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# Reduction of couplings in the MSSM

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We present an application of the reduction of couplings program in the minimal supersymmetric Standard Model (MSSM). We investigate if a functional relation between $\alpha_1$ and $\alpha_2$ gauge couplings can be realized which is Renormalization Group Invariant (RGI). Following the same procedure for the top and bottom Yukawa couplings we end up with a prediction of a narrow window for $\tan\beta$, which is one of the basic parameters that determine the light Higgs mass.

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## 1. Introduction

The ultimate goal of Particle Physics community is to describe the fundamental interactions in nature as a unified one. Superstring theory is one of the relevant candidates to achieve this. However this unified picture must be able to give plausible explanations for the large number of free parameters of the Standard Model (SM). As a matter of fact this is a very difficult project if not impossible. So at least we can try to relate some parameters, achieving some partial reduction of couplings. In general imposing a symmetry is a natural way to reduce the number of independent couplings of a theory. Grand Unified Theories (GUTs) which support SU(5) symmetry is an example of how to reduce three gauge couplings into a unified one. Besides this great achievement SU(5) GUT can also relate Yukawa couplings via the prediction of the ratio $M_\tau/M_b$. However imposing larger symmetries seems not to help, because of the new degrees of freedom that are introduced.

In order to avoid such difficulties we can adopt a more general approach. We try to reduce the number of independent couplings by imposing relations among them. The crucial point is that these relations are such that renormalizability is preserved and are independent of the normalization point. This method was initially developed for the complete reduction from n+1 coupling parameters $\alpha_0, \alpha_1, ..., \alpha_n$ to a description in terms of $\alpha_0$ only, the so-called program of reduction of coupling parameters [1]. The basic requirement is that the original as well as the reduced theory have to satisfy the corresponding renormalization group equations. The last years great progress has been made applying this method and looking for RGI relations [2, 3, 4, 5, 6, 7, 8, 9] holding below the Planck scale up to GUT or lower scales.

Application of this procedure to dimensionless couplings of supersymmetric GUTs has led to the correct prediction of top quark mass in the finite and in the minimal N = 1 supersymmetric SU(5) GUTs [2, 3]. The most impressive aspect of the RGI relations is that they are valid to all orders of perturbation theory, a fact that can be realized by exploring the uniqueness of these relations at 1-loop level [1]. Besides this we can also find RGI relations that guarantee finiteness to every order in perturbation theory [10, 11].

Here we would like to apply the program of reduction of coupling parameters to minimal schemes such as MSSM. We explore the possibility to relate $\alpha_1$ and $\alpha_2$ gauge couplings. We continue applying this method to the Yukawa sector, relating top quark and bottom quark Yukawa couplings [12]. As a result we finally achieve to give a narrow range of values for $\tan\beta$, that permit us to predict the mass of the Higgs boson. Recently application of the above program relating top quark and bottom quark Yukawa couplings with $\alpha_3$ gauge coupling has led to a prediction of the Higgs mass with great success [13].

## 2. General Method of Reduction

Our aim is to express $\alpha_1, \alpha_2, ..., \alpha_n$ coupling parameters as functions of $\alpha_0$ so that a model involving a single coupling constant parameter $\alpha_0$ is obtained, which is again invariant under the renormalization group. So we can write the functions

$$\alpha_j = \alpha_j(\alpha_0) \qquad j = 1, ..., n. \tag{2.1}$$



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We also assume that the functions $\alpha_j(\alpha_0)$ should vanish in the weak coupling limit

$$\lim_{\alpha_0 \to 0} \alpha_j(\alpha_0) = 0.$$

Invariance of the Green's functions $G(p_i, M, \alpha_0, \alpha_1, ..., \alpha_n)$ of the original system under renormalization group implies the Callan-Symanzik equations

$$\left( M\frac{\partial}{\partial M} + \sum_{j=0}^{n} \beta_j \frac{\partial}{\partial \lambda_j} + \gamma \right) G(p_i, M, \alpha_0, \alpha_1, ..., \alpha_n) = 0$$

where $M, \beta_j, \gamma$ are the renormalization mass, the beta functions and the anomalous dimensions correspondingly. Similarly for the Green's functions $G'(p_i, M, \alpha_0, \alpha_1(\alpha_0), ..., \alpha_n(\alpha_0))$ of the reduced system we have

$$\left( M\frac{\partial}{\partial M} + \beta' \frac{\partial}{\partial \alpha_0} + \gamma' \right) G'(p_i, M, \alpha_0, \alpha_1(\alpha_0), ..., \alpha_n(\alpha_0)) = 0.$$

We can see that G$'$ is obtained from G by substituting the functions (2.1)

$$G' = G(\alpha_0, \alpha_1(\alpha_0), ..., \alpha_n(\alpha_0))$$

so differentiating with respect to $\alpha_0$ we obtain

$$\frac{\partial G'}{\partial \alpha_0} = \frac{\partial G}{\partial \alpha_0} + \sum_{j=1}^{n} \frac{\partial G}{\partial \alpha_j} \frac{d\alpha_j}{d\alpha_0}.$$

The above equations as well as linear independence of the Green's functions and their derivatives lead to the relations

$$\beta' = \beta_0, \qquad \gamma' = \gamma, \qquad \beta' \frac{d\alpha_j}{d\alpha_0} = \beta_j.$$

Hence the functions (2.1) must satisfy the following differential equations, the reduction equations

$$\beta_j = \beta_0 \frac{d\alpha_j}{d\alpha_0}. \tag{2.2}$$

A crucial point here is that the system (2.2) forms a necessary and sufficient condition for reducing the original system by the functions $\alpha_j(\alpha_0)$.

## 3. Reduction of a system with two coupling constants

For simplicity we assume that the original system has two coupling parameters, $\alpha_0$ and $\alpha_1$. We will examine if we can reduce $\alpha_1$ in favor of $\alpha_0$. The corresponding beta-functions can be written at lowest order as

$$\beta_0 = b_0 \alpha_0^2 + ....$$

$$\beta_1 = c_1 \alpha_1^2 + c_2 \alpha_0 \alpha_1 + c_3 \alpha_0^2....$$

which cover a wide range of models. The reduction equation which we have to solve is

$$\beta_1 = \beta_0 \frac{d\alpha_1}{d\alpha_0}. \tag{3.1}$$

Assuming power series solution we can expand $\alpha_1$ as

$$\alpha_1 = p_0^{(1)} \alpha_0 + \sum_{n=1} p_n^{(1)} \alpha_0^{(n+1)}.$$

Substituting the above expression into the (3.1) at lowest order we end up with a quadratic equation

$$c_1 p_0^2 + (c_2 - b_0) p_0 + c_3 = 0$$

which can be easily solved calculating the corresponding determinant.

## 4. Application to the MSSM

### 4.1 Relating $\alpha_1$ and $\alpha_2$ gauge couplings

We will explore the possibility to reduce $\alpha_2$ gauge coupling in favor of $\alpha_1$ ($\alpha_i = g_i^2/4\pi$). Assuming that there is a relation between them, a function $\alpha_2(\alpha_1)$, we have to solve the following reduction equation

$$\beta_2 = \beta_1 \frac{d\alpha_2}{d\alpha_1} \tag{4.1}$$

where

$$\beta_2 \equiv \frac{d\alpha_2}{dt} = \frac{b_2}{2\pi}\alpha_2^2, \qquad \beta_1 \equiv \frac{d\alpha_1}{dt} = \frac{b_1}{2\pi}\alpha_1^2$$

are the $\beta$ functions for the $\alpha_2$ and $\alpha_1$ gauge couplings correspondingly, $b_2 = 1$ and $b_1 = 11$ are the $\beta$ function coefficients and $t = \ln E$. Assuming that the differential equation (4.1) has a power series solution, we can expand $\alpha_2$ at lowest order in perturbation theory as

$$\alpha_2 = c_0 \alpha_1$$

where $c_0$ is a constant. Substituting this relation to the reduction equation (4.1) we are led to

$$c_0 = \frac{\beta_2}{\beta_1} = \frac{b_2 \alpha_2^2}{b_1 \alpha_1^2} = \frac{b_2 c_0^2 \alpha_1^2}{b_1 \alpha_1^2} \Rightarrow$$

$$c_0(c_0 b_2 - b_1) = 0 \Rightarrow$$

$$c_0 = 0, \qquad c_0 = 11.$$

Hence $\alpha_2$ can be written as a function of $\alpha_2$ as

$$\alpha_2 = 11\alpha_1.$$

We can check now if the above result is compatible with the experimental values

$$\frac{1}{\alpha_{em}(Mz)} = \frac{1}{\alpha_1(Mz)} + \frac{1}{\alpha_2(Mz)} \Rightarrow$$

$$\alpha_{em}(Mz) = \frac{11}{12}\alpha_1(Mz).$$

We know that

$$\sin^2\theta_w(Mz) = \frac{\alpha_{em}(Mz)}{\alpha_2(Mz)} \Rightarrow$$

$$\sin^2\theta_w(Mz) = \frac{11}{12}\frac{\alpha_1(Mz)}{11\alpha_1(Mz)} = \frac{1}{12} = 0.08333$$

which is unacceptable because $\sin^2\theta_w(Mz)_{exp} = 0.23151 \pm 0.00017$. Concluding, the reduction of $\alpha_2$ and $\alpha_1$ couplings in the context of the MSSM is not possible.

