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Why neutrinos are different

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We review how a model based on two extra dimensions (ED) can provide an elegant solution for various issues in the Standard Model (SM) of particle physics. In particular, we focus here on neutrino sector and show that large mixing, Majorana nature, inverted hierarchy and partial suppression of neutrinoless double beta decay could be related to each other. As a concrete illustration, we provide a complete set of fitted parameters (for the quark, the scalar and the neutrino sectors).

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1. Introduction

Neutrinos are very peculiar particles for they are very much lighter than the lightest fermion of the Standard Model (SM) of particle physics. While we only possess an upper bound on their masses, we know they must be massive (at least two of them) to account for neutrino oscillations which are parametrized by the PMNS matrix. These oscillations result from a mismatch between the weak and the mass eigenstates and are already well-known in the quark sector of the SM where these mixing are summarized in the CKM matrix. Still, the two matrices are quite different. Indeed, while the CKM matrix is mainly diagonal (small mixing), the PMNS matrix is highly non diagonal (large mixing). This, with the smallness of their masses, makes us claim that neutrinos are different and calls for an elucidation.

An additional motivation for the investigation of the neutrino sector rests on a couple of important open questions, *i.e.* what is the absolute mass scale for neutrinos ? what is the hierarchy (normal or inverted) ? what is their nature (Dirac or Majorana particles) ? and finally, what about CP violation in this sector ?

A model that justifies the masses and mixing patterns of the neutrino sector should provide us with (partial) predictions for these issues. Here we propose a model based on extra dimensions (ED) which has already been useful to describe quark sector and gave us a good "range" for the SM scalar mass [1, 2, 3, 4, 5]. Moreover, this model in $4+2$ dimensions has predicted [6] the θ_{13} angle before his measurement at Daya Bay [7].

Here, we focus on the neutrino sector but for easy reference we summarize the general purposes of the model in section 2. Then, in section 3 we present possibilities to provide neutrinos with a mass in $4+2D$ and show how a difference in nature (with respect to the quarks) can be connected with other properties. In section 4, we present some recent results (for details see [8]) and finally in section 5, we conclude.

2. The Model

In our model, we add two ED to the usual $3+1$ dimensions. For technical reasons we compact them on a sphere (for a recent analysis of that, see [9]). In addition, we introduce a Nielsen-Olesen vortex structure on it, which is exactly what describes type-II superconductors and corresponds to a tube of magnetic flux at the North Pole (origin).

If fermion fields enter the game, the index theorem tells us that the number of flux units in the tube is equal to the number of chiral fermions in the spectrum [10]. Thus, we can obtain 3 families in $4D$ from one field in $6D$. Up to this point, everything could seem to be introduced *ad hoc*, but a new thing is hidden behind this: the 3 families are not free with respect to each other any more. In this case, their wavefunctions on the sphere are related and completely set by the dynamics (coupling to the vortex). While at large distance from the vortex core, they tend to have quite similar profiles, they possess very distinct behaviours around the origin. More precisely we have respectively a constant, a linear and a quadratic behaviour at the origin (see Figure 1). This becomes very interesting if we then introduce a localized scalar field at the origin to play the role of the SM scalar [4]. Indeed, a Yukawa term of the form $\lambda H\bar{\Psi}\Psi$ in $6D$ will produce effective $4D$ masses for each family as a "convolution" between both fermion and scalar wavefunctions. Thus

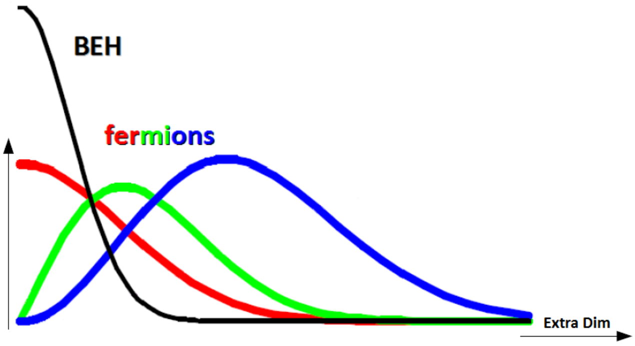

Figure 1: Fermion zero modes profiles (red, green, blue) and SM scalar (or BEH scalar) profile (black picked at the origin) in the extra dimensions. We see at the origin the characteristic behaviours of the fermions: constant (red) which will have the largest overlap with the scalar, linear (green) with a medium overlap and quadratic (blue) with the smallest overlap.

we automatically end up with mass matrix of the form:

$$\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \tag{2.1}$$

with $m_1 \ll m_2 \ll m_3$, which is precisely the observed hierarchy in the quarks and charged leptons sectors (see Figure 1).

