PROCEEDINGS OF SCIENCE

Oscillatory modes of $q\,\overline{q}\,'$ in mesons and $q\,q\,'q\,''$ in baryons ($\leftrightarrow$ antibaryons)

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The embedding of QCD in the unifying gauge group spin10 will be discussed . The construction of oscillatory modes of $q\,\overline{q}\,'$ and $q\,q\,'q\,''$ –modes in mesons and baryons is presented as an abbreviated outline .

Proceedings of the Corfu Summer Institute 2014 "School and Workshops on Elementary Particle Physics and Gravity",
3-21 September 2014
Corfu, Greece

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PoS(CORFU2014)048

1. Introduction

This contribution is a an account of 'Oscillatory modes of valence quarks in hadrons – mesons , baryons and antibaryons' .

The traceback of derivations , which led to the elaboration of several Fortran programs in a new endeavor in collaboration with Sonia Kabana, is also contained , ascending in time , in refs. 1 = [1-2013] , 2 = [2-2013] , 3 = [3-2013] (56 slide-pages) , 4 = [4-2013] (150 slide-pages) .

The development of oscillatory modes of valence quarks and antiquarks in mesons was subject of lectures/exercises by one of us (P.M.) in 1978 . Derivations are given in condensed form in ref. 5 = [5-1980].

This set up the problem of extension to (three) valence quarks in baryons (and antiquarks in antibaryons), which took a while, to be answered in ref. 5 = [5-1980], where the discussion of baryons was restricted to only two light flavors : u, d . It was noted in Table 2 , p. 267, op.cit. ref. 5 , that starting with the main oscillatory quantum number N = 2 there were (nonstrange) baryon states missing in the PDG tables of 1980 .

In order to bring up to date the phenomenological situation and the comparison with the meson and baryon states for 3 light flavors of valence quarks with candidate resonances listed by the PDG ref. 6 = [6-2012] an extension to a notefile, shown in part, but only for baryons and antibaryons at the occasion of two seminars in 2013 : 24. April at the Los Alamos National Laboratory and 1. - 8. May at Caltech was made in ref. 2 = [2-2013].

A useful tool in assessing the $N = 1,3$ u, d, s baryons is the PDG review, ref. 7 = [7-2012] .

The use of a broken extended symmetry combining $SU2\left(\sum_q \vec{S}_q\right) \times SO3\left(\sum_q \vec{L}_q\right) \to SU6$ was vigorously pursued up to the end of the 60-ies, whence the three valence quarks wave function is conceived in their c.m. frame. Other orbital 3-quark states were also considered, as e.g. the $p_{\,z} \to \infty$ states, where a boosted $SU6_{\,w}$ symmetry is introduced instead of $SU6$. In this respect the main quantum number N was not related to genuine oscillator degrees of freedom but instead inferred from recurrences along Regge trajectories from the relation

$$\alpha\,' M^2\,(\,J\,) \;=\; J \,+\, J_{\,0} \;\text{ with }\; \alpha\,'\Delta\,M^{\,2} \;=\; \Delta\,J \;=\; \Delta\,N \tag{1.1}$$

In eq. 1.1 $\vec{J} = \vec{L} + \vec{S}$ denotes total angular momentum.

The literature quoted in ref. 5 = [5-1980] is very limited, to which we add here papers by R. Dashen and M. Gell-Mann, ref. 8 = [8-1965] , by G. Zweig , ref. 9 = [9-1968] and J. Schwinger , ref. 10 = [10-1964] , illustrating the search for local field variables underlying the strong interactions at that time.

One result from these investigations in trying to identify then known baryon resonances, was the finding that states were missing.

The counting of oscillatory modes of light flavored u, d, s quarks is a new investigation, which began in 2013, with a report to the ICNFP2013 conference in ref. 1 = [1-2013] .

On 12. July 2014 a new *extension* of Fortran programs was established, compiled and executed for the first time at CERN. This gave rise to an updated résumé of results in ref. 3 = [3-2013] , (op.cit.) .

2. The local colored fields associated with oscillatory modes – a prerequisite substrate of oscillatory modes

The initial consideration were devoted to clarify what field content – if any – would be admitted , consistent with locality and microscopic causality without revealing an associate gauge transformation associated color quantum number in the spectrum of hadronic states . In this connection the *minimum* field content versus resonance states within the well identifyable hadrons was aligned through quark and antiquark (spin $\frac{1}{2}$ - fields)

$$\begin{array}{lll} \left(q_{\,c\,fl} \right)_{A} (x) & & q\,,\,\overline{q}^{\,\prime} \quad \text{flavored mesons} \\ & \leftrightarrow & \\ \left(\overline{q}_{\,\dot{c}\,fl} \right)_{\dot{A}} (x) & & q\,,\,q^{\,\prime},q^{\,\prime\prime} \ \text{flavored baryons} \\ \multicolumn{3}{l}{c\,,\dot{c}^{\,\prime} \,=\, 1,2,3 \ : \text{color - } ; \ A\,,\,\dot{A}^{\,\prime} \,=\, 1,\cdots,4 \ : \ \text{spinor indices}} \\ \multicolumn{3}{l}{x \,=\, t\,,\,\vec{x} \ : \ \text{space-time variables}} \end{array} \tag{2.1}$$

We repeat eq. 2.2 below for clarity

$$\begin{array}{lll} \left(q_{\,c\,fl} \right)_{A} (x) & & q\,,\,\overline{q}^{\,\prime} \quad \text{flavored mesons} \\ & \leftrightarrow & \\ \left(\overline{q}_{\,\dot{c}\,fl} \right)_{\dot{A}} (x) & & q\,,\,q^{\,\prime},q^{\,\prime\prime} \ \text{flavored baryons} \\ \multicolumn{3}{l}{c\,,\dot{c}^{\,\prime} \,=\, 1,2,3 \ : \text{color - } ; \ A\,,\,\dot{A}^{\,\prime} \,=\, 1,\cdots,4 \ : \ \text{spinor indices}} \\ \multicolumn{3}{l}{x \,=\, t\,,\,\vec{x} \ : \ \text{space-time variables}} \end{array} \tag{2.2}$$

In eq. 2.2 the labels and variables pertaining to the local quark and antiquark fields are displayed . On the phenomenological side denoted quark (antiquark) flavored mesons and baryons , besides the PDG listings of $\sim$ 1970 , the assignment of resonances to Regge trajectories – excepting the Pomeron – were used , first as known in 1970 . I quote the textbook by Collins and Squires ref. 11 [11-1968] for a coherent presentation of Regge theory .

