

Modeling of path delay in the neutral atmosphere: a paradigm shift

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Computation of propagation effects in the neutral atmosphere, namely path delay, extinction, and bending angle is a trivial task provided the 4D state of the atmosphere is known. Unfortunately, the mixing ratio of water vapor is highly variable and it cannot be deduced from surface measurements. That fact led to a paradigm that considers path delay and extinction in the atmosphere as a priori unknown quantities that have to be evaluated from the radio astronomy data themselves. Development of our ability to model the atmosphere and to digest humongous outputs of these models that took place over the course of the 21st century changed the game. Using the publicly available output of operational numerical weather model GEOS run by NASA, we are in a position to compute path delay through the neutral atmosphere for any station and for any epoch from 1979 through now with accuracy of $45 \text{ ps} \cdot \text{cosec elevation}$. We are in a position to compute extinction with accuracy better than 10 per cents. We are in a position to do it routinely, in a similar way how we update apparent star positions for precession and nutation. Moreover, we are in a position to do it now. As a demonstration of current capabilities, I have computed time series of path delays for all radiotelescopes that I was aware of (220 sites) since 1979 with a step 3-6 hours. Results of the validation tests are presented. A new paradigm of data analysis assumes that we know the atmosphere propagation effects a priori with the accuracy higher than one could deduce them from radio astronomy observations.

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1. Introduction

Radio waves travel billions years from typical sources observed with VLBI. At the very end of their journey, at the last millisecond, they propagate through the Earth's atmosphere. While multi-frequency observations allow us to evaluate the contribution of the ionized component of the Earth's atmosphere, modeling propagation effects in the neutral atmosphere poses a challenge. There are four effects: 1) refraction, i.e. trajectory bending, 2) delay in the atmosphere, 3) attenuation, and 4) atmosphere emission. If we know the state of the atmosphere, we can compute these quantities. The crux of the problem is that the state of the 4D atmosphere cannot be deduced from surface measurements. The reason of that is water is underwent phase transition at the atmospheric layers that are above the surface, and as a result, the mixing ratio of water vapor is highly volatile.

2. Old paradigm

The old paradigm assumed we do not know the instantaneous state of the atmosphere. In order to circumvent the lack of knowledge, some greatly simplified or just wrong a priori models were used for computation of path delay in the past. In particular, Saastamoinen (1972) model became very popular. That model, in a way how it was implemented, assumed that water vapor present in the atmosphere had thermodynamic properties of dry air. A typical error of path delay computed that way was 300–1300 ps, i.e. 5–15% of the total delay. In order to improve modeling, the astronomical data themselves were suggested to be used for solving residual parameters of the propagation model. It is assumed in the framework of this paradigm that the path delay $\tau(t, e, A)$ can be¹ decomposed into two azimuth-independent components:

$$\tau(t, e, A) = \tau_d(t) \cdot m_d(e) + t_w(t) \cdot m_w(e), \quad (2.1)$$

where $\tau_d(t)$ is the so-called dry component, $m_d(e)$ is the so-called dry mapping function, i.e. a derivative with respect to elevation; $t_w(t)$ and $m_w(e)$ is wet path delay and its derivative with respect to elevation angle; t , e , and A stands for time, elevation angle above the horizon, and azimuth.

For evaluation of extinction in the atmosphere measurements of antenna brightness temperature at different elevations were often made (tipping curves).

3. New paradigm

Advances in numerical weather models made it possible to evaluate parameters of the atmosphere using various ground, air-born, and space-born measurements that are assimilated into a dynamic model of the atmosphere. The output of these models defines the parameters of the state of the atmosphere on a 4-dimensional grid. Three parameters are important for reduction of astronomy observations: air temperature T , total atmospheric pressure P , and partial pressure of water vapor P_w . Thus, the state of the atmosphere in the new paradigm is considered known. At the moment, there are several centers in the world that produce numerical models of the atmosphere. I used models produced by the NASA Global Modeling and Assimilation Office (GMAO)² for this study.

¹Unfortunately, it is often forgotten, that such a decomposition is a only a simplified approximation within its range of applicability.

