

2p-2h excitations in neutrino scattering: angular distribution and frozen approximation

I. Ruiz Simo^{*†}, C. Albertus,^a J.E. Amaro,^a M.B. Barbaro,^b J.A. Caballero,^c and T.W. Donnelly^d

^a *Departamento de Física Atómica, Molecular y Nuclear, and Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Granada 18071, Spain*

^b *Dipartimento di Fisica, Università di Torino and INFN, Sezione di Torino, Via Pietro Giuria 1, 10125 Torino, Italy*

^c *Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, Apartado 1065, 41080 Sevilla, Spain*

^d *Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

E-mail: ruizsig@ugr.es

We study the phase-space dependence of 2p-2h excitations in neutrino scattering using the relativistic Fermi gas model [1]. We follow a similar approach to Refs. [2, 3], but focusing in the phase-space properties, comparing with the non-relativistic model of [4]. A careful mathematical analysis of the angular distribution function for the outgoing nucleons is performed. Our goals are to optimize the CPU time of the 7D integral to compute the hadron tensor in neutrino scattering, and to conciliate the different relativistic and non relativistic models by describing general properties independently of the two-body current. For some emission angles the angular distribution becomes infinite in the Lab system, and we derive a method to integrate analytically around the divergence. Our results show that the frozen approximation, obtained by neglecting the momenta of the two initial nucleons inside the integral of the hadron tensor, reproduces fairly the exact response functions for constant current matrix elements.

*16th International Workshop on Neutrino Factories and Future Neutrino Beam Facilities
25 -30 August, 2014
University of Glasgow, United Kingdom*

^{*}Speaker.

[†]Special thanks to L. Alvarez-Ruso for invitation

1. Introduction

The analysis of neutrino oscillation experiments requires having under control all the nuclear effects, which are inherent to any ν -nucleus scattering event. These effects play a major role in modifying the free ν -nucleon cross section, and a good understanding of them is clearly mandatory in order to reduce the systematic uncertainty of the models employed in the experimental analysis. There is strong theoretical evidence [5, 6, 7] about a significant contribution from multi-nucleon knockout to the inclusive Charged Current (CC) cross section around and above the quasielastic (QE) peak region.

There are, at least, three different microscopic models [5, 6, 7, 8] which are based in the relativistic Fermi Gas. But they differ from each other in several assumptions and different nuclear ingredients for the interaction. Therefore, it is really difficult to disentangle the origin of any discrepancy in the final results.

Other models [9, 10] are based on the available phase space just assuming a constant transition matrix element and fitting it to the experimental cross section.

Finally, our goal is to reduce the computational time needed in this kind of calculations, or at least to establish what assumptions previously made by other authors are really good enough in order to estimate accurately and fast the contribution of these multi-nucleon processes to the inclusive channel.

2. 2p-2h phase space in the Relativistic Fermi Gas model

The hadron tensor for the 2p-2h channel in a fully relativistic framework is given by:

$$W_{2p2h}^{\mu\nu} = \frac{V}{(2\pi)^9} \int d^3 p'_1 d^3 p'_2 d^3 h_1 d^3 h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \Theta(p'_1, p'_2, h_1, h_2) \delta^3(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q}) \quad (2.1)$$

where m_N is the nucleon mass, V is the volume of the system and we have defined the product of step functions,

$$\Theta(p'_1, p'_2, h_1, h_2) \equiv \theta(p'_1 - k_F) \theta(p'_2 - k_F) \theta(k_F - h_1) \theta(k_F - h_2) \quad (2.2)$$

which encodes the nuclear model.

Finally, the function $r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2)$ is the elementary ‘‘hadron’’ tensor for the transition of a nucleon pair with given initial $(\mathbf{h}_1, \mathbf{h}_2)$ and final $(\mathbf{p}'_1, \mathbf{p}'_2)$ momenta, summed up over spin and isospin, given schematically in terms of the antisymmetrized two-body currents by:

$$r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) = \frac{1}{4} \sum_{\sigma, \tau} j^\mu(1', 2'; 1, 2)_A^* j^\nu(1', 2'; 1, 2)_A \quad (2.3)$$

The above multidimensional integral (2.1) can be done either numerically or its dimensions can be further reduced under some approximations [4, 11, 12]. We do not know exactly the origin of the discrepancies between the available models. These could be due to different two-body currents, to local Fermi Gas model rather than global one, or other effects. It is difficult to make any concluding

assessment right now, but all those models should agree at the level of phase space function $F(\omega, q)$, obtained assuming a constant $r^{\mu\nu}$:

$$F(\omega, q) \equiv \int d^3 h_1 d^3 h_2 d^3 p'_1 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \Theta(p'_1, p'_2, h_1, h_2) \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \quad (2.4)$$

where $r^{\mu\nu} = 1$ and $\mathbf{p}'_2 = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q} - \mathbf{p}'_1$ integrating out the delta function of momentum conservation.

The remaining delta function enables analytical integration over the modulus of \mathbf{p}'_1 :

$$F(\omega, q) = 2\pi \int d^3 h_1 d^3 h_2 d\theta'_1 \sin\theta'_1 \frac{m_N^2}{E_1 E_2} \sum_{\alpha=\pm} \frac{m_N^2 p_1'^2}{|E'_2 p'_1 - E'_1 \mathbf{p}'_2 \cdot \hat{\mathbf{p}}'_1|} \Theta(p'_1, p'_2, h_1, h_2) \Big|_{p'_1=p_1'^{(\alpha)}} \quad (2.5)$$

and the sum inside the integral sign runs over the two solutions $p_1'^{(\pm)}$ of the energy conservation delta function (see appendix C on Ref. [1]).

2.1 Frozen nucleon approximation

The frozen nucleon approximation is just a particular case of the mean-value theorem in several variables

$$\int_a^b f(x) dx = f(c)(b-a) \quad \text{with } c \in [a, b] \quad (2.6)$$

$$\int_{\mathcal{V}} f(\mathbf{r}) d^n \mathbf{r} = f(\mathbf{c}) \int_{\mathcal{V}} d^n \mathbf{r} = f(\mathbf{c}) \mathcal{V} \quad \text{with } \mathbf{c} \in \mathcal{V} \quad (2.7)$$

In our case we are going to skip the two 3D integrations over the holes momenta ($\mathbf{h}_1, \mathbf{h}_2$) by fixing them to some, in principle unknown, values inside the Fermi sphere, while keeping the integration over the emission angle and constraining it by energy conservation. Of course, the obvious particularization of the mean-value theorem (2.7) is:

$$F(\omega, q) = \int d^3 h_1 d^3 h_2 d^3 p'_1 f(\mathbf{h}_1, \mathbf{h}_2, \mathbf{p}'_1) = \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3 p'_1 f(\langle \mathbf{h}_1 \rangle, \langle \mathbf{h}_2 \rangle, \mathbf{p}'_1) \quad (2.8)$$

where ($\langle \mathbf{h}_1 \rangle, \langle \mathbf{h}_2 \rangle$) are the two unknown hole momenta inside the Fermi sphere.

Up to now the whole discussion is exact. What makes the difference between an excellent approximation to the true result or a poor one is just the selection of the ‘‘average’’ hole momenta ($\langle \mathbf{h}_1 \rangle, \langle \mathbf{h}_2 \rangle$). Our choice to call it *frozen nucleon approximation* will be $(\vec{0}, \vec{0})$. There are mainly two arguments which favor this choice:

- For high $q \gg k_F > h_i$, one can assume both initial nucleons at rest.
- If one does not assume the above statement, one has to determine the average angles for the two holes. And this cannot be done without computing the full 7D integral (see section 2.2).

Now, we can define the *frozen approximation* phase-space function $\bar{F}(\omega, q)$:

$$\bar{F}(\omega, q) = \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3 p'_1 \delta(E'_1 + E'_2 - \omega - 2m_N) \Theta(p'_1, p'_2, 0, 0) \frac{m_N^2}{E'_1 E'_2} \quad (2.9)$$

where now $\mathbf{p}'_2 = \mathbf{q} - \mathbf{p}'_1$.

The above phase-space function (2.9) allows us to define the angular distribution in the *frozen approximation*:

$$\bar{F}(\omega, q) = \left(\frac{4}{3} \pi k_F^3 \right)^2 2\pi \int_0^\pi d\theta'_1 \Phi(\theta'_1) \quad (2.10)$$

$$\Phi(\theta'_1) = \sin \theta'_1 \int dp'_1 p_1'^2 \delta(E'_1 + E'_2 - \omega - 2m_N) \Theta(p'_1, p'_2, 0, 0) \frac{m_N^2}{E'_1 E'_2} \quad (2.11)$$

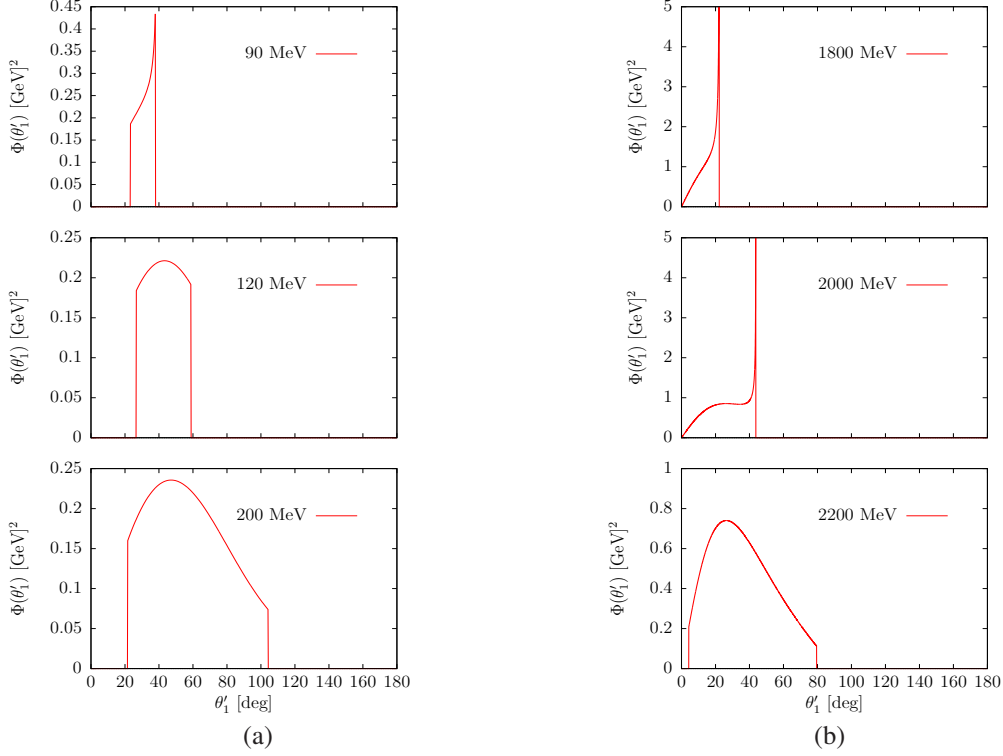


Figure 1: Angular distributions $\Phi(\theta'_1)$ for $q = 500$ MeV/c (left panel) and for $q = 3$ GeV/c (right panel) in the frozen nucleon approximation, for three different values of ω as a function of the emission angle θ'_1 .

The problem stands on the divergence of the angular distribution for some kinematics. This can be seen in some of the panels of figure 1 for the frozen nucleon approximation. For other values of the holes momenta, the position of the pole is different, but it is still present. This makes crucial to determine analytically the angular interval where the integration has to be performed. This was done explicitly on section VI of Ref. [1].