The first projection of investigations to be pursued was formulated in ref. 12 = [12-1970] which led to the following logical possibilities

1) local quark and antiquark fields – as displayed in eq. 2.2 – do carry color and *necessarily* appear together with an octet of gauge connection fields

$$\boxed{\begin{array}{l} \left(\mathscr{W}_{\mu} \left(\mathscr{D} \right) \right)_{\alpha\beta} (x) \,=\, W_{\mu}^{\,r} (x) \left(d_{\,r} \right)_{\alpha\beta} \ \leftrightarrow \\ \mathscr{W}_{\mu} \left(\mathscr{D} \right) \,=\, -\,\mathscr{W}_{\mu} \left(\mathscr{D} \right)^{\dagger} \\ d_{\,r} \,=\, -\,d_{\,r}^{\,\dagger} \,=\, \frac{1}{i}\,J_{\,r} \,\in\, Lie \left(\mathscr{D} \right) \\ \left[\, d_{\,p}\,,\,d_{\,q} \,\right] \,=\, f_{\,pqr}\,d_{\,r} \\ r\,,\,p\,,\,q \,=\, 1,\,\cdots,\,dim\,\mathscr{G} \ ; \ \alpha\,,\,\beta \,=\, 1,\,\cdots,\,dim\,\mathscr{D} \end{array}} \tag{2.3}$$

1) (continued) : We repeat eq. 2.3 below for clarity

$$\boxed{\begin{array}{l} \left(\mathscr{W}_{\mu} \left(\mathscr{D} \right) \right)_{\alpha\beta} (x) \,=\, W_{\mu}^{\,r} (x) \left(d_{\,r} \right)_{\alpha\beta} \ \leftrightarrow \\ \mathscr{W}_{\mu} \left(\mathscr{D} \right) \,=\, -\,\mathscr{W}_{\mu} \left(\mathscr{D} \right)^{\dagger} \\ d_{\,r} \,=\, -\,d_{\,r}^{\,\dagger} \,=\, \frac{1}{i}\,J_{\,r} \,\in\, Lie \left(\mathscr{D} \right) \\ \left[\, d_{\,p}\,,\,d_{\,q} \,\right] \,=\, f_{\,pqr}\,d_{\,r} \\ r\,,\,p\,,\,q \,=\, 1,\,\cdots,\,dim\,\mathscr{G} \ ; \ \alpha\,,\,\beta \,=\, 1,\,\cdots,\,dim\,\mathscr{D} \end{array}} \tag{2.4}$$

In eq. 2.4 $\mathscr{D}$ denotes an irreducible unitary representation of the gauge group $G = SU3_{\,c}$, $Lie(\mathscr{D}\,)$ the corresponding antihermitian-matrix representation of its Lie algebra. Here $\mathscr{D}$ is the 3 , $(\overline{3})$ representation for q , $\overline{q}\,'$ fields respectively. The hermitian gauge potentials $W_{\mu}^{\,a}\,(\,x\,)$ form the corresponding gauge connection field as shown in eq. 2.4 .

2) The main oscillatory modes : Given 1) to be valid, the first oscillatory modes to be faced are the light flavored valence $q\,\overline{q}\,'$ u, d , s mesons . The associated spectroscopy showed first of all a severe restriction concerning the local *and* global *exact* gauge invariance pertaining not just to the quark-antiquark degrees of freedom , but more importantly those related to gauge potentials and their field strengths , shown in eq. 2.5 below . What is meant here is the exact gauge invariance , which is necessary to completely remove color and its counting from the hadron resonances in question.

Here is a good place to quote a general collection of work concerning the construction of QCD in ref. 13 = [13-2014]

$$\begin{array}{c}
\text{gauge bosons :}\;\; \mathscr{L}_{B} \;=\; -\,\frac{1}{4\,g^{\,2}}\,B^{\,\mu\nu\,r}\,B_{\,\mu\nu}^{\,r} \\
B_{\,\mu\nu}^{\,r} \;=\; \partial_{\,\mu}\,W_{\,\nu}^{\,r} \,-\, \partial_{\,\nu}\,W_{\,\mu}^{\,r} \,+\, f_{\,rst}\,W_{\,\mu}^{\,s}\,W_{\,\nu}^{\,t} \longleftarrow \\
\left(\,W_{\,\mu}^{\,r} \;\equiv\; -\,v_{\,\mu}^{\,r}\,\right) \\
r,s,t \;=\; 1,\cdots,dim\,(\,G \;=\; SU3_{\,c}\,) \;=\; 8 \\
\text{Lie algebra labels,}\left[\tfrac{1}{2}\,\lambda^{\,r}\,,\,\tfrac{1}{2}\,\lambda^{\,s}\right] \;=\; i\,f_{\,rst}\,\tfrac{1}{2}\,\lambda^{\,t} \\
\text{perturbative rescaling :} \\
W_{\,\mu}^{\,r} \;=\; g\,W_{\,\mu\;pert}^{\,r}\;,\;B_{\,\mu\nu}^{\,r} \;=\; g\,B_{\,\mu\nu\;pert}^{\,r}
\end{array} \tag{2.5}$$

3) Given 1) and 2) the absence of a sizable weak interaction channel mediated through the neutral current called for the GIM mechanism , in 1970 in ref. 14 = [14-1970] and the existence of charm as a fourth flavor somewhat more massive than three light ones .

Together with the studies extending the light quark flavors , the search was undertaken to detect the neutral current weak interaction through the neutrino or antineutrino channels . This led to the discovery of the neutral current by the Gargamelle collaboration at CERN in 1973 , ref. 15 = [15-1973] .

4) the gauge group forms a *minimal* collection including spontaneously broken subgroups

The consequence of the discovery of the neutral current weak interaction comparable in strength with the charged current one is , that beyond the exactly unbroken subgroup of $SU3_{\,c}$ the electroweak part – $SU2_{\,L} \,\times\, U1_{\,\mathscr{Y}}$ – form together a lowest level of combined exact and broken group

$$\begin{array}{l}
G_{\,min} \;=\; SU3_{\,c} \,\times SU2_{\,L} \,\times U1_{\,\mathscr{Y}} \\
Q_{\,em}\,/\,|\,e\,| \;=\; I_{\,3\,w} \,+\, \mathscr{Y}
\end{array} \tag{2.6}$$

In eq. 2.6 $I_{\,3\,w}$ denotes the third component of weak isospin , $\mathscr{Y}$ the weak hypercharge commuting with $SU2_{\,L}$, $Q_{\,em}$ the electric charge and $|\,e\,|$ the charge of the proton in rational units .

PoS(CORFU2014)048

Volume 46B, number 1 PHYSICS LETTERS 3 September 1973

OBSERVATION OF NEUTRINO-LIKE INTERACTIONS WITHOUT MUON OR ELECTRON IN THE GARGAMELLE NEUTRINO EXPERIMENT

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Received 25 July 1973

Events induced by neutral particles and producing hadrons, but no muon or electron, have been observed in the CERN neutrino experiment. These events behave as expected if they arise from neutral current induced processes. The rates relative to the corresponding charged current processes are evaluated.