²The output of the models is available at <http://gmao.gsfc.nasa.gov>

Table 1: GMAO models of the atmosphere

MERRA:	Since 1979.01.01	72 lev $\times 0.5^\circ \times 0.67^\circ \times 6^h$	Latency: 40 ^d
GEOS FPIT	Since 2000.01.01	72 lev $\times 0.5^\circ \times 0.67^\circ \times 3^h$	Latency: 12 ^h
GEOS FP	Since 2011.09.01	72 lev $\times 0.25^\circ \times 0.3125^\circ \times 3^h$	Latency: 12 ^h

4. Computation of path delay

Electromagnetic waves in the medium propagate with group velocity v_m that is always smaller than the speed of light in vacuum c . Propagating through the medium that has variable refractivity defined as $r = \frac{c-v_m}{v_m}$, the electromagnetic wave changes its direction. Refractivity is a simple function of P , T , and P_w that depends on coefficients measured in laboratory. According to Aparicio & Laroche (2011), refractivity in radio range can be computed with accuracy $\sim 0.1\%$.

There are several ways to solve for ray trajectory in a continuous medium with known refractivity field. I prefer an approach based on solving the variational problem: the actual trajectory that the electromagnetic wave travels is the one that minimizes travel time (Fermat principle). This principle was first formulated empirically in 1662 but nowadays can be derived directly from Maxwell equations (see, for instance *Landau and Lifshitz*, 1988). A general solution of this problem of calculus of variations was found by Euler in 1744 and it is reduced to solving a non-linear system of differential equations of the 4th order with mixed initial values. The coefficients of equations depend on the refractivity and its gradient.

Considering propagation of electromagnetic wave through a continuous medium requires representation of refractivity $r(h, \lambda, \varphi_{gd}, t)$ as a continuous differentiable function. The mathematical model that I used for such a representation is an expansion of refractivity into a 4D tensor product of B-spline functions of the m th degree B_p^m :

$$r(h, \lambda, \varphi_{gd}, t) = \sum_{i=1-m}^{i=d_1-1} \sum_{j=1-m}^{j=d_2-1} \sum_{k=1-m}^{k=d_3-1} \sum_{l=1-m}^{l=d_4-1} f_{ijkl} B_i^m(h) B_j^m(\lambda) B_k^m(\varphi_{gd}) B_l^m(t). \quad (4.1)$$

The property of B-spline, minimal support, reduces the problem of finding the coefficients f_{ijkl} to solving a system of algebraic equations with an m -diagonal matrix, which can be solved extremely efficiently at modern computers.

Representation of the refractivity fields in the form of the tensor product of B-splines of the 3rd degree ($m=3$), reduces differential equations that are a solution of the variational problem to a system of tri-diagonal algebraic non-linear equations. That system is efficiently solved by iterations starting from the 1st approximation that radio waves propagate along the straight line. Two iterations are sufficient to reduce errors of a numerical solution below 1 ps.

After determining the trajectory of the radio wave from the emitter to the receiver, we can easily find the time delay in the neutral atmosphere along the trajectory $\eta(\xi), \zeta(\xi)$ by integrating refractivity along the path:

$$\tau_{na} = \frac{1}{c} \int_0^\infty \left((1 + r(\xi, \eta, \zeta)) \sqrt{1 + \left(\frac{d\eta}{d\xi}\right)^2 + \left(\frac{d\zeta}{d\xi}\right)^2} - 1 \right) d\xi, \quad (4.2)$$

where the Cartesian coordinate system ξ, η, ζ is chosen in such a way that axis ξ is along the straight line from the receiver to the emitter, and axes η, ζ are orthogonal to that direction.

Attenuation in the atmosphere can be computed by evaluating the integral that resembles 4.2:

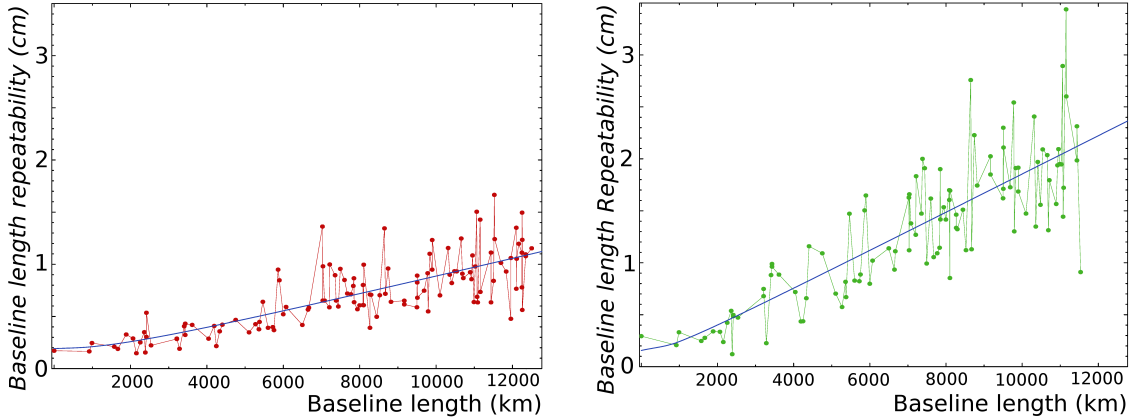
$$a(\xi_0) = \int_{\xi_0}^{\infty} \alpha(P, P_w, T, f) \sqrt{1 + \left(\frac{d\eta}{d\xi}\right)^2 + \left(\frac{d\zeta}{d\xi}\right)^2} d\xi, \quad (4.3)$$