Once the problem of the divergence has been correctly addressed, the results for the phase space function in the frozen approximation can be compared with the full integral in figure 2. Here we can appreciate the quality of the approximated result for a wide range of momentum transfers. The main discrepancies arise on the low energy-transfer region in each plot. But the phase space function is well-reproduced in the rest of the interval and, furthermore, we have skipped 6 additional integrals (over holes momenta), thus reducing significantly the computation time. These

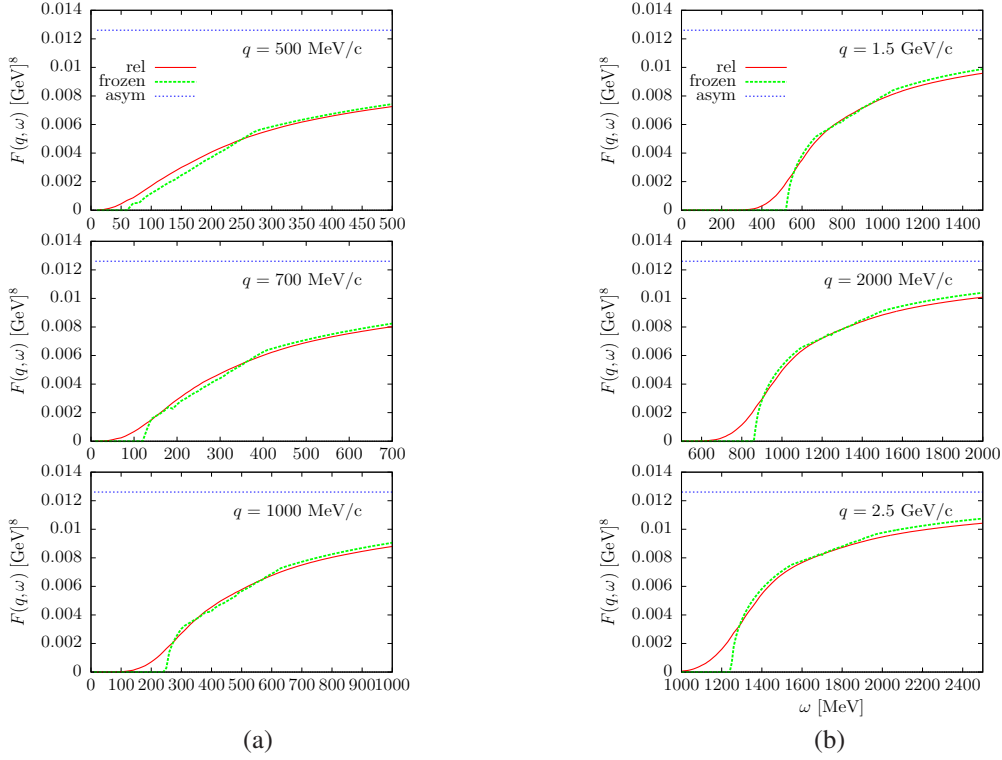


Figure 2: Comparison between frozen nucleon approximation and full integral for low and intermediate momentum transfers (left panel) and for high momentum transfers (right panel).

results indicate that this approximation is especially well-suited for Montecarlo event generators, particularly if the goal is to estimate quite accurately the cross section for multi-nucleon knockout in the shortest time as possible.

2.2 Other initial configurations

We have chosen up to now the *frozen nucleon approximation*, but other initial $(\langle \mathbf{h}_1 \rangle, \langle \mathbf{h}_2 \rangle)$ configurations could have been selected as well. We have considered six configurations depicted in fig. 3a. The configurations with total initial momentum of the pair equal to zero (U,D or T,-T) would, in principle, yield a result very close to that of the *frozen approximation*. This can be easily observed in figure 4a.

In the first comparison, Fig. 3b, we show the contribution of two pairs of nucleons with the same momentum $h_1 = h_2 = 200 \text{ MeV}/c$, and both parallel, pointing upward (U) and downward (D) with respect to the z axis, that is, the direction of \mathbf{q} . The contribution of the UU configuration is smaller than average, while the DD is larger. This is so because in the UU case the total momentum p' in the final state is large. By momentum conservation, the momenta p'_1 and p'_2 must also be large. Therefore, these states need a large excitation energy, and they start to contribute for high ω transfer. In the DD configuration, the total momentum p' is smaller, so the final momenta p'_1 and

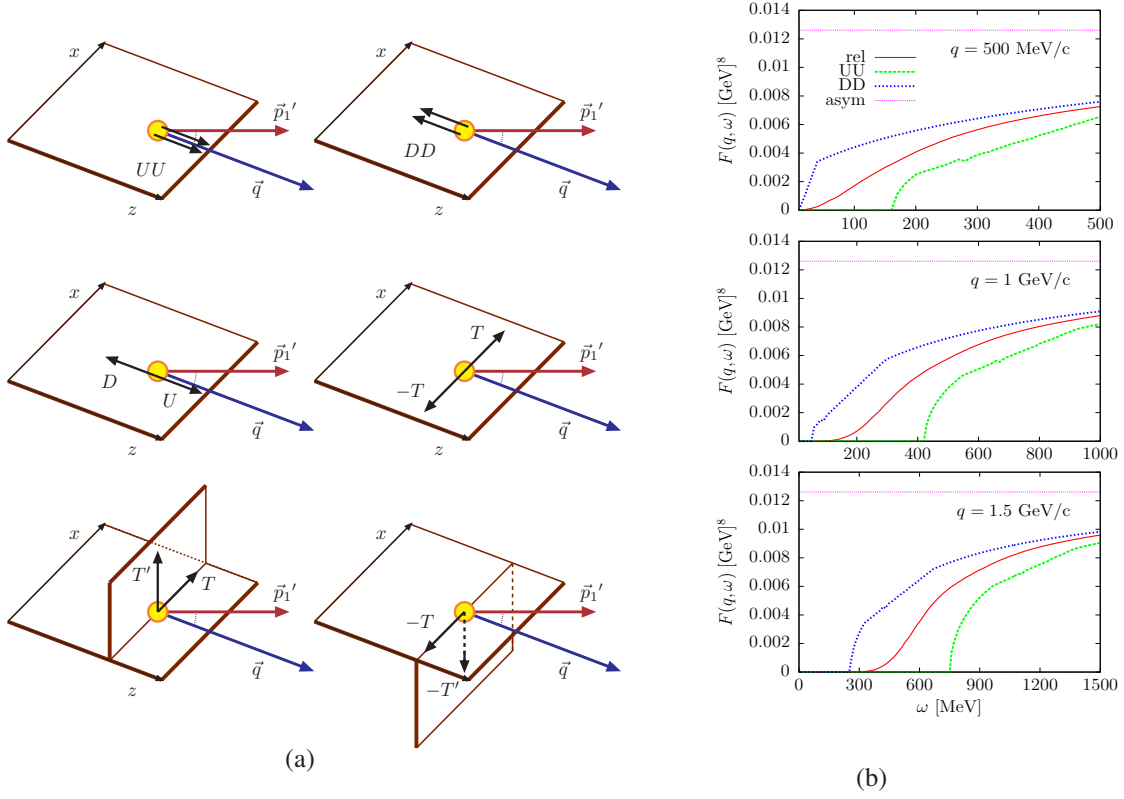


Figure 3: Geometry employed for emission of a pair of nucleons with initial parallel momenta (UU,DD), anti-parallel (UD,T-T) and perpendicular (TT',-T-T') on the left panel. In the right panel a comparison between average momentum approximation and full integral is shown. Initial momenta are both 200 MeV/c.

p_2' can also be small, with small required excitation energy. Therefore, they start to contribute at lower ω .

In the example of Fig. 4a, two anti-parallel configurations are shown. In the UD case, one nucleon is moving upward and the other downward the z axis with total momentum zero of the pair. This situation is similar to that of a pair of highly correlated nucleons with large relative momentum [13]. Since the total momentum is zero, the final 2p-2h state has total momentum q , exactly the same that it would have in the *frozen nucleon approximation*. Therefore, the contribution of this configuration is similar to the average. The same conclusions can be drawn in the case of the configuration $T, -T$, with one nucleon moving along the x axis (transverse direction) and the other along $-x$ with opposite momentum. The contribution of this pair is exactly the same as that of the UD configuration in the total phase-space function.

Finally, we show in Fig. 4b two intermediate cases that are neither parallel nor anti-parallel configurations. They consist of two pairs of transverse nucleons moving along mutually perpendicular directions. In the first case, we consider a T nucleon and a second T' nucleon moving in the y axis

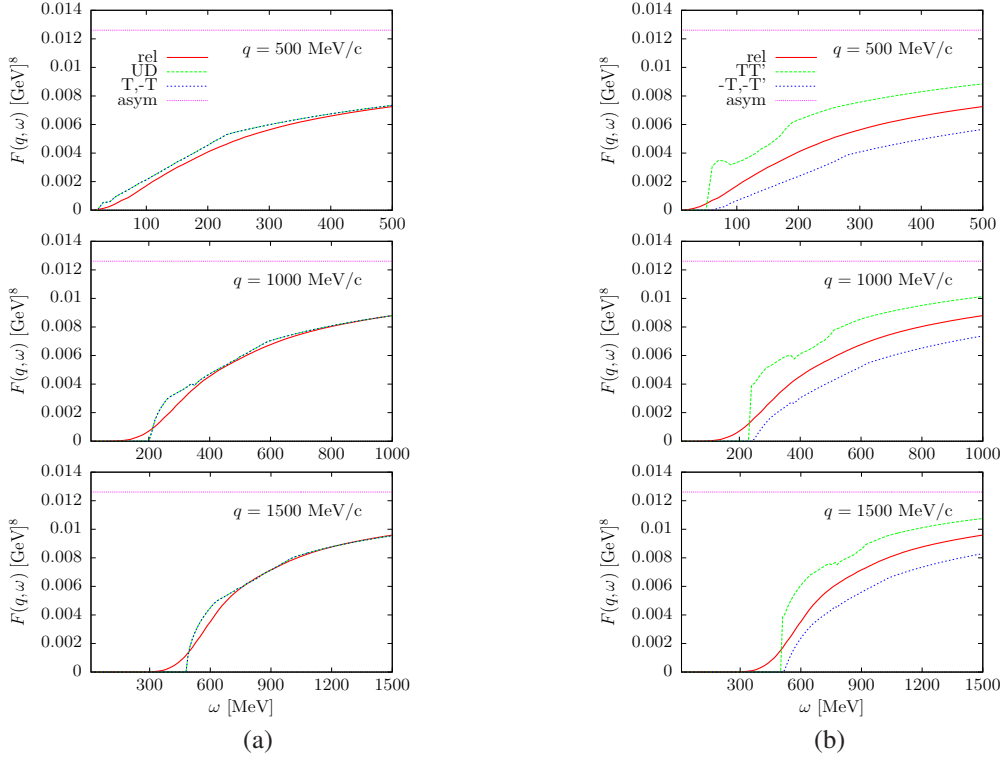


Figure 4: Comparison between average momentum approximation and full integral. In the left panel the initial momenta are both 200 MeV/c with total momentum of the pair equal to zero. In the right one the initial momenta are both 200 MeV/c pointing in orthogonal directions.

out of the scattering plane. The contribution of the TT' pair is large, while the one of the opposite case, $-T, -T'$, is small. On the average, they are close to the total result.

3. Conclusions

We have performed a detailed study of the two-particle- two-hole phase-space function, which is proportional to the nuclear two-particle emission response function for constant current matrix elements. Our final goal was to obtain accurate enough results without calculating the 7D integral. The frozen nucleon approximation (1D integral), that is, neglecting the momenta of the initial nucleons for high momentum transfer, seems to be a quite promising approach to reduce the computation time without missing significant accuracy. The CPU time of the 7D integral has been reduced significantly. We are presently working on an implementation of the present method with a complete model of the MEC operators.

4. Acknowledgements

This work was supported by DGI (Spain), Grants No. FIS2011-24149 and No. FIS2011-

28738-C02-01; by the Junta de Andalucía (Grants No. FQM-225 and No. FQM-160); by the Spanish Consolider-Ingenio 2010 programmed CPAN, in part (M. B. B.) by the INFN project MANYBODY and in part (T. W. D.) by U.S. Department of Energy under Cooperative Agreement No. DE-FC02-94ER40818. C. A. is supported by a CPAN postdoctoral contract.

References

- [1] I. Ruiz Simo, C. Albertus, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, *Relativistic effects in two-particle emission for electron and neutrino reactions*, Phys. Rev. D **90**, 033012 (2014) [nucl-th/1405.4280].
- [2] A. De Pace, M. Nardi, W.M. Alberico, T.W. Donnelly, A. Molinari, *The 2p-2h electromagnetic response in the quasielastic peak and beyond*, Nucl. Phys. A **726** (2003) 303-326 [nucl-th/0304084].
- [3] J.E. Amaro, C. Maieron, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, *Pionic correlations and meson-exchange currents in two-particle emission induced by electron scattering*, Phys. Rev. C **82**, 044601 (2010) [nucl-th/1008.0753].
- [4] J.W. Van Orden, T.W. Donnelly, *Mesonic Processes in Deep Inelastic electron Scattering from Nuclei*, Annals Phys. **131** (1981) 451-493.
- [5] M. Martini, M. Ericson, G. Chanfray, J. Marteau, *A Unified approach for nucleon knock-out, coherent and incoherent pion production in neutrino interactions with nuclei*, Phys. Rev. C **80** (2009) 065501 [nucl-th/0910.2622].
- [6] J. Nieves, I. Ruiz Simo, M. J. Vicente Vacas, *Inclusive Charged-Current Neutrino-Nucleus Reactions*, Phys. Rev. C **83** (2011) 045501 [hep-ph/1102.2777].
- [7] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, C. F. Williamson, *Meson-exchange currents and quasielastic neutrino cross sections in the SuperScaling Approximation model*, Phys. Lett. B **696** (2011) 151 [nucl-th/1010.1708].
- [8] J. E. Amaro, M. B. Barbaro, J. A. Caballero and T. W. Donnelly, *Meson-exchange currents and quasielastic antineutrino cross sections in the SuperScaling Approximation*, Phys. Rev. Lett. **108** (2012) 152501 [nucl-th/1112.2123].
- [9] O. Lalakulich, K. Gallmeister and U. Mosel, *Many-Body Interactions of Neutrinos with Nuclei - Observables*, Phys. Rev. C **86** (2012) 014614 [Erratum-ibid. C **90** (2014) 2, 029902] [nucl-th/1203.2935].
- [10] J. T. Sobczyk, *Multinucleon ejection model for Meson Exchange Current neutrino interactions*, Phys. Rev. C **86** (2012) 015504 [hep-ph/1201.3673].
- [11] W. M. Alberico, M. Ericson and A. Molinari, *The Role of Two Particles - Two Holes Excitations in the Spin - Isospin Nuclear Response*, Annals Phys. **154** (1984) 356.
- [12] A. Gil, J. Nieves and E. Oset, *Many body approach to the inclusive (e, e' -prime) reaction from the quasielastic to the Delta excitation region*, Nucl. Phys. A **627** (1997) 543 [nucl-th/9711009].
- [13] I. Korover, N. Muangma, O. Hen, R. Shneor, V. Sulkosky, A. Kelleher, S. Gilad and D. W. Higinbotham *et al.*, *Probing the Repulsive Core of the Nucleon-Nucleon Interaction via the $4\text{He}(e, e'pN)$ Triple-Coincidence Reaction*, Phys. Rev. Lett. **113** (2014) 022501 [nucl-ex/1401.6138].