### 4.2 Relating $\alpha_t$ top quark and $\alpha_b$ bottom quark Yukawa couplings

Following the same procedure we assume that $\alpha_t$ Yukawa coupling can be related with the $\alpha_b$ Yukawa coupling ($\alpha_i = h_i^2/4\pi, i = t, b$), so they must satisfy the reduction equation

$$\beta_t = \beta_b \frac{d\alpha_t}{d\alpha_b} \Rightarrow$$

$$\frac{d\alpha_t}{d\alpha_b} = \frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t + \alpha_\tau - \frac{7}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}$$

where $\beta_t$ and $\beta_b$ are the $\beta$ functions of top quark Yukawa coupling and bottom quark Yukawa coupling correspndingly. We can for simplicity neglect the contribution from the $\tau$ lepton, $\alpha_\tau$, and the small difference between $\frac{13}{15}$ and $\frac{7}{15}$, so we are led to

$$\frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)} \tag{4.2}$$

Assuming again power series solution of the reduction equation we can expand top quark Yukawa coupling at lowest order as

$$\alpha_t = d_0\alpha_b$$

where $d_0$ is a constant. The derivative of the ratio of the two Yukawa couplings must be zero

$$\frac{d}{dt}(\frac{\alpha_t}{\alpha_b}) = 0 \Rightarrow$$

$$\frac{1}{\alpha_t^2}(\alpha_b\beta_t - \alpha_t\beta_b) = 0 \Rightarrow$$

$$\frac{\alpha_t}{\alpha_b} = \frac{\beta_t}{\beta_b}.$$

Substituting the above result into eqn.(4.2) we obtain

$$\frac{\alpha_t}{\alpha_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)} \Rightarrow$$

$$6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3 = 6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3 \Rightarrow$$

$$\alpha_t = \alpha_b.$$

That is, if we start with $\alpha_t$ and $\alpha_b$ equal at an energy scale, equality will persist at all energies.

The next thing to do is to solve numerically the one-loop coupled differential equations of top and bottom Yukawa couplings taken account the $\alpha_\tau$ contribution and the difference between the numerical factors, to see if such a relation like the previous one can exist. First, we solve the differential equations for the gauge and Yukawa couplings in the SM. And then at $M_{SUSY}$ we impose the next boundary conditions for some values of $\tan\beta$ that keeps the ratio $\alpha_t/\alpha_b$ constant for all energies

$$\alpha_{t_{SM}}(M_{SUSY}) = \alpha_{t_{MSSM}}(M_{SUSY})\sin^2\beta$$

$$\alpha_{b_{SM}}(M_{SUSY}) = \alpha_{b_{MSSM}}(M_{SUSY})\cos^2\beta$$

$$\alpha_{\tau_{SM}}(M_{SUSY}) = \alpha_{\tau_{MSSM}}(M_{SUSY})\cos^2\beta$$

In Fig.1 we plot the ratio of the two Yukawa couplings under investigation $h_t/h_b$ (a) and the derivative of their ratio (b) as a function of energy, for several values of $\tan\beta$, $M_{SUSY} = 1$ TeV, $m_b(M_Z) = 2.82$ GeV and $m_t = 172$ GeV. We can see that for the range of values for $\tan\beta$ between 52.25-58.55, the derivative of the ratio is very close to zero.

In Fig.2 in (a) and (c) we plot the ratio $h_t/h_b$ as well as the derivative of the ratio in (b) and (d) as a function of energy for $\tan\beta = 56$. In (a) and (b) we can see three curves corresponding to $M_{SUSY} = 1$, 5 and 10 TeV (we have kept the masses of top and bottom quarks at their central values). In (c) and (d) we have taken three values for the bottom mass $m_b(M_Z) = 2.75$, 2.82 and 2.89 GeV, keeping the top mass at its central value and $M_{SUSY} = 1$ TeV.