where $\alpha(P, P_w, T, f)$ is the specific attenuation coefficient that has a strong dependency on frequency f . In particular, attenuation of the signal at the ground is $a(0)$.

Finally, atmosphere brightness temperature T_{atm} is found by solving the radiative transfer equation, which is reduced to evaluation of the integral

$$T_{atm} = \int_0^{\infty} T(\xi, \eta, \zeta) \cdot e^{-a(\xi)} \sqrt{1 + \left(\frac{d\eta}{d\xi}\right)^2 + \left(\frac{d\zeta}{d\xi}\right)^2} d\xi. \quad (4.4)$$

Figure 1: Baseline length repeatability for a case when residual zenith path delay is solved for **(Left)** and when no path delay was estimated **(Right)**. The baseline length repeatability extrapolated to the Earth's diameter is 1.12 cm for the first case and 2.32 cm for the second case.



At a given station, given epoch, integrals τ_{na} , a , and T_{atm} are functions of azimuth and elevation. For logistical reasons it is convenient to compute these quantities on a 2D azimuth-elevation regular grid (that does not have to be uniform), expand them into the B-spline basis and store the coefficients. This approach disentangles computation of integrals 4.2–4.4 from radio astronomy data analysis.

5. Results

I computed azimuth-elevation coefficients for path delay expansions for all 220 VLBI stations that participated in radio astronomy observations since 1979 through present using GMAO numerical weather models GEOS-FPIT and MERRA. The input dataset has rather significant size, 36Tb, but still manageable.

In order to evaluate the accuracy of path delays computation, I ran two solutions: with estimating residual path delay in zenith direction and without estimating. For each baseline I estimated the weighted root mean square (wrms) of deviations of baseline lengths with respect to the linear model — the so-called baseline length repeatability test. The baseline length repeatability dependence on baseline length L (blue lines in Figure 1) was fitted to $R(L) = \sqrt{A^2 + (B \cdot L)^2}$, where A and B are fitted coefficients. When the baseline length between two stations is approaching to the Earth's diameter, its vector is approaching to the local vertical at both stations. Therefore, the wrms of station position uncertainties can be evaluated as $R(L = \text{diam})/\sqrt{2}$. Comparing the baseline length repeatabilities extrapolated to the Earth's diameter from two solutions, I derived the additional variance in vertical station positions due to errors in a priori path delays derived from the output of numerical weather models: 1.45 cm (48 ps).

Another approach to evaluate errors of a priori path delay is to compute the rms of estimates of residual path delay in zenith direction. It varies from 0.6 cm at polar station NYALES20 to 2.2 cm at tropical station SC-VLBA with an average value of 1.25 cm (37 ps). It is instructive to note that the average zenith total path delay is 251.4 cm and the average zenith wet path delay is 12.3 cm. This allows me to conclude that total path delay can be computed with accuracy 0.5%, and the contribution of water vapor to path delay can be computed with accuracy 10%.