Now let us see what happens for neutrinos in this context.

3. Neutrinos masses in 4+2*D*

The standard tricks to give neutrinos a mass imply the introduction of right-handed (RH) partners ν_R. Then, according to their nature, we can either, if they are Dirac particles, mimic the mechanism used for other fermions in the SM or, if they are Majorana particles, utilize a kind of see-saw mechanism. The first possibility, while perfectly legitimate, forces us to tune Yukawa couplings with the aim of reaching the small scale of neutrino masses (and this will be true in any number of dimensions). On the contrary, see-saw mechanisms give quite an elegant justification to this small scale for it originates from a natural suppression by a high scale (see-saw scale) where new physics is not unlikely at all.

Still, this solution might seem compromised since 6*D* Majorana particles do not exist (see *e.g.* [11]). Nevertheless, $\bar{N}^c N$ terms are not forbidden (for fields which are neutral under all gauge groups) and as we will see, it is worth studying them.

To figure this out, let us have a look at Table 1. Here, we have decomposed "Dirac" ($\bar{\Phi}\Psi$) and "Majorana" ($\bar{\Phi}^c\Psi$) mass terms in their chiral components (both in 4*D* and 6*D*). For this purpose, we remind the reader that a 6*D* Dirac spinor is 8-components and can be decomposed into 2 Weyl spinors both 4-components with 6*D* "chirality" Ψ_+ and Ψ_- defined as the eigenvalue of Γ_7 which is a 8×8 equivalent of γ_5 matrix (see [11]). Moreover, we can associate 4*D* chirality to the components of these 6*D* Weyl spinors. Without entering into details, let us just give the complete decomposition:

$$\Psi_{6D} = (\Psi_{+R}\,\Psi_{+L}\,\Psi_{-L}\,\Psi_{-R})^T.$$

	$4D$	$6D$
"Dirac"	$\bar{\Phi}\Psi = \Phi_R^\dagger\Psi_L + \Phi_L^\dagger\Psi_R$	$\bar{\Phi}\Psi = \Phi_-^\dagger\Psi_+ + \Phi_+^\dagger\Psi_-$
"Majorana"	$\bar{\Phi}^c\Psi = \Phi_L^T\varepsilon\Psi_L + \Phi_R^T\varepsilon\Psi_R$	$\bar{\Phi}^c\Psi = \Phi_+^T E\Psi_- + \Phi_-^T E\Psi_+$

Table 1: "Dirac" and "Majorana" mass terms chiral decomposition in $4D$ and $6D$. $\varepsilon = i\sigma_2$ is the 2×2 completely antisymmetric matrix and $E \equiv \sigma_1 \otimes \varepsilon$.

As we see in Table 1, in $4D$ a "Dirac" term couples spinors of opposite chiralities while a "Majorana" one couples spinors with the same chirality. In 6D dimensions on the contrary, both "Dirac" and "Majorana" terms couples spinors with opposite $6D$ chiralities (as expected). But if we develop this last term a bit further we find:

$$\bar{\Phi}^c\Psi = \Phi_{+R}^T\varepsilon\Psi_{-R} + \Phi_{+L}^T\varepsilon\Psi_{-L} + \Phi_{-L}^T\varepsilon\Psi_{+L} + \Phi_{-R}^T\varepsilon\Psi_{+R} \tag{3.1}$$

where effective $4D$ Majorana mass terms appear. So, even if the "Majorana" mass term has a completely different behaviour in $6D$, the result in the effective $4D$ theory is an effective Majorana mass term in the usual sense.

Note that if we develop the same way the "Dirac" term it leads to:

$$\bar{\Phi}\Psi = \Phi_{-L}^\dagger\Psi_{+R} + \Phi_{-R}^\dagger\Psi_{+L} + \Phi_{+R}^\dagger\Psi_{-L} + \Phi_{+L}^\dagger\Psi_{-R} \tag{3.2}$$

which are nothing but regular $4D$ Dirac terms.