We have searched for the neutral current (NC) and charged current (CC) reactions:

$$\text{NC} \quad \nu_\mu/\bar{\nu}_\mu + \text{N} \to \nu_\mu/\bar{\nu}_\mu + \text{hadrons}, \tag{1}$$

$$\text{CC} \quad \nu_\mu/\bar{\nu}_\mu + \text{N} \to \mu^-/\mu^+ + \text{hadrons} \tag{2}$$

which are distinguished respectively by the absence of any possible muon, or the presence of one, and only one, possible muon. A small contamination of $\nu_e/\bar{\nu}_e$ exists in the $\nu_\mu/\bar{\nu}_\mu$ beams giving some CC events which are easily recognised by the e^-/e^+ signature. The analysis is based on 83 000 ν pictures and 207 000 $\bar{\nu}$ pictures taken at CERN in the Gargamelle bubble chamber filled with freon of density 1.5×10^3 kg/m^3 ‡. The dimensions of this chamber are such that most

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‡ A more detailed account of the analysis of this experiment appears in a paper to be submitted to Nuclear Physics.

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Figure 1: Titlepage of ref. [15-1973]

5) Quest for a unified gauge theory – of chargelike gauges – neglecting gravity

After pioneering work in this direction by by Jogesh Pati and Abdus Salam ref. 16 = [16-1973] , Howard Georgi and Sheldon Glashow ref. 17 = [17-1974] and Feza Gursey , Pierre Ramond and Pierre Sikivie ref. 18 = [18-1976] , we initiated a systematic investigation with this goal with Harald Fritzsch by the end of 1973 giving rise to a minimal such enveloping gauge group, spin10, in ref. 19 = [19-1975] . To stay in line with the experimental discovery of charm, as predicted in ref. 14 = [14-1970] in point 3) , we recall the so called 'November revolution' in 1974 , ref. 20 = [20-1974] and ref. 21 = [21-1974] .

6) Reflections on the nature of interactions in the environment of gauge-invariance and gauge-breaking within unified gauge field theory(-ies)

Given the *assumed* validity of the logical chains in points 1) - 5) I have to conclude it established , that the nature of unifyed gauge theories is singled out . It is however impossible at present, to specify the deduced subtle properties of these theories in precise mathematical terms . The restriction to gravitationless theories in uncurved 3 + 1 - dimensional space-time is of course quite restrictive .

Recent wider discussions of these and related points can be found in ref. 22 = [22-2012] , ref. 23 = [23-2014] and ref. 24 = [24-2014] .

2.1 Oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'\;;\,q,q\,' =$ u. d, s in mesons

The first elaboration of the mesonic oscillatory modes was not undertaken in 1970, the year of the project formulation in ref. 12 = [12-1970] . This I started during the last months of my stay at Caltech until end of February 1976 . It took two years to bring about a written (published) report – ref. 25 = [25-1978] – at the occasion of a lectture given at the 1978 Karslruhe Summer Institute . Ref. 25 could recently be updated with the help of the representatives of inspirehep.net , after a package of reprints at an unexpected location in Bern were found .

And this goes as follows : we report the derivations leading to the asymptotic form for the eigenvalues of the masssquare operator in the meson c.m. frame from refs. 5 = [5-1980] and 25 = [25-1978] extending eq. 1.1 below

$$\begin{array}{l} \alpha\,' M^2\;(\,J\,)\;=\;J\,+\,J_{\,0}\;\;\text{with}\;\;\alpha\,'\Delta\,M^{\,2}\;=\;\Delta\,J\;=\;\Delta\,N \\ \left(\alpha\,'\right)^{-1}\;=\;1.06\;\pm\;5\%\;\text{GeV}^{\,2} \\ \alpha\,'\;:\;\text{slope of Regge trajectories excepting the Pomeron} \end{array} \tag{2.7}$$

In order to keep the systematics of barycentric coordinates as defined for general $N_{\,c}\;\equiv\;\mathbb{N}$, also appropriate for baryons discussed in ref. 5 = [5-1980] , we set for the kinematically simpler $q\,\overline{q}\,'$ mesons (ref. 5 was achieved after obstacles had been overcome, which postponed it to 1980)

$$\begin{array}{l} \vec{z}\;=\;\vec{z}_{\,1}\;=\;\frac{1}{\sqrt{2}}\;(\,\vec{x}_{\,1}\;-\;\vec{x}_{\,2}\,)\;\textbf{and} \\ \vec{y}\;=\;\vec{x}_{\,1}\;-\;\vec{x}_{\,2}\;=\;\sqrt{2}\,\vec{z} \end{array} \tag{2.8}$$

Next we consider the Lagrangean

$$\begin{array}{l} \mathscr{L}_{\,q\overline{q}\,'}\;=\;-\;\left[\begin{array}{r} m_{\,1}\;(\,\vec{y}\,;M_{\,1}\,)\;\sqrt{1\;-\;\vec{v}_{\,1}^{\,2}} \\ +\,m_{\,2}\;(\,\vec{y}\,;M_{\,2}\,)\;\sqrt{1\;-\;\vec{v}_{\,2}^{\,2}} \end{array}\right] \\ M_{\,1,2}\;:\;\text{masses of}\;\;q_{\,1}\;,\;\overline{q}_{\,2}\;\;\text{respectively} \\ \vec{v}_{\,1,2}\;=\;(\,d\;/\;dt\,)\;\vec{x}_{\,1,2}\;\;;\;\;\text{in the meson c.m. system} \\ t\;:\;\text{overall synchronized time in the c.m. system ,} \\ \qquad\text{using units such that}\;\;c\;=\;1 \end{array} \tag{2.9}$$

In the next step we choose the chiral limit as a valid approximation at large distances , to be specified subsequently. Then eq. 2.9 simplifies through the relations

$$\begin{array}{l} m_{\,1}\;(\,\vec{y}\,;\,M_{\,1}\;\rightarrow\;0\,)\;\rightarrow\;m_{\,1}\;(\,\vec{y}\,) \\ m_{\,2}\;(\,\vec{y}\,;\,M_{\,2}\;\rightarrow\;0\,)\;\rightarrow\;m_{\,2}\;(\,\vec{y}\,) \\ m_{\,1}\;(\,\vec{y}\,)\;=\;m_{\,2}\;(\,\vec{y}\,)\;=\;m\;(\,\vec{y}\,) \\ \vec{v}_{\,1}\;=\;-\,\vec{v}_{\,2}\;=\;\vec{v} \end{array} \tag{2.10}$$

and $\mathscr{L}_{q\overline{q}\,'}$ in eq. 2.9 becomes

$$\begin{array}{l} \mathscr{L}_{q\overline{q}\,'} \ = - \ \overline{m} \ (\ \vec{y} \ ) \ \sqrt{1 \ - \ \vec{v}^{\,2}} \\ (\ \overline{m} \ = \ m_{\,1} \ + \ m_{\,2} \ = \ 2 \, m \ ) \ (\ \vec{y} \ ) \\ \vec{v} \ = \ \frac{1}{2} \ \dot{\vec{y}} \ ; \ ^{\cdot} \ = \ d \ / \ dt \end{array} \tag{2.11}$$

We expand the derivations following ref. 5 = [5-1980] . The canonical momentum relative to $\frac{1}{2}\ \vec{y}$ becomes

$$\begin{array}{l} \vec{p} \ = \ \vec{p}_{\,1} \ - \ \vec{p}_{\,2} \ = 2 \ \vec{p}_{\,c.m.} \ = \ \left(\ \mathscr{L}_{q\overline{q}\,'} \ \right)_{, \ \vec{v}} \ = \ \overline{m} \ \dfrac{\vec{v}}{\sqrt{1 \ - \ \vec{v}^{\,2}}} \\ \mathscr{H}_{(2)} \ = \ \vec{v} \ \left(\ \mathscr{L}_{q\overline{q}\,'} \ \right)_{, \ \vec{v}} \ - \ \mathscr{L}_{q\overline{q}\,'} \ = \ \dfrac{\overline{m}}{\sqrt{1 \ - \ \vec{v}^{\,2}}} \\ \vec{p}^{\,2} \ = \ \overline{m}^{\,2} \ \dfrac{v^{\,2}}{1 \ - \ v^{\,2}} \ = \ \mathscr{H}_{(2)}^{\,2} \ - \ \overline{m}^{\,2} \ ; \ \ v^{\,2} \ = \ \vec{v}^{\,2} \ \ \longrightarrow \\ \vec{p}^{\,2} \ + \ \overline{m}^{\,2} \ = \ \mathscr{H}_{(2)}^{\,2} \end{array} \tag{2.12}$$

It follows from the relations in eq. 2.12 that $\mathscr{H}_{(2)}$ is a constant of motion in both classical and quantum mechanical interpretations of the two body $q\,\overline{q}\,'$ system considered . The Euler-Lagrange equations become

$$\begin{array}{l} \dot{\vec{p}} \ = \ \mathscr{H}_{(2)} \ \ddot{\vec{v}} \ = \ \mathscr{H}_{(2)} \ \frac{1}{2} \ \ddot{\vec{y}} \ = \ \left(\ \mathscr{L}_{q\overline{q}\,'} \ \right)_{, \ \frac{1}{2} \ \vec{y}} \\ \quad = - \ \sqrt{1 \ - \ \vec{v}^{\,2}} \ 2 \ grad_{\,\vec{y}} \ \overline{m} \ = \ - \ \mathscr{H}_{(2)}^{\,-1} \ \overline{m} \ grad_{\,\vec{y}} \ \overline{m}^{\,2} \\ \longrightarrow \ \mathscr{H}_{(2)} \ \frac{1}{2} \ \ddot{\vec{y}} \ = \ - \ \left(\ \mathscr{H}_{(2)} \ \right)^{-1} \ 2 \ \overline{m} \ grad_{\,\vec{y}} \ \overline{m} \\ \qquad = \ - \ \left(\ \mathscr{H}_{(2)} \ \right)^{-1} \ grad_{\,\vec{y}} \ \overline{m}^{\,2} \end{array} \tag{2.13}$$

We can see how the mass-square dynamical variable arises in the classical interpretation of the equations of motion introducing the notation

$$\mathscr{M}_{\,meson}^{\,2} \ = \ \mathscr{H}_{(2)}^{\,2} \tag{2.14}$$

Then eq. 2.13 takes the form

$$\mathscr{M}_{\,meson}^{\,2} \ \ddot{\vec{y}} \ = \ - \ 2 \ grad_{\,\vec{y}} \ \overline{m}^{\,2} \tag{2.15}$$

The oscillatory modes inherit the central scale of QCD for light (u, d , s-) flavors of quark as implied by the trace anomaly – ref. 26 = [26-1975] , in the adopted chiral limit through the parameter Λ , not to be confused with $\Lambda_{\,QCD}$, valid in the perturbatively accessible region, in the Ansatz for the mass function for large values of $| \ \vec{y} \ |$, of dimension mass-square

$$\overline{m}^{\,2} \ (\ \vec{y} \ ) \sim \ \longrightarrow_{|\vec{y}| \to \infty} \ \sim \frac{1}{4} \ \Lambda^{\,2} \ | \ \vec{y} \ |^{\,2} \ \left[\ 1 \ + \ O \left(\frac{M_{\,q}}{\Lambda \ | \ \vec{y} \ |} \right) \right] \tag{2.16}$$

In eq. 2.16 $M_{\,q}$ denote the physical quark-mass parameters Inserting the asymptotic large $| \ \vec{y} \ |$ part of the mass-square function $\overline{m}^{\,2} \ (\ \vec{y} \ )$ into eq. 2.15 we obtain

$$\begin{array}{l} \mathscr{M}_{\,meson}^{\,2} \ \ddot{\vec{y}} \ = - \ \sim \ \Lambda^{\,2} \ \vec{y} \ \ \leftrightarrow \ \ \omega_{\,cl} \ = \ \dfrac{\Lambda}{\mathscr{H}_{(2)}} \\ \omega_{\,cl}^{\,2} \ = \ \dfrac{\Lambda^{\,2}}{\mathscr{M}^{\,2}} \end{array} \tag{2.17}$$

We here turn to the quantum mechanical description following from the Lagrangean given in eq. 2.11

$$\widehat{\vec{p}} \;=\; \tfrac{1}{i}\,\nabla_{\,\frac{1}{2}\,\vec{y}} \;=\; \left(\mathscr{L}_{\,q\overline{q}\,'} \right)_{,\,\vec{v}} \;=\; \left(\overline{m}\;\frac{\vec{v}}{\sqrt{1\,-\,v^{\,2}}} \right)_{ordered} \tag{2.18}$$

In eq. 2.18 the suffix *ordered* shall indicate that the operators $\overline{m}$ and $\dfrac{\vec{v}}{\sqrt{1\,-\,v^{\,2}}}$, written as product, do not commute, which necessitates the ordering guaranteeing a self-adjoint operator beeing represented by the product .

2.1.1 Counting oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'\,;\,q,q\,'$ = u. d, s in mesons

We first determine the number density at given main quantum number N , which amounts to calculate the power of the set of occupation numbers $n_{\,1}\,,\,n_{\,2}\,,\,n_{\,3}$ of the associated 3 oscillators .

p is given by the number of partitions

$$p\,(N) = \{n_1, n_2, n_3 |\, n_1 + n_2 + n_3 = N \;;\; n_{1,2,3} = 0, 1, 2 \cdots N\} \tag{2.19}$$

multiplied with the multiplicity of $SU6(spin\,\times\,N_{\,fl}) = 36$.

The power of the set $p\,(\,N\,)$ is readily written as a sum over $n_{\,3}$

$$\begin{array}{l} p\,(\,N\,) \;=\; \sum_{n_3=0}^{N}\, p\,(\,n_1, n_2\,;\,\nu\,) \;=\; \sum_{\,\nu=0}^{\,N}\, p\,(\,n_{\,1}\,,\,n_{\,2};\,\nu\,) \\ \nu \;=\; N\, - n_{\,3} \;=\; 0, 1, \cdots, N \;;\; n_{\,1,2} \;=\; 0.1, 2\, \cdots\, N \\ p\,(\,n_{\,1}\,,\,n_{\,2}\,;\,\nu\,) \;=\; \{\,n_{\,1}\,,\,n_{\,2}\,|\;\; n_{\,1}\,+\,n_{\,2} \;=\; \nu\,\} \;=\; \nu\,+\,1 \end{array} \tag{2.20}$$

Thus $p\,(\,N\,)$ defined in eq. 2.19 becomes

$$\begin{array}{rl} p\,(\,N\,) & = \sum_{\,\nu=0}^{\,N}\,(\,\nu\,+\,1\,) \;=\; \sum_{\,\nu_+=1}^{\,N+1}\,\nu_+ \\ & = \frac{1}{2}\,(\,N\,+\,1\,)\,(\,N\,+\,2\,) \\ \nu_+ & = \;\nu\,+\,1 \end{array} \tag{2.21}$$

and $z(\,N\,) = 36\,p\,(\,N\,)$ is

$$\begin{array}{l} z\,(\,N\,) \;=\; \partial\,\rho\,/\,\partial\,\left(\alpha\,'\,M^{\,2}\right) \;=\; 18\,(\,N\,+\,1\,)\,(\,N\,+\,2\,) \\ \alpha\,'\,M^{\,2} \;=\; \alpha\,'\,\Delta\,M^{\,2}\,+\,N_{\,0} \;=\; N\,+\,N_{\,0} \end{array} \tag{2.22}$$

We give here the derivation yielding the sum of squares of integers from the difference of sums of cubes

$$S_{\,2}\,(\,N^{\,*}\,) \;=\; \sum_{\,\nu=0}^{\,N^{\,*}}\,\nu^{\,2} \;,\; S_{\,3}\,(\,N^{\,*}\,) \;=\; \sum_{\,\nu=0}^{\,N^{\,*}}\,\nu^{\,3} \tag{2.23}$$

which makes use of the recursive relation

$$\begin{array}{l} S_{\,3}\,(\,N^{\,*}\,+\,1\,) - S_{\,3}\,(\,N^{\,*}\,) \;= \\ \quad = \sum_{\,\nu=0}^{\,N^{\,*}}\,\left[\,(\,\nu\,+\,1\,)^{\,3}\,-\,\nu^{\,3}\,\right] \\ \quad = (\,N^{\,*}\,+\,1\,)^{\,3} \\ \quad = 2\,S_{\,2}\,(\,N^{\,*}\,) + \frac{3}{2}\,N^{\,*}(\,N^{\,*}\,+\,1\,) + N^{\,*} + 1 \end{array} \tag{2.24}$$

Then it follows

$$S_2\,(N^*) = \tfrac{1}{6}\,N^*(N^* + 1)\,(2N^* + 1) \tag{2.25}$$

We display the function $S_2\,(N^*)$, $N^* = 1$ to 5 in table 1 below

N^*	$S_2\,(N^*)$
1	1
2	5
3	14
4	30
5	55

(2.26)

Table 1

The sum of the squares $S_2\,(N^*)$ is in the present context an *auxiliary* function necessary for the calculation of the total number of meson states with $N \leq N^*$.

We thus find for the number of u,d,s meson states with $N \leq N^*$

$$\begin{aligned}
Z\,(N^*) &= \\
&= 18\,\left(S_2\,(N^* + 1) + \tfrac{1}{2}\,(N^* + 1)\,(N^* + 2)\right) \\
&= 18\begin{pmatrix} \tfrac{1}{6}\,(N^*+1)\,(N^*+2)\,(2N^*+3) \\ +\tfrac{1}{2}\,(N^*+1)\,(N^*+2) \end{pmatrix} \\
&= 18\left((N^*+1)\,(N^*+2)\left(\tfrac{1}{3}\,(N^*+1)\right)\right) \\
&= 6\,(N^* + 1)\,(N^* + 2)\,(N^* + 3)
\end{aligned} \tag{2.27}$$

The main results of this subsection are contained in eqs. 2.22 for the number density as a function of the main oscillator quantum number N: $z\,(N) = \partial\rho\,/\,\partial\left(\alpha'\,M^2\right)$ and 2.27 for the total number of oscillatory modes below and including the limiting main quantum number N^* : $Z\,(N^*)$, both equations recapitulated for u, d, s mesons $q\,\overline{q}\,'$ below

$$\begin{aligned}
z\,(N) &= \partial\rho\,/\,\partial\left(\alpha'\,M^2\right) = 18\,(N + 1)\,(N + 2) \\
\alpha'\,M^2 &= \alpha'\,\Delta M^2 + N_0 = N + N_0
\end{aligned} \tag{2.28}$$

$$Z\,(N^*) = 6\,(N^* + 1)\,(N^* + 2)\,(N^* + 3) \tag{2.29}$$

3. Oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'$; $q, q\,' =$ u. d, s in mesons (continued)

We recall here the relations in eqs. 2.10 and 2.11 , repeated below

$$\begin{array}{l} m_1 \left(\vec{y} \, ; M_1 \to 0 \right) \to m_1 \left(\vec{y} \right) \\ m_2 \left(\vec{y} \, ; M_2 \to 0 \right) \to m_2 \left(\vec{y} \right) \\ m_1 \left(\vec{y} \right) = m_2 \left(\vec{y} \right) = m \left(\vec{y} \right) \\ \vec{v}_1 = - \vec{v}_2 = \vec{v} \end{array} \tag{3.1}$$

$\mathscr{L}_{q\overline{q}\,'}$ in eq. 2.11 becomes

$$\begin{array}{l} \mathscr{L}_{q\overline{q}\,'} = - \overline{m} \left(\vec{y} \right) \sqrt{1 - \vec{v}^{\,2}} \\ \left(\overline{m} = m_1 + m_2 = 2\, m \right) \left(\vec{y} \right) \\ \vec{v} = \frac{1}{2} \dot{\vec{y}} \; ; \; \dot{} = d \,/\, dt \end{array} \tag{3.2}$$

Next we recall eq. 2.18 repeated below

$$\widehat{\vec{p}} = \frac{1}{i} \nabla_{\frac{1}{2} \vec{y}} = \left(\mathscr{L}_{q\overline{q}\,'} \right)_{, \vec{v}} = \left(\overline{m} \, \frac{\vec{v}}{\sqrt{1 - v^2}} \right)_{ordered} \tag{3.3}$$

Eq. 2.12 adapted to the quantum mechanical logic becomes

$$\begin{array}{l} \widehat{\vec{p}} = \widehat{\vec{p}}_1 - \widehat{\vec{p}}_2 = 2 \widehat{\vec{p}}_{c.m.} = \left(\mathscr{L}_{q\overline{q}\,'} \right)_{, \vec{v}} = \overline{m} \left. \dfrac{\vec{v}}{\sqrt{1 - \vec{v}^2}} \right|_{ordered} \\ \widehat{\mathscr{H}}_{(2)} = \vec{v} \left(\mathscr{L}_{q\overline{q}\,'} \right)_{, \vec{v}} - \mathscr{L}_{q\overline{q}\,'} = \left. \dfrac{\overline{m}}{\sqrt{1 - \vec{v}^{\,2}}} \right|_{ordered} \end{array} \tag{3.4}$$

$$\begin{array}{l} \widehat{\vec{p}}^{\,2} = \overline{m}^{\,2} \left. \dfrac{v^2}{1 - v^2} \right|_{ordered} = \widehat{\mathscr{H}}^{2}_{(2)} - \overline{m}^{\,2} \; ; \; v^2 = \vec{v}^{\,2} \longrightarrow \\ \\ \widehat{\vec{p}}^{\,2} + \overline{m}^{\,2} = \widehat{\mathscr{H}}^{2}_{(2)} = \widehat{\mathscr{M}}^{\,2} \; ; \; \widehat{\vec{p}}^{\,2} = - \Delta_{\frac{1}{2} \vec{y}} = - 4 \, \Delta_{\vec{y}} \\ \overline{m}^{\,2} \left(\vec{y} \right) \sim \underset{|\vec{y}| \to \infty}{\longrightarrow} \sim \frac{1}{4} \Lambda^2 \, | \, \vec{y} \, |^{\,2} \left[1 + O \left(\dfrac{M_q}{\Lambda \, | \, \vec{y} \, |} \right) \right] \end{array} \tag{3.5}$$

Eq. 3.5 shows the main result of this section , in particular the second order wave equation (on the second line)

$$\widehat{\mathscr{M}}^{\,2} = \left[- 4 \, \Delta_{\vec{y}} + \frac{1}{4} \Lambda^2 \, | \, \vec{y} \, |^{\,2} \right] \tag{3.6}$$

If we determine it from the positive parity Λ trajectory from the present PDG tables [6-2012]

$$\begin{array}{rlll} \Lambda \, , \, J^P : & \frac{1}{2}^{\,+} & \frac{5}{2}^{\,+} & \frac{9}{2}^{\,+} \\ M_j : & 1.115683 & 1.820 & 2.350 \\ M_j^2 : & 1.2447485 & 3.3124 & 5.5225 \\ \frac{1}{2} \, \Delta \, M^{\,2} : & & 1.034 & 1.105 \end{array} \tag{3.7}$$

and average the two half mass square difference entries in the last line of eq. 3.7 with weights two to one we obtain

$$1\,/\,\alpha^{\,\prime} = \tfrac{1}{3}\,\left(M_{2}^{\,2} - M_{1}^{\,2}\right) + \tfrac{1}{6}\,\left(M_{3}^{\,2} - M_{2}^{\,2}\right) \sim 1.06\ \mathrm{GeV}^{\,2} \tag{3.8}$$

We remark that in ref. 6 = [6-2012] $\Lambda\,\frac{9}{2}^{\,+}$ has only three stars , and furthermore the trajectory contains only three entries , whereas I think to remember that it contained four sometimes back [1]. $\Lambda\,\frac{13}{2}^{\,+}$ would extrapolate to 2.755 GeV using eq. 3.7 .

In order to exhibit the oscillator variables we substitute rescaled coordinates and derivatives relative to the spatial variable $\vec{y}$

$$\vec{y} = \lambda^{\,-1}\,\vec{\zeta}\ ,\ \nabla_{\vec{y}} = \lambda\,\nabla_{\vec{\zeta}} \tag{3.9}$$

The differential operator on the right hand side of eq. 3.8 becomes

$$-\,4\,\Delta_{\vec{y}} + \tfrac{1}{4}\,\Lambda^{2}\,|\,\vec{y}\,|^{\,2} = -\,4\,\lambda^{\,2}\,\Delta_{\vec{\zeta}} + \tfrac{1}{4}\,\lambda^{\,-2}\Lambda^{2}\,|\,\vec{\zeta}\,|^{\,2} \tag{3.10}$$

The parameter λ , of dimension mass, shall be chosen such that

$$4\,\lambda^{\,2} = \tfrac{1}{4}\,\lambda^{\,-2}\,\Lambda^{2} \ \rightarrow\ 2\,\lambda = \tfrac{1}{2}\,\lambda^{\,-1}\,\Lambda \ \rightarrow\ 4\,\lambda^{\,2} = \Lambda \tag{3.11}$$

Substituting the last equation in eq. 3.11 in eq. 3.10 we obtain

$$-\,4\,\Delta_{\vec{y}} + \tfrac{1}{4}\,\Lambda^{2}\,|\,\vec{y}\,|^{\,2} = \Lambda\,\left[\,-\,\Delta_{\vec{\zeta}} + |\,\vec{\zeta}\,|^{\,2}\,\right] \tag{3.12}$$

The deliberations leading eventually to the definition of $\widehat{\mathscr{M}}^{\,2}$ given in eq. 3.8 go back to ref. 12 = [12-1970] in 1970, published in 1971. They take shape as oscillatory modes for light flavored mesons first in 1975 and in published form in ref. 25 = [25-1978] in 1978, oscillatory modes for light flavored mesons and baryons were first presented in ref. 5 = [5-1980] .

3.1 Oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}^{\,\prime}\ ;\ q,q^{\,\prime}\ =$ u. d, s in mesons (proper)

The dimensionless rescaled spatial variables $\vec{\zeta}$ defined through eqs. 3.9 and 3.11

$$\vec{\zeta} = \lambda\,\vec{y}\ ;\ \lambda = \tfrac{1}{2}\,(\,\Lambda\,)^{\,\frac{1}{2}} \tag{3.13}$$

and their canonically conjugate momenta

$$\widehat{\vec{p}}_{\vec{\zeta}} = \tfrac{1}{i}\,\nabla_{\vec{\zeta}} = \tfrac{1}{i}\,\partial_{\vec{\zeta}} \tag{3.14}$$

in components

$$\zeta^{\,m}\ ,\ \widehat{p}_{\,n} = \partial_{\,\zeta^{\,n}}\ ;\ [\ \widehat{p}_{\,n}\ ,\ \zeta^{\,m}\ ] = \tfrac{1}{i}\,\delta_{\,n}^{\ \ m}\ ; m,n = 1,2,3 \tag{3.15}$$

generate the 3 oscillator absorption and creation operators

[1] "Tempora mutantur nos et mutamus in illis ."

$$
\begin{array}{rl}
a^{\,m} \,=\, \frac{1}{\sqrt{2}} \,\left(\, \zeta^{\,m} \,+\, i\,\widehat{p}_{\,m} \,\right)\, & a^{\,*\,m} \,=\, \frac{1}{\sqrt{2}} \,\left(\, \zeta^{\,m} \,-\, \partial_{\,\zeta^{\,m}} \,\right) \\
\,=\, \frac{1}{\sqrt{2}} \,\left(\, \zeta^{\,m} \,+\, \partial_{\,\zeta^{\,m}} \,\right)\, , & \,=\, \frac{1}{\sqrt{2}} \,\left(\, \zeta^{\,m} \,-\, \partial_{\,\zeta^{\,m}} \,\right) \\
m \,=\, 1,2,3 &
\end{array}
\tag{3.16}
$$

obeying the commutation rules

$$
\left[\, a^{\,m} \,,\, a^{\,*\,n} \,\right] \,=\, \delta^{\,mn}\,\P \;\; ; \;\; \left[\, a^{\,m} \,,\, a^{\,n} \,\right] \,=\, \left[\, a^{\,*\,m} \,,\, a^{\,*\,n} \,\right] \,=\, 0
\tag{3.17}
$$

The oscillator algebra displayed in eq. 3.17 is common to any (3) canonical pairs of operators , associated with a threedimensional uncurved space, as is the case here. It shows directly the U3-invariance of the commutation rules.