Figure 2: Atmosphere brightness temperature at elevation 45° at SARDINIA at 86.304 GHz

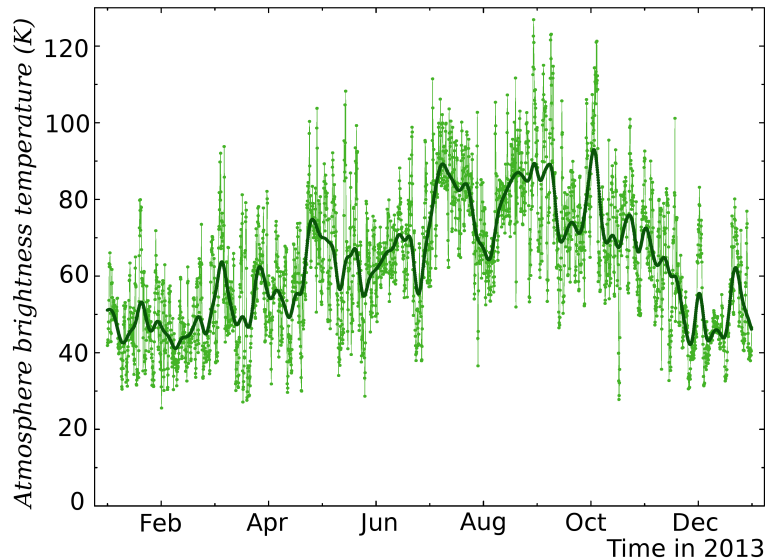
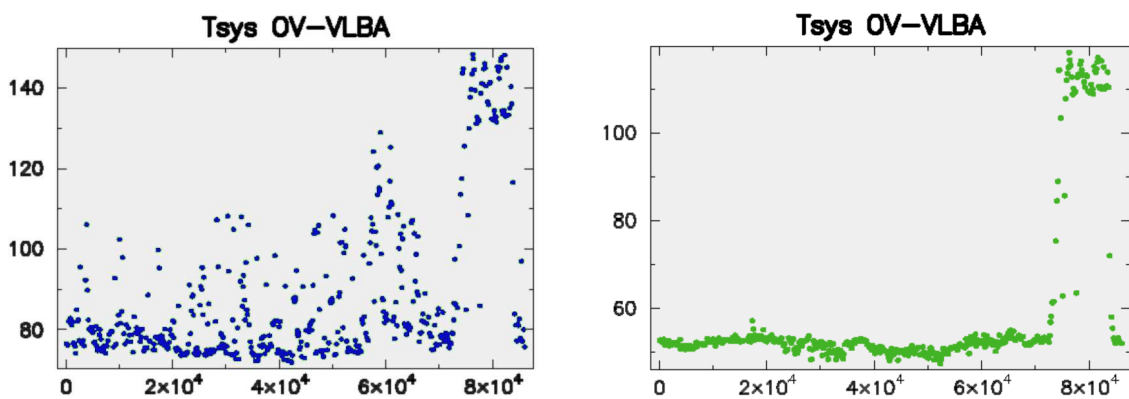


Figure 2 shows the estimates of the atmosphere brightness temperature at SARDINIA at 86.304 GHz. This figure may help us to make a rough prediction of the atmosphere contribution to station performance. We see that a naive surmise that the performance in winter will be always significantly better than in summer is a gross simplification.

Using estimates of the atmosphere attenuation from the output of numerical weather models is straightforward: we just calibrate fringe amplitudes by dividing them by e^{-a} factor and thus relate them to the top of the atmosphere.

There are also certain benefits of being able to compute atmosphere brightness temperature using the output of numerical weather model. The difference between system temperature T_{sys} and T_{atm} is the sum of receiver temperature T_{rec} and the spill-over term that does not depend on time, but depends on elevation and, in a less extent, on azimuth. If the receiver works properly, T_{rec} is stable. Stacking T_{sys} and T_{atm} over the time range when T_{rec} is stable, we can separate the spillover term from T_{rec} and develop an empirical model for it. Analysis of T_{rec} time series allows us to identify the period of time when it had anomalies (see Figure 3). This allows us to clean the data for bad estimates of T_{sys} caused by radio interference and restore missing T_{sys} by extrapolating T_{rec} .

Figure 3: K-band system temperature in K as a function of time in sec (**Left**) and receiver temperature (**Right**) after subtraction of atmosphere brightness temperature computed using the output of numerical weather model GEOS-FPIT.



It is difficult to estimate errors of atmosphere brightness temperature and attenuation in the atmosphere directly. It is reasonable to expect that in the frequency range where the dominating contribution to the atmosphere attenuation is water vapor, their errors are close to the uncertainty of wet path delay, i.e. around 10%. Estimates of accuracies demonstrate that certain old-fashioned steps of data calibration, such as inserting geodetic blocks into schedules and measuring tipping curves, are nowadays unnecessary, since even better accuracies can be achieved by utilizing the output of numerical weather models.

References

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