Now let us be a bit more precise and apply this in the context of our model. As mentioned earlier, we can get k $4D$ chiral zero modes (in this case $k=3$) from one $6D$ fermionic field. Depending on the type of interactions with the vortex we can get either LH or RH modes. An important feature of these modes is the kind of redundancy we get in the $4D$ description, *i.e.* Ψ_{+L} and Ψ_{-L} describe the same effective $4D$ spinor (the same holds for RH spinors). On the other hand, the wavefunctions in the ED are different. With (θ,φ) the usual spherical coordinates we can write for the zero modes ($n=1,2,3$):

$$\Psi_n^{\text{LH}} \sim \begin{pmatrix} 0 \\ \chi_L(x;n)e^{i\varphi(3-n)}f(\theta;n) \\ \chi_L(x;n)e^{i\varphi(1-n)}g(\theta;n) \\ 0 \end{pmatrix} \quad \text{and} \quad \Psi_n^{\text{RH}} \sim \begin{pmatrix} \chi_R(x;n)e^{i\varphi(1-n)}g(\theta;n) \\ 0 \\ 0 \\ \chi_R(x;n)e^{i\varphi(3-n)}f(\theta;n) \end{pmatrix}$$

The exponential factors relate to the "winding" and are very important. Indeed, we see that each family has a different winding number, and this will in general produce $\exp[i\varphi(n-n')]$ factors in the 2-fermions interactions which in turn will lead to selection rules when integration over φ is performed to compute the effective $4D$ Lagrangian. For the quark sector where mass comes from a "Dirac" term like in eqn. (3.2), we will get a diagonal mass matrix $m_{nn'} \sim \delta_{nn'}$ (as in eqn. (2.1) where it was not justify). On the other hand, for hypothetical Majorana neutrinos, eqn. (3.1) gives $m_{nn'} \sim \delta_{n+n',4}$ which corresponds to an off-diagonal matrix.

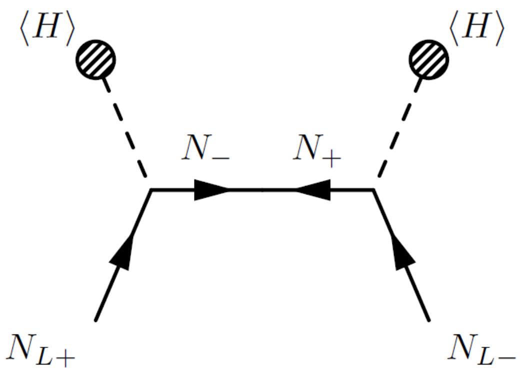

Figure 2: See-saw diagram for the light neutrino mass generation.

Here we will not enter into the details of the concrete realisation of the see-saw mechanism that can be found in [6], but let us comment quickly on it. Exactly like in $4D$, the LH neutrinos come from $L = (N_L\ E_L)^T$ the SU(2) doublet of LH leptons and interact with a new neutral field N with a $6D$ "Majorana" mass ($\sim M\bar{N}^c N$) and the SM scalar H through a Yukawa coupling of the type $\bar{N}LH + h.c.$ Then, the Majorana nature of N_L results from the diagram shown in Figure 2. Now we have to remember that N is a $6D$ field which corresponds to an infinite tower of Kaluza-Klein (KK) modes[1]. In other words, we have to sum over an infinite number of exchanged modes. Hopefully there is a natural cut-off: N being neutral it can not *a fortiori* interact with the vortex and so instead of being stuck near the origin, it is free to propagate all over the sphere. Associated to that are delocalized, oscillating wavefunctions and the heavier are the modes, the quicker are the oscillations. Thus for sufficiently high modes the convolution with localized profiles (of L and H) tends to become negligible. A discussion about the absolute mass scale that results from this can be found in [6].

With this, it is quite natural for us to obtain a mass matrix for neutrino of the type:

$$M_\nu \sim \begin{pmatrix} & & m \\ & \mu & \\ m & & \end{pmatrix}$$

with $m \gg \mu$, or after a trivial diagonalisation:

$$M_\nu^D \sim \begin{pmatrix} m & & \\ & -m & \\ & & \mu \end{pmatrix}$$

From this simple exercise, we can already extract a few important consequences. The model predicts an inverted hierarchy ($|m_1|, |m_2| = m \gg |m_3| = \mu$), a large (maximal) mixing angle ($\pi/2$), and finally neutrinoless beta decay (thanks to the Majorana nature of neutrinos) but with a partial suppression (thanks to the opposite sign of m_1 and m_2 and the relative smallness of μ).