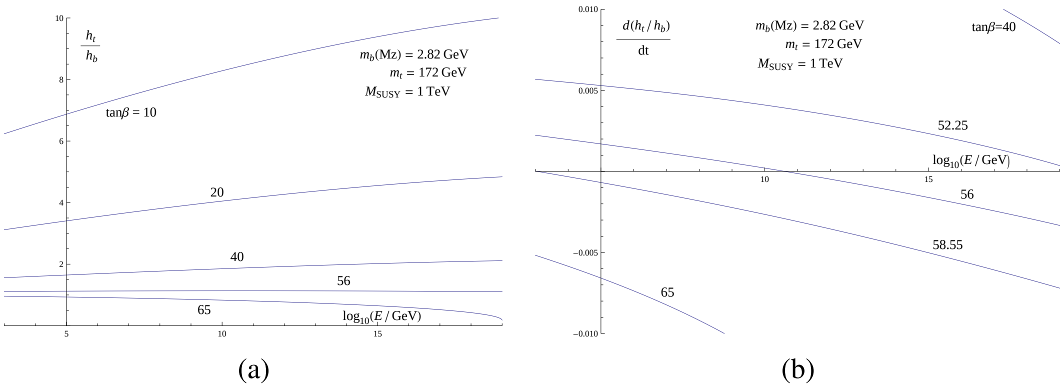



**Figure 1:** (a) The ratio of top and bottom quark Yukawa couplings $h_t/h_b$ and (b) the derivative of their ratio as a function of energy. Several values of $\tan\beta$ have been taken in addition with $M_{SUSY} = 1$ TeV, $m_t = 172$ GeV and $m_b(M_Z) = 2.82$ GeV.

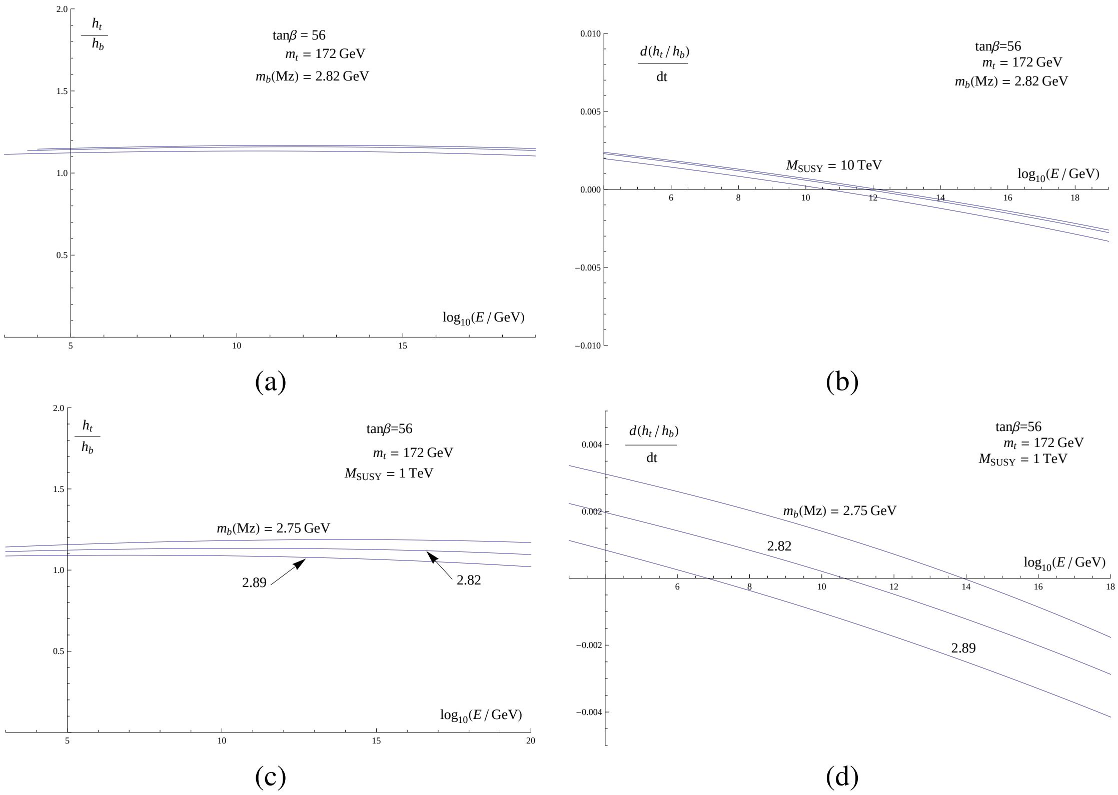
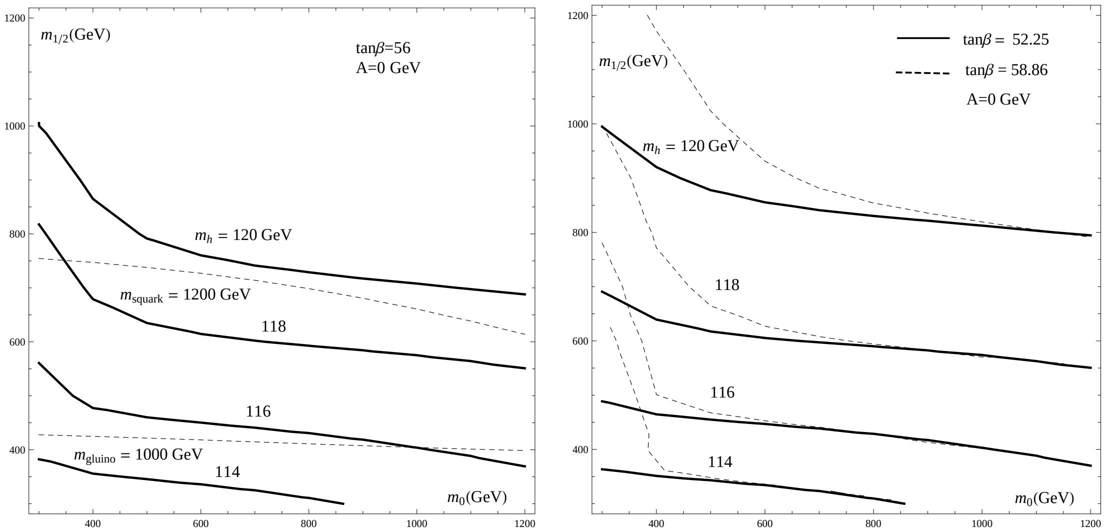



**Figure 2:** (a) The ratio of top and bottom quark Yukawa couplings $h_t/h_b$ and (b) the derivative of their ratio as a function of energy for $M_{SUSY} = 1$, 5 and 10 TeV. (c) The ratio $h_t/h_b$ and (d) the derivative of the ratio as a function of energy for $M_{SUSY} = 1$ TeV and three values of the bottom mass that vary in the experimental error region.

Having used a Fortran code for the calculation of higgs particle spectrum in the MSSM (SUSPECT [14])[1], we plot in the plane of sfermions and gauginos, $(m_0, m_{1/2})$, contours of constant mass values for the lightest supersymmetric Higgs $m_h$. In Fig.3 in (a) we plot these contours for several values of the lightest supersymmetric Higgs $m_h = 114, 116, 118, 120$ GeV. We also choose the values $A = 0$ GeV and $\tan\beta = 56$. The contours with the dashed line correspond to a gluino mass of 1 TeV and to (the lightest) squark mass of 1.2 TeV correspondingly. In (b) we plot contours of constant mass values for the lightest supersymmetric Higgs $m_h$ for two values of $\tan\beta$: 58.55 and 52.25.




(a) (b)

**Figure 3:** (a) Contours of constant pole mass for the lightest supersymmetric Higgs $m_h$ in the plane of $(m_0, m_{1/2})$ for initial value $A = 0$ GeV and for $\tan\beta = 56$. The contours with the dashed line correspond to a gluino mass of 1 TeV and to (the lightest) squark mass of 1.2 TeV correspondingly. In (b) we plot contours of constant $m_h$ (pole) mass in the plane of $(m_0, m_{1/2})$ for initial value $A = 0$ GeV and for two values of $\tan\beta$: 58.55 and 52.25.

## 5. Conclusions

The program of reduction of couplings is a very powerful method that relates arbitrary coupling parameters. As a result we obtain a new reduced theory which has an increased predictive power. This method has been applied to supersymmetric GUTs and lately to the MSSM with great success. In our work relating top and bottom quark Yukawa couplings, we give a narrow range of values for $\tan\beta$ that can permit us to give a prediction for the lightest supersymmetric Higgs particle in the MSSM.

[1] We run the program choosing the mSUGRA model, running the renormalization group equations in 2-loop level and evaluating the pole masses. Also we choose $\mathrm{sign}(\mu) = +1$.

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