What is very special is the relation for the dynamical form of the mass square operator $\widehat{\mathscr{M}}^{\,2}$ as given in eq. 3.12 , which becomes

$$
\begin{array}{l}
\widehat{\mathscr{M}}^{\,2} \,=\, \Lambda \,\left[\, -\,\Delta_{\,\vec{\zeta}} \,+\, \vec{\zeta}^{\,2} \,\right] \,=\, 2\,\Lambda\,\left[\, \widehat{N} \,+\, \frac{3}{2}\,\P \,\right] \\
\widehat{N} \,=\, \sum_{\,m=1}^{\,3} \,\widehat{N}_{\,m} \;\; ; \;\; \widehat{N}_{\,n} \,=\, a^{\,*\,n}\,a^{\,n} \,-\, \frac{1}{2}\,\P
\end{array}
\tag{3.18}
$$

For an individual n we have

$$
\begin{array}{rl}
2\,a^{\,*\,n}\,a^{\,n} \,= & \left(\, \zeta^{\,n} \,-\, \partial_{\,\zeta^{\,n}} \,\right)\,\left(\, \zeta^{\,n} \,+\, \partial_{\,\zeta^{\,n}} \,\right) \\
\,= & -\,\partial_{\,\zeta^{\,n}}^{\,2} \,+\, \left(\, \zeta^{\,n} \,\right)^{\,2} \,-\, \P
\end{array}
\tag{3.19}
$$

which proves the correctness of the first relation in eq. 3.18 .

4. Identifying $2\,\Lambda$ with the inverse slope of Regge trajectories other than the Pomeron

Since the relation in eq. 3.18 is only valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N \,= n_{\,1} \,+\, n_{\,2} \,+\, n_{\,3} \,=\, 0,1,2,\cdots$, where $n_{\,k}$, $k \,=\, 1,2,3$ denote the eigenvalues of the individual counting operators $\widehat{N}_{\,k}$, we can reparametrize within the same approximation accuracy eq. 3.18 in the form

$$
\widehat{\mathscr{M}}^{\,2} \,==\, 2\,\Lambda\,\left[\, \widehat{N} \,+\, \widehat{N}_{\,0} \,\right]
\tag{4.1}
$$

In eq. 4.1 the operator $\widehat{N}_{\,0}$ contains all effects from short distances and parametrically depends on quark masses . It does not commute with the asymptotically dominating number operators $\widehat{N}$, $\widehat{N}_{\,k}$; $k \,=\, 1,2,3$. We deal with the 'intercept'-related perturbations through the operator $\widehat{N}_{\,0}$ as characterized in eq. 4.1 and the text specifying its details in the sense of perturbations of *large* eigenvalues of $\widehat{N}$, the dominant operator for large eigenvalues N of the mass square operator $\widehat{\mathscr{M}}^{\,2}$, adopting the ansatz for the eigenvalues of $\widehat{\mathscr{M}}^{\,2}$

$$
\mathscr{M}^{\,2} \,\sim\, 2\,\Lambda\,\left(\, N \,+\, N_{\,0} \,\right) \;\; ; \;\; N \,=\, 0,1,2\cdots \text{ yet } \gg\, 1
\tag{4.2}
$$

We compare the structure of $\mathscr{M}^{\,2}$ in eq. 4.2 with the relation to this quantity along a Regge trajectory taken approximately linear

$$\begin{array}{l} \alpha\,(\,\mathscr{M}^{\,2}\,)\;=\;\alpha^{\,\prime}\,\mathscr{M}^{\,2}\,+\,\alpha_{\,0}\;\rightarrow\;\alpha^{\,\prime}\;:\quad \text{universal slope of} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{Regge trajectories} \\ \alpha\,(\,\mathscr{M}^{\,2}\,)\;\rightarrow\;J\;\rightarrow\;N\;\;;\;\;\mathscr{M}^{\,2}\;=\;\dfrac{1}{\alpha^{\,\prime}}\;\;(\,N\,-\,\alpha_{\,0}\,) \end{array} \tag{4.3}$$

Comparing the two expressions for $\mathscr{M}^{\,2}$ in eqs. 4.2 and 4.3 , we obtain the sought identification

$$2\,\Lambda\;=\;\frac{1}{\alpha^{\,\prime}}\;\sim\;1.06\;\pm\;0.05\;\text{GeV}^{\,2} \tag{4.4}$$

4.1 First results and illustrations from the count of oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'$; $q,q\,'$ = u. d, s for mesons and baryons

$N^{\,*}$	Z Baryons	Z Mesons
0	56	36
1	266	144
2	2310	360
3	4090	720

(4.5)

Table 2

In Table 2 we give a comparison of the number of oscillatory modes up to $N^{\,*}$; $N^{\,*}\;=\;0.1.2.3$ for baryons and mesons . The numbers for baryons are from ref. 1 = [1-2013] .

The oscillator structure for mesons shown in eqs. 3.16 - 3.19 , which gives rise to the count of partitions, as charecterized in eqs. 4.1 - 4.3,

is valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N\;=n_{\,1}\,+\,n_{\,2}\,+\,n_{\,3}\;=\;0,1,2,\cdots$, where $n_{\,k}$, $k\;=\;1,2,3$ denote the eigenvalues of the individual counting operators $\widehat{N}_{\,k}$.

The count of these (3-) partitions is illustrated for the simpler case of (2-) partitions ($N\;=\;n_{\,1}\,+\,n_{\,2}\,+\,n_{\,3}\;=\;0,1,2,\cdots$) in Fig. 2 , below . It is independent of the 'intercept'-induced shifts which give rise to the parameters $N_{\,0}\;\leftrightarrow\;\alpha_{\,0}$ shown in eqs. 4.2 and 4.3 .