[1] To be more precise we have an infinite number of modes labelled by a non zero integer λ and with masses $M^2 + \lambda^2/R^2$ where R is the sphere radius (see [6]).

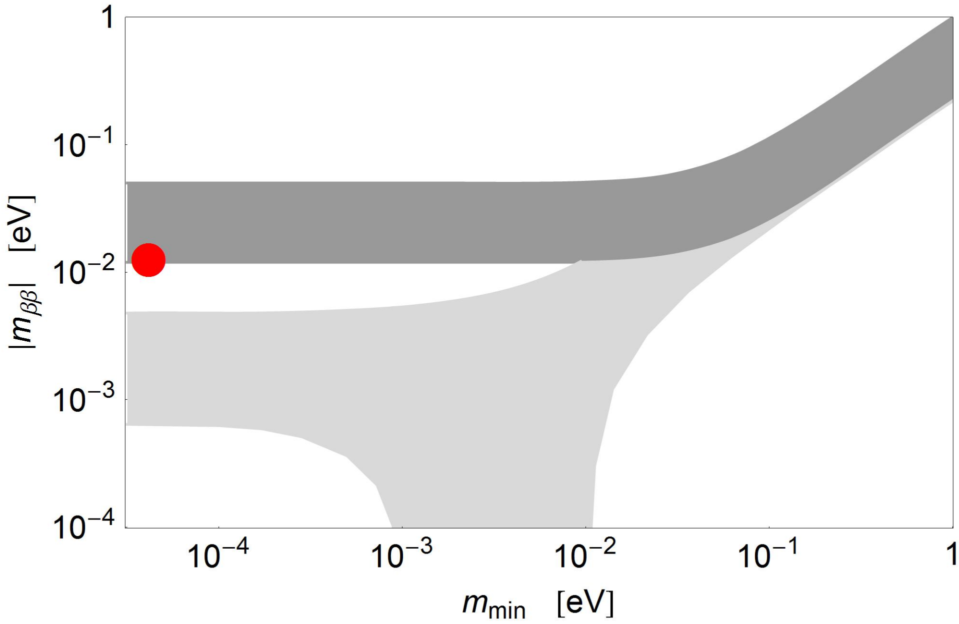

Figure 3: Effective Majorana mass versus the minimal absolute neutrino mass. 99% allowed regions for normal (light grey) and inverted (dark grey) hierarchies. We are on the border (thick red dot).

Obviously all these predictions must be moderated. Indeed, here we have only the main character of the mass matrices (the first order if one wants). For instance there is no mixing in the quark sector and the complete mixing pattern in the leptonic sector is far from being reproduced. To describe this we need to introduce new interactions but this is beyond the scope of this short note (see [8] and references therein). Rather, we will present the numerical results obtained in a recent realisation of the model. As we will see, we have reproduced all the observed parameters with a decent agreement. It is worth noting that our main predictions are trivially realized in this example.

4. Results

Details and numerical values of the parameters for our complete realisation of the model can be found in [8]. The Table 2 gives fitted versus experimental values of the mass parameters.

Let us shortly comment on the neutrino sector. First of all, the measured parameters (*i.e.* the Δm^2's and the mixing angles (or the PMNS matrix)) are fitted pretty well. We can see in particular one of the large mixing that comes from the generic form of M_ν at first order — the other comes from charged lepton sector. Then our two other predictions are realized here: we have $m_1, m_2 \gg m_3$ (inverted hierarchy) and a suppression of neutrinoless beta decay (parametrized with the effective Majorana mass $\langle m_{\beta\beta}\rangle$). This appears more clearly on Figure 3 where $\langle m_{\beta\beta}\rangle$ is plotted versus the minimal absolute neutrino mass. The grey sectors represent the 99% allowed regions for normal hierarchy (light grey) and inverted hierarchy (dark grey). Unlike the normal hierarchy, there is a lower bound on $\langle m_{\beta\beta}\rangle$ for the inverted hierarchy and we see (thick red dot) that we are just on the border. This sensitivity will be reached by the "phase 3" of the GERDA experiment.

