive cross section for electron scattering off nuclear target at the energy ~1 GeV. The experimental data [12] are collected for carbon using 961-MeV beam scattered at 37.5 degrees. The position of quasielastic peak is shown by arrow.

2. Ideal case

Consider an unrealistic but illustrative example of a monochromatic neutrino beam of unknown energy ~1 GeV. To avoid model dependence of the discussion, we can refer to electron scattering data, such as those from Ref. [12] presented in Fig. 2, and analyze them using the methods applied in neutrino physics.

At the kinematics of interest, the electron mass can be safely neglected, and the energy reconstruction formula (1.3) simplifies to

$$E_e^{\rm rec} = \frac{2(M-\varepsilon)E'_e + M'^2 - (M-\varepsilon)^2}{2\big[M-\varepsilon-E'_e(1-\cos\theta)\big]}, \tag{2.1}$$

where E'_e is the scattered electron's energy.

Because the beam is monochromatic, we can readily interpret the data at given scattering angle and identify the region dominated by QE mechanism of interaction, without using information on the topology of events. In the data shown in Fig. 2, collected at $\theta = 37.5^\circ$ for the carbon target, this region corresponds to E'_e between approximately 658 and 888 MeV, and the QE peak is located at $E'_e = 768 \pm 5$ MeV, with the uncertainty resulting from the 10-MeV binning of the data.

Applying $\varepsilon = 25$ MeV, determined for 500-MeV electron scattering at 60° off carbon [13], and the E'_e value at the QE peak, we obtain from (2.1) the reconstructed energy 960 ± 7 MeV, in excellent agreement with the true energy 961 MeV. Should the energy be reconstructed for all the QE events of E'_e between 658 and 888 MeV, the corresponding $E_e^{\rm rec}$ values would vary between 803 and 1143 MeV.

While we observe an excellent agreement between the true and reconstructed energy in the considered example, it is important to check whether this feature holds true at other kinematical setups. Comparisons performed at different beam energies, using the data from Refs. [12, 14], are presented in Table 1. It clearly appears that the lower the beam energy, the worse the accuracy of energy reconstruction. These results are a consequence of the fact that a fixed parameter ε cannot reproduce the position of the QE peak over a broad range of beam energies or scattering angles, largely due to the final-state interactions. I will return to this issue in Sec. 4.

Table 1: Analysis of energy reconstruction method using experimental ${}^{12}_{6}\mathrm{C}(e,e')$ data [12, 14]. Knowing the scattering angle θ and the position of the quasielastic peak E'_e, the reconstructed energy $E_e^{\rm rec}$ is obtained using the average separation energy $\varepsilon = 25$ MeV, and compared to the true value of beam energy $E_e^{\rm true}$. The last row gives the ε value which should be applied for the reconstructed energy to match the true one.

θ	(deg)	37.5	37.1	36.0	36.0
E'_e	(MeV)	768 ± 5	615 ± 5	487.5 ± 5	287.5 ± 2.5
$E_e^{\rm rec}$	(MeV)	960 ± 7	741 ± 7	571 ± 6	333 ± 3
$E_e^{\rm true}$	(MeV)	961	730	560	320
ε	(MeV)	26 ± 5	16 ± 5	16 ± 3	13 ± 3

Reversing the problem, we could ask what ε is required to reconstruct the energy exactly. Such determined values, listed in the last row of Table 1, sizably change for the selected cases.

The first problem encountered in reconstruction of the neutrino energy is how to find the value of the average separation energy in Eq. (1.3). The answer turns out to depend not only on the nuclear target, but also on the kinematics of the experiment. It is important to note that focusing our analysis on the top of the QE peak, we have selected the kinematics where influence of Fermi motion is minimized, and the observed effects are predominantly related to the energy distribution of nucleons inside nucleus.

3. Realistic case

Additional complications appear for polychromatic beams, used—out of necessity—in modern neutrino experiments.

On the one hand, at fixed scattering angle, different neutrino energies yield different positions of the QE peak, allowing energy reconstruction from Eq. (1.3). On the other hand, E_ℓ at the QE peak for some neutrino energy may correspond to position of the Δ resonance peak for another energy, see e.g. Fig. 4 in Ref. [15] or Fig. 3. Therefore, information on the event topology is essential for its classification as a QE one.

Typically, neutrino experiments [16, 17, 18] define QE events as those events which do not contain detected pions. In theoretical terms, such a broad class comprises true QE interactions induced by the one- and two-body currents, pion production followed by its absorption in nuclear medium, and detector-dependent background of undetected pions, but not those QE interactions in which (detected) pions are produced as a result of final-state interactions.

The effect of absorbed or undetected pions has been analyzed within the Giessen Boltzmann-Uehling-Uhlenbeck (GiBUU) transport model by Leitner and Mosel [19]. The results of Ref. [19] show that the reconstructed energy of such events is typically lower than the true one by $\sim$300–350 MeV. This conclusion comes as no surprise when we recall that for pion production, it is more appropriate to apply the Δ resonance mass M_Δ rather than M' in the energy reconstruction formula (1.3), and this difference changes the reconstructed energy approximately by

$$\frac{M_\Delta^2 - M'^2}{2M} \approx 340 \text{ MeV}.$$

Being based on the assumption that a single nucleon takes part in the interaction, the energy reconstruction formula (1.3) becomes inaccurate for two- or more-nucleon ("multinucleon") final states. It is known for 30 years [20] that such states may result from three different processes: one-body interactions involving (i) initial- or (ii) final-state correlations, or (iii) two-body interactions, such as those involving meson-exchange currents [21]. However, as *ab initio* cross section calculations consistently taking them into account are not available yet, their influence on energy reconstruction has been analyzed relying on effective approaches, following the puzzling results of the MiniBooNE experiment [17, 22]

The calculations of Martini *et al.* [23], subsequently modified by introducing relativistic corrections [24], are based on the local Fermi gas model with the random-phase approximation (RPA) effects taken into account using the formalism of Marteau [25]. In this approach, in-medium modifications of the Δ resonance are implemented following Ref. [26], and the multinucleon contributions to the response functions are extracted from the results of Alberico *et al.* [20] for Fe(e,e') scattering, and extrapolated to neutrino interactions with the carbon nucleus and to the broader kinematic region of MiniBooNE. Comparisons of the double differential CC QE cross sections obtained by Martini *et al.* [24] with the MiniBooNE data show excellent agreement in the neutrino mode and good agreement in the antineutrino mode. Considering multinucleon effects on energy reconstruction, Martini *et al.* [27] find that they redistribute the strength of the reconstructed flux from the peak to the tails, affecting the extracted oscillation parameters.

The approach of Nieves *et al.* [28, 29], extended to high energy in Ref. [30], is to use the local Fermi gas model as a starting point and to add the RPA modifications. It differs from that of Martini *et al.* by employing effective interactions, the parameters of which have been fixed in earlier studies of photon, electron, and pion scattering off nuclei. While the obtained ν_μ ($\bar{\nu}_\mu$) cross sections are lower by $\sim$10% ($\sim$15%) then those calculated by Martini *et al.*, Nieves *et al.* also find very good (good) agreement with the MiniBooNE data, in view of the reported normalization uncertainties. Analyzing the procedure of unfolding of the total CC QE cross sections [29, 31], Nieves *et al.* observe significant distortion of their energy dependence when the effect of multinucleon final states on reconstructed energy is neglected.

The result of Amaro *el al.* [32] are obtained using the superscaling approach extended to account for the contribution of the two-body currents coming from the vector part of meson-exchange currents only. However, the efforts to include also the axial part are ongoing [33].

The GiBUU model is supplemented by the two-nucleon knockout contribution to the CC QE cross section, obtained from a fit to the MiniBooNE data performed by Lalakulich *et al.* [34]. According to its predictions [34], the effect of two-nucleon processes on energy reconstruction is more relevant in the low-energy region, whereas the role of pion-related backgrounds is more pronounced at higher energy.

The effect of absorbed or undetected pions and multinucleon final states on determination of the oscillation parameters has been quantitatively analyzed by Coloma and Huber [35] for an experimental setup similar to that of T2K. The authors of Ref. [35] have observed a sizable bias that could not be removed even by taking full advantage of the near detector, which clearly shows the importance of an accurate modeling of nuclear effect in precise oscillation studies.

The available results suggest that the uncertainties of the CC QE cross sections are $\sim$10% for neutrinos and $\sim$15% for neutrinos. Those numbers can be considered rather optimistic, having in mind that the existing estimates are not based on *ab initio* calculations and are not uncorrelated.

4. Relevance of final-state interactions and a new method of energy reconstruction

Studying the impact of nuclear model applied in the analysis on the extraction of neutrino-oscillation parameters, Coloma *et al.* [36] have compared the results obtained using the GiBUU model and the GENIE Monte Carlo generator [37] for an experiment similar to T2K. They have observed that when final-state interactions (FSI) between the struck nucleon and the $(A-1)$-nucleon system are not taken into account, the obtained CC QE event distributions as a function of reconstructed neutrino energy are in very good agreement. However, the effects of FSI turns out to introduce a 10% shift between the results of GiBUU and GENIE, which translates into a difference between the extracted values of oscillation parameters as large as several standard deviations [36].

The accuracy of description of nuclear effects, including those related to FSI, can be tested against electron scattering data. Considering the carbon target, the authors of Ref. [11] have performed an extensive comparison of the calculations based on a generalization of the spectral function (SF) formalism [38] to the available data, and analyzed consequences of their approach—involving no adjustable parameters—for neutrino energy reconstruction in CC QE interactions. In the absence of interaction mechanisms other than QE scattering induced by one-body currents, the uncertainty of the calculated (e,e') cross sections has been estimated at 5%, with a 5-MeV uncertainty of the predicted positions of the QE peaks.

As an example of the results of Ref. [11], Fig. 3 shows a comparison of the experimental data [39] to the predictions of the relativistic Fermi gas (RFG) model and the SF results calculated neglecting the effects of FSI and accounting for them. Note that while the RFG model is routinely employed in data analysis by modern neutrino experiments, it cannot reproduce the heights, shapes, and positions of the QE peaks, even though the beam energy is higher than 1 GeV.

In the SF approach, on the one hand, initial-state correlations between nucleons lower the cross section and render the shape of the QE peak asymmetric [40]. On the other hand, FSI bring about a redistribution of the strength from the peak to the tails [41] and a shift of the cross section, resulting from a modification of the struck nucleon's energy. Those effects altogether allow to reproduce the experimental data with remarkable accuracy, see Fig. 3(c). While the approach of Ref. [11] is not implemented in any Monte Carlo generator so far, the additional package to GENIE [42], validated

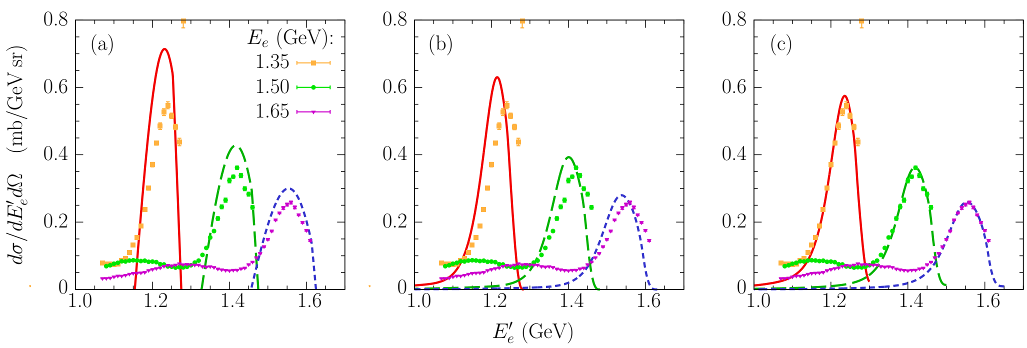

Figure 3: Double differential ${}^{12}_{6}\mathrm{C}(e,e')$ cross sections at scattering angle 13.5° and beam energy 1.35, 1.50, and 1.65 GeV. The experimental data [39] are compared to the calculations for (a) the relativistic Fermi gas model and the spectral function approach (b) without and (c) with FSI effects [11].

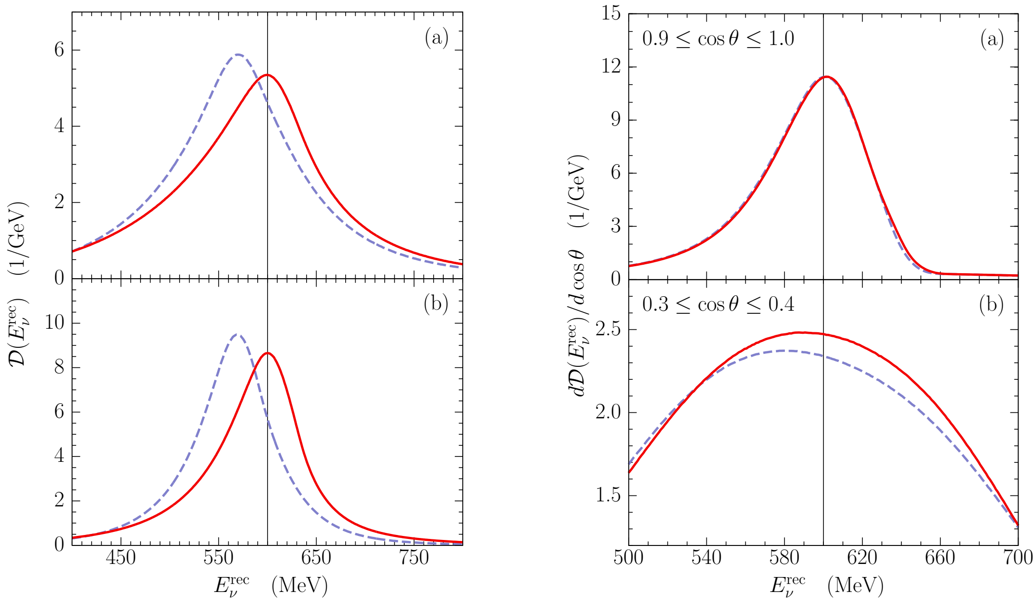

Figure 4: *Left panel*: Comparison of the reconstructed energy distributions calculated with (solid lines) and without (dashed lines) final-state interactions in (a) ν_μ and (b) $\bar{\nu}_\mu$ interactions at the true energy 600 MeV. *Right panel*: Contributions of different $\cos\theta$ bins to the $E_\nu^{\rm rec}$ distribution at $E_\nu = 600$ MeV. The energy reconstruction with the separation energy ε depending on muon kinematics (solid lines) is compared with the standard method with constant ε (dashed lines).

through comparisons with electron scattering data, allows to use the SF model in simulations of CC QE events.

Requiring the $E_\nu^{\rm rec}$ distribution to be peaked at the true energy of several hundred MeV, the authors of Ref. [11] have determined the separation energy ε, appearing in the energy reconstruction formula (1.3), to be 19 (6) MeV in the case of neutrino (antineutrino) interactions. At that kinematics, the effects of FSI on the $E_\nu^{\rm rec}$ distributions are its broadening and a ∼30-MeV shift of the maximum, as shown in the left panel of Fig. 4. Note that the 13-MeV difference between the separation energies in neutrino and antineutrino scattering, compared to 4 MeV applied in the analysis of the MiniBooNE and MINERvA experiments, results from various Coulomb effects, and should not be neglected in the context of measurements of CP violation.

Analyzing electron scattering data in Table 1 and Fig. 3, we have observed that a constant value of the separation energy is not able to accurately reproduce the positions of QE peaks over broad kinematic regions, such as those relevant to neutrino oscillation studies. To improve the accuracy of energy reconstruction, one could treat the separation energy in Eq. (1.3) as a function of the charged lepton's kinematics [11],

$$\varepsilon = \varepsilon(E_\ell, \cos\theta).$$

This function can be determined, for example, requiring $E_\nu^{\rm rec}$ to be the most probable neutrino energy that can produce a charged lepton of energy E_ℓ at the angle θ. Although $\varepsilon(E_\ell, \cos\theta)$ cannot be obtained independently of theoretical model, its uncertainties can be kept under control by detailed tests against electron scattering data at the kinematics of interest.

The difference between this novel method of energy reconstruction and the standard one, with a constant value of ϵ, is shown in the right panel of Fig. 4, comparing the corresponding contributions of different $\cos\theta$ bins to the $E_\nu^{\rm rec}$ distribution calculated for the true energy $E_\nu = 600$ MeV. At $0.9 \leq \cos\theta \leq 1.0$, the standard and new methods are in good agreement, yielding the distributions peaked at 601 and 602 MeV, respectively. However, at higher muon production angles, the decreasing accuracy of the the standard reconstruction method can be observed, and at $0.3 \leq \cos\theta \leq 0.4$, the peaks of the standard and new $E_\nu^{\rm rec}$ distributions are located at 581 and 591 MeV, respectively. This effect can be traced back to the shift resulting from FSI, which exhibits angular dependence. The separation energy fixed to reproduce the true energy at the dominant kinematics ($\cos\theta \sim 0.83$) turns out not to be appropriate at the subdominant ones. Note, however, that $0.9 \leq \cos\theta \leq 1.0$ and $0.3 \leq \cos\theta \leq 0.4$ play non-negligible role in CC QE scattering, contributing 8.2 and 6.2% of the cross section, respectively, compared to the dominant 10.9% contribution from $0.8 \leq \cos\theta \leq 0.9$.

As the new method of energy reconstruction accounts, by construction, for the shift produced by FSI, it is able to bring the reconstructed energy into better agreement with its true value.

5. Summary

Due to the great progress in the experimental precision over the past few years, nuclear effects have become one of the main sources of systematic uncertainties in neutrino oscillation studies. Of particular importance is their influence on widely-employed energy reconstruction from the charged lepton kinematics in CC QE events.

In this paper, I have argued that electron scattering data point to kinematic dependence of the nucleon separation energy appearing in the standard energy reconstruction formula. To discuss the role of CC QE-like backgrounds, I have reviewed selected results available in the literature. In the end, the effects of final-state interactions have been analyzed, and a possible improvement of the energy reconstruction method has been considered.

The puzzling tensions between the recent cross section measurements [16, 17, 18] have attracted a sizable attention of the theoretical community, and a number of approaches developed to describe nuclear response to other probes has been extended to neutrino interactions. While our understanding of nuclear effects relevant to neutrino scattering has clearly improved, many problems still await a quantitative explanation, preferably by *ab initio* calculations. The main difficulty of theoretical model building—unique to the neutrino case—stems from the flux average over polychromatic beams, which couples many interaction channels into events of the same topology [43].

For many nuclear targets, precise data for (e,e') cross sections are available over the broad kinematics of interest, allowing for detailed tests and uncertainty estimates of theoretical approaches. Hopefully, a model meeting the needs of the next generation of oscillation measurements will emerge, permitting fully trustable analysis of experimental data.

Acknowledgments

I would like to express my gratitude to Makoto Sakuda for numerous discussions, from which the ideas presented in this paper are born. The comments of Omar Benhar and Camillo Mariani are also gratefully acknowledged. This work was supported by JSPS under Grant No. PE 13056 and partly under the Grant-in-Aid for Scientific Research (B) No. 23340073.

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