For the so defined (n-) partition count , the unit spatial grid is independent of the operator $\widehat{N}_{\,0}$, displayed in eq. 4.1 . It is reached in momentum space from eq. 4.4 , which yields the unit in momentum space for each of the 3 components of $\vec{p}$ in the case of mesons .

$$[\,p\,]\;=\;\left(\alpha^{\,\prime}\right)^{\,-1/2}\;=\;(\;1.0296\;\pm\;2.5\%\;)\;\text{GeV} \tag{4.6}$$

The comparison of the number density of states per unit mass-square $z\,(\,N\,)$ between baryons and mesons is illustrated in Fig. 3 below

Fig. 3 serves to show, that the density per unit mass-square count reveals more baryon states than mesonic ones, for all values of N .

Nevertheless in production cross-sections as well as in thermal equilibrium mesons are dominantly produced, for low energies or temperatures .

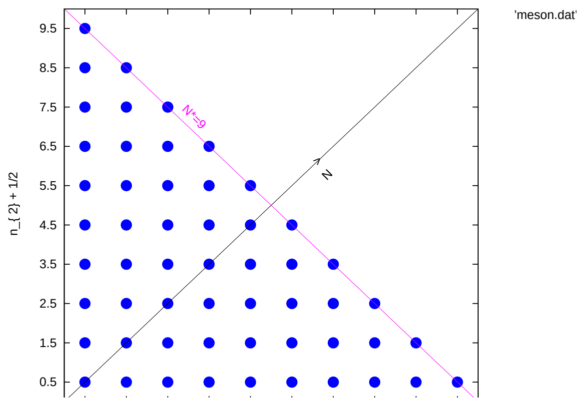

Figure 2: Illustration of partitions $N = n_1 + n_2$ simplifying the ones relevant for mesons $N = n_1 + n_2 + n_3$

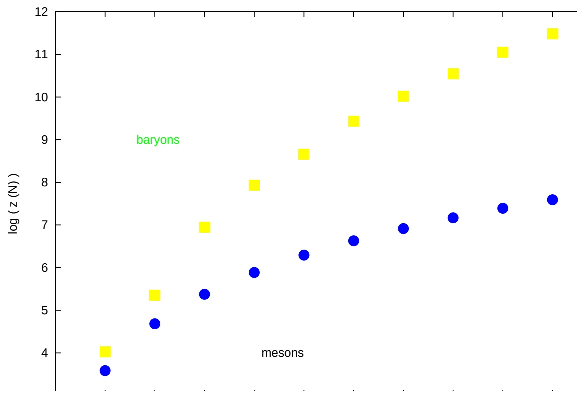

Figure 3: Density of states $z\,(N)$ in logarithmic scale per unit mass-square

To this end we calculated the exponentially weighted density per unit mass-square for baryons

$$\partial\, \rho_{\,baryon} \,/\, \partial \left(\alpha\,' M^{\,2} \right) \times \exp\,(\,-\,M \,/\, T\,) \tag{4.7}$$

for $T = 0.2$ GeV displayed in Fig. 4 below

The interpolation between the 4 points displayed in Fig. 4 is done using cubic splines , whereby we have approximated

$$\frac{1}{\alpha\,'} \sim 1.0\ \mathrm{GeV}^{\,2} \tag{4.8}$$

5. Outlook

The present outline of oscillatory modes of – valence quarks and antiquarks $q\,\overline{q}\,'$ in mesons and $q, q\,', q\,''$ in baryons and $\overline{q}, \overline{q}\,', \overline{q}\,''$ in antibaryons – is very incomplete due to limited time . I wish to conclude emphasizing the following points :

Ou-1) Spectral densities for mesons baryons and antibaryons

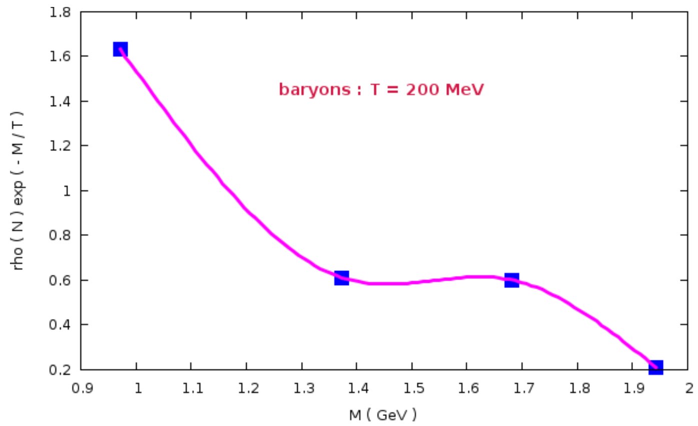

Figure 4: Density of states $z\,(\,N\,)$ per unit mass-square for baryons wighted with the exponential factor $exp\,(\,-\,M/T\,)\,;\,T$: Temperature

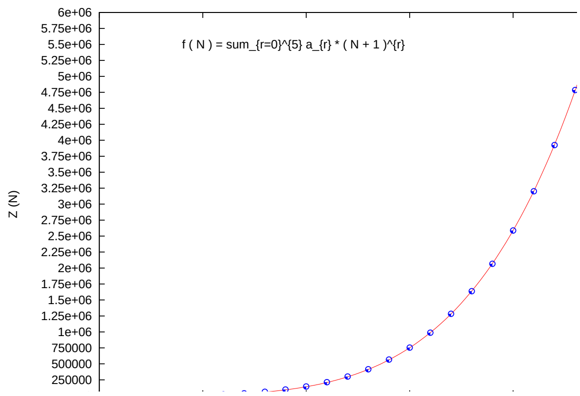

Figure 5: # (ℕ)

are growing proportional to $N^{\,2}$ and $N^{\,5}$ for large values of N, for mesons , and baryons (antibaryons) respectively .

This is *compatible* with the requirement of an underlying local gauge theory – neglecting gravity . As a consequence it follows that any form of consistent string-theory is *inequivalent* to the oscillatory modes considered, independent of the fact that no concrete properties of modes involving valence gauge bosons (of QCD) have as yet been derived .

Ou-2) More details concerning triple-quark modes in baryons have been presented in ref. 1 = [1-2013] . The inconsistencies addressed in point Ou-1) between string-theories and oscillatory modes discussed here and in ref. 1 = [1-2013] have been presented in more detail in ref. 28 = [28-2012] . We display the density per mass-square for valence quark u, d, s - baryons from ref. 1 = [1-2013] in Fig. 5 below

Ou-3) Extending the Count of oscillatory modes we look forward to proceed along the path of counting oscillatory modes of light flavored mesons and baryons , eventually to elaborate *in addition* oscillatory modes of valence gauge bosons , as it is outlined here in its present incomplete stage . The goal is to obtain main consequences for the description of thermal equilibrium as applying to the hadronic phase of QCD .

What becomes clear now is that the alleged thermal properties at temperatures not exceeding 200 MeV including the limit of chemical freeze-out at small or vanishing chemical potentials involve a newly posed assessment of feeding through the decay products of the heavy (heavier) baryons and antibaryons. This is a serious problem affecting mainly pions and kaons . It is conceivable that the difficulties in accounting for the observed low nucleon abundances at RHIC and Alice at LHC heavy ion collisions are a consequence of incomplete feeding corrections in the sense sketched above .

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