Parameter	Fitted value	Experimental value
The scalar-boson mass		
m_H	125 GeV	$125.5 \pm 0.2(\text{stat.}) \pm 0.6(\text{syst.})$ $125.7 \pm 0.3(\text{stat.}) \pm 0.3(\text{syst.})$
Quark masses at Z scale		
m_d	0.01 GeV	(0.00282 ± 0.00048) GeV
m_s	0.051 GeV	$(0.057^{+0.018}_{-0.012})$ GeV
m_b	2.86 GeV	$2.86^{+0.16}_{-0.06}$ GeV
m_u	0.023 GeV	$0.00138^{+0.00042}_{-0.00041}$ GeV
m_c	0.72 GeV	$0.638^{+0.043}_{-0.084}$ GeV
m_t	172 GeV	172.1 ± 1.2 GeV
Quark mixing matrix		
$\lvert U_{\text{CKM}} \rvert$	$\begin{pmatrix} 0.979 & 0.207 & 0.0015 \\ 0.206 & 0.9730 & 0.046 \\ 0.011 & 0.049 & 0.999 \end{pmatrix}$	$\begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$
Charged-lepton masses		
m_e	0.00061 GeV	0.0004866 GeV
m_μ	0.089 GeV	0.1027 GeV
m_τ	1.74 GeV	1.746 GeV
Neutrino masses		
m_1	$5.46 \cdot 10^{-2}$ eV	–
m_2	$5.53 \cdot 10^{-2}$ eV	–
m_3	$4.17 \cdot 10^{-5}$ eV	–
Δm^2_{21}	$7.96 \cdot 10^{-5}$ eV^2	$(7.50 \pm 0.185) \cdot 10^{-5}$ eV^2
Δm^2_{13}	$2.98 \cdot 10^{-3}$ eV^2	$(2.47^{+0.069}_{-0.067}) \cdot 10^{-3}$ eV^2
Lepton mixing matrix		
$\lvert U_{\text{PMNS}} \rvert$	$\begin{pmatrix} 0.76 & 0.63 & 0.13 \\ 0.39 & 0.58 & 0.72 \\ 0.52 & 0.52 & 0.68 \end{pmatrix}$	$\simeq \begin{pmatrix} 0.795-0.846 & 0.513-0.585 & 0.126-0.178 \\ 0.205-0.543 & 0.416-0.730 & 0.579-0.808 \\ 0.215-0.548 & 0.409-0.725 & 0.567-0.800 \end{pmatrix}$
$\langle m_{\beta\beta} \rangle$	0.013 eV	$\lesssim 0.3$ eV
J	0.019	$\lesssim 0.036$
θ_{12}	39.7°	$\simeq (31.09^\circ - 35.89^\circ)$
θ_{23}	46.5°	$\simeq (35.8^\circ - 54.8^\circ)$
θ_{13}	7.2°	$\simeq (7.19^\circ - 9.96^\circ)$

Table 2: Fitted versus experimental values of the mass parameters. Note: the experimental values are not the most recent ones but the ones we used at the time of the realisation which corresponds more or less with the epoch of the Corfu 2013 Workshop. See [8] for references.

We will not comment on CP violation here (summarized in the Jarlskog determinant J in Table 2) because recent analysis of this issue has shown that the prediction is not stable.

5. Conclusions

We have presented a model in which two ED are added to the four usual ones and compactified on a sphere. Thanks to a vortex structure implemented in this extra space, one fermion field gives rise to 3 families in the effective $4D$ theory. These families are no longer independent from each other, rather their are linked by dynamics in the ED. This already helped to clarify the hierarchy structure in the quark sector and here we have shown how neutrino sector can be treated. There, a see-saw mechanism is used to explain naturally their small mass scale and this leads (quite generically) to a completely different behaviour. Moreover, we end up with a bunch of striking predictions: an inverted hierarchy and a Majorana nature for neutrinos but with a suppressed neutrinoless beta decay.

It is worth noting that this is not the only interesting feature of the model and other matters have been studied in this context both before and after the Workshop. See for instance [9] for a recent update on some questions in the gauge sector.

Acknowledgments

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