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Neutrino Mixing from SUSY breaking*

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We propose a mechanism to generate the neutrino mixing matrix from supersymmetric threshold corrections. Flavor violating soft breaking terms induce flavor changing self-energies that give a finite renormalization to the mixing matrix. The described threshold corrections get enhanced in case of quasi-degenerate neutrino masses. In this scenario, we adjust potentially arbitrary soft breaking parameters in a way to reproduce the observed neutrino mixing at one loop working with non-minimal flavor violating soft parameters. To incorporate small neutrino masses already at tree-level via a type I seesaw mechanism, we extend the Minimal Supersymmetric Standard Model with singlet Majorana neutrinos. The radiative corrections do not decouple with the scale of Supersymmetry and persist when the spectrum is shifted to higher values. Moreover, the mixing matrix renormalization with flavor-changing self-energies is not restricted to supersymmetric theories and give similar results in any theory with new flavor structures.

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1. Introduction

The Standard Model (SM) of elementary particle physics provides excellent predictions of fundamental properties of nature and elementary processes. Most parameters of the theory are related to flavor and have to be determined by measurement: fermion masses and mixing angles. In general, they are arbitrary parameters of the theory originating in the Yukawa couplings of the fermions to the SM Higgs doublet. Gauge interactions do not change the flavor, therefore the (weak) gauge interaction basis sets the flavor basis. Masses are generated after spontaneous symmetry breaking through the Yukawa couplings

$$-\mathscr{L}^{\mathrm{SM}}_{\mathrm{Yuk}} = Y^{\mathrm{d}}_{ij}\bar{Q}_{\mathrm{L},i}\cdot H d_{\mathrm{R},j} - Y^{\mathrm{u}}_{ij}\bar{Q}_{\mathrm{L},i}\cdot \tilde{H} u_{\mathrm{R},j} + Y^{\mathrm{e}}_{ij}\cdot \bar{L}_{\mathrm{L},i}\cdot H e_{\mathrm{R},j} - Y^{\nu}_{ij}\bar{L}_{\mathrm{L},i}\cdot \tilde{H}\nu_{\mathrm{R},j} + \mathrm{h.\,c.}\,, \tag{1.1}$$

where $H = (h^+, h^0)^{\mathrm{T}}$ is the Higgs doublet and $\tilde{H} = \mathrm{i}\,\tau_2 H^*$ the charge conjugated version of it. The quark and lepton left-handed doublets are given by $Q_{\mathrm{L}} = (u_{\mathrm{L}}, d_{\mathrm{L}})^{\mathrm{T}}$ and $L_{\mathrm{L}} = (\nu_{\mathrm{L}}, e_{\mathrm{L}})^{\mathrm{T}}$, respectively. The right-handed SM fermions are labeled obviously. For later purpose we have already introduced right-handed neutrinos ν_{R} and their Yukawa couplings to left-handed leptons. Right-handed neutrinos are complete singlets under the SM gauge group. Fermion masses are given by the mass matrices $m^f = vY^f/\sqrt{2}$ with the vacuum expectation value v of the Higgs field, $\langle h^0\rangle = v/\sqrt{2} = 174\,\mathrm{GeV}$ and $f = \mathrm{u,d,e},\nu$. Generation indices i, j count the number of generations and diagonalization of the mass matrices m^f transform into the mass eigenbasis. We can do so with bi-unitary transformations, such that

$$Y^f \to S^f_{\mathrm{L}} Y^f \left(S^f_{\mathrm{R}}\right)^\dagger = \hat{Y}^f = \text{diagonal}. \tag{1.2}$$

We find the fermion mixing matrices as they appear in the weak charged current due to misalignment of the Yukawa couplings as the Cabibbo–Kobayashi–Maskawa [1,2] (CKM) matrix $V_{\mathrm{CKM}} = S^{\mathrm{u}}_{\mathrm{L}}\left(S^{\mathrm{d}}_{\mathrm{L}}\right)^\dagger$ and the Pontecorvo–Maki–Nakagawa–Sakata [3,4] (PMNS) matrix $U_{\mathrm{PMNS}} = S^{\mathrm{e}}_{\mathrm{L}}\left(S^{\nu}_{\mathrm{L}}\right)^\dagger$.

The observed mixing patterns for quarks and leptons are quite different. Where the CKM matrix is close to the unit matrix consistent with small mixing, the PMNS matrix shows a rather anarchic mixing pattern with no clear hierarchy in the elements. This unlike behavior of quark versus lepton mixing can be displayed very visually showing the sizes of the magnitudes

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} \bullet & \cdot & \cdot \\ \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet \end{pmatrix}, \qquad |U_{\mathrm{PMNS}}| = \begin{pmatrix} \bullet & \bullet & \cdot \\ \bullet & \bullet & \bullet \\ \cdot & \bullet & \bullet \end{pmatrix}.$$

The structure of the CKM matrix suggests the possibility of generating quark mixing via higher orders in perturbation theory: the flavor violating contributions are small and of the size of typical one-loop corrections. The PMNS matrix on the other side does not allow for lepton mixing as a genuine loop effect at first sight: the leptonic mixing angles are just too large. However, the contributions to the mixing matrix renormalization can be drastically enhanced by the neutrino mass spectrum as will be discussed in the following after a brief overview of a radiative description of quark mixing. We follow the procedure for the supersymmetric renormalization of the CKM matrix [5] and apply it to the lepton case where the contributions are generically smaller because only slepton–electroweakino loops are present. As we will show, the neutrino corrections can be significantly enhanced and therefore dominating over any tree-level mixing pattern.

2. Radiative Flavor Violation in the MSSM

A radiative origin of quark masses and flavor mixing was already proposed in the early days of the SM [6] and later on applied in the context of grand unified theories [7–9]. Especially supersymmetric models give the opportunity to radiatively generate masses and Yukawa couplings via soft breaking terms [10–14]. The soft breaking Lagrangian of supersymmetric theories generically carry arbitrary flavor structures in addition to the flavor structure of the SM which is transferred from Eq. (1.1) to the *superpotential* of the Minimal Supersymmetric Standard Model (MSSM):

$$\mathscr{W}_{\mathrm{MSSM}} = \mu H_{\mathrm{d}} \cdot H_{\mathrm{u}} - Y^{\mathrm{d}}_{ij} H_{\mathrm{d}} Q_{\mathrm{L},i} \bar{D}_{\mathrm{R},j} + Y^{\mathrm{u}}_{ij} H_{\mathrm{u}} \cdot Q_{\mathrm{L},i} \bar{U}_{\mathrm{R},j} - Y^{\mathrm{e}}_{ij} H_{\mathrm{d}} \cdot L_{\mathrm{L},i} \bar{E}_{\mathrm{R},j} + Y^{\nu}_{ij} H_{\mathrm{u}} \cdot L_{\mathrm{L},i} \bar{N}_{\mathrm{R},j}, \tag{2.1}$$

where the fields are chiral superfields and the number of Higgs doublets is doubled in the MSSM. Charge conjugated right-handed matter fermions fit into the left-chiral superfields $\bar{U}_{\mathrm{R}} = \{\tilde{u}^*_{\mathrm{R}}, u^{\mathrm{c}}_{\mathrm{R}}\}$, $\bar{D}_{\mathrm{R}} = \{\tilde{d}^*_{\mathrm{R}}, d^{\mathrm{c}}_{\mathrm{R}}\}$, $\bar{E}_{\mathrm{R}} = \{\tilde{e}^*_{\mathrm{R}}, e^{\mathrm{c}}_{\mathrm{R}}\}$ and $\bar{N}_{\mathrm{R}} = \{\tilde{\nu}^*_{\mathrm{R}}, \nu^{\mathrm{c}}_{\mathrm{R}}\}$, whereas $Q_{\mathrm{L}} = \{\tilde{q}_{\mathrm{L}}, q_{\mathrm{L}}\}$ and $L_{\mathrm{L}} = \{\tilde{\ell}_{\mathrm{L}}, \ell_{\mathrm{L}}\}$ represent the weak doublets. The Higgs doublets $H_{\mathrm{u}} = (H^+_{\mathrm{u}}, H^0_{\mathrm{u}})^{\mathrm{T}}$ and $H_{\mathrm{d}} = (H^0_{\mathrm{d}}, H^-_{\mathrm{d}})^{\mathrm{T}}$ give masses to the up-type and down-type fermions, respectively.

Supersymmetry (SUSY) is softly broken via the following soft breaking terms

$$\begin{aligned} -\mathscr{L}^{\mathrm{MSSM}}_{\mathrm{soft}} = {} & \tilde{q}^*_{\mathrm{L},i} \left(\tilde{m}^2_Q\right)_{ij} \tilde{q}_{\mathrm{L},j} + \tilde{u}^*_{\mathrm{R},i} \left(\tilde{m}^2_u\right)_{ij} \tilde{u}_{\mathrm{R},j} + \tilde{d}^*_{\mathrm{R},i} \left(\tilde{m}^2_d\right)_{ij} \tilde{d}_{\mathrm{R},j} \\ & + \tilde{\ell}^*_{\mathrm{L},i} \left(\tilde{m}^2_\ell\right)_{ij} \tilde{\ell}_{\mathrm{L},j} + \tilde{e}^*_{\mathrm{R},i} \left(\tilde{m}^2_e\right)_{ij} \tilde{e}_{\mathrm{R},j} + \tilde{\nu}^*_{\mathrm{R},i} \left(\tilde{m}^2_\nu\right)_{ij} \tilde{\nu}_{\mathrm{R},j} \\ & + \left[h_{\mathrm{d}} \cdot \tilde{\ell}_{\mathrm{L},i} A^{\mathrm{e}}_{ij} \tilde{e}^*_{\mathrm{R},j} + \tilde{\ell}_{\mathrm{L},i} \cdot h_{\mathrm{u}} A^{\nu}_{ij} \tilde{\nu}^*_{\mathrm{R},j} + h_{\mathrm{d}} \cdot \tilde{q}_{\mathrm{L},i} A^{\mathrm{d}}_{ij} \tilde{d}^*_{\mathrm{R},j} + \tilde{q}_{\mathrm{L},i} \cdot h_{\mathrm{u}} A^{\mathrm{u}}_{ij} \tilde{u}^*_{\mathrm{R},j} + \mathrm{h.\,c.} \right]. \end{aligned} \tag{2.2}$$

We omitted soft breaking terms relevant for gauginos and Higgses. The soft breaking masses $\tilde{m}^2_f$ as well as the trilinear couplings A^f have *a priori* no restriction in their flavor structure which causes problems in flavor changing neutral current observables. So either one imposes Minimal Flavor Violation (MFV) which reduces dangerously large contributions to those observables under the assumption that only SM Yukawa couplings transport flavor information. In that view, the trilinear couplings are $A^f = a_f Y^f$ with some SUSY-scale parameter a_f that is flavor-universal. On the other hand, we can seek for a symmetry that forbids flavor transitions in the SM sector at tree-level and generates all flavor violation radiatively (RFV). In that view, all the SM Yukawa couplings are simultaneously diagonal. Such symmetries may be U(2) flavor symmetries as proposed and studied in [5, 15–20]. The Yukawa couplings vanish except for the third generation

$$Y^f = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_f \end{pmatrix}, \tag{2.3}$$

and the first two generation Yukawa couplings and the mixing have to be generated radiatively.

We want to focus on the supersymmetric renormalization of the CKM matrix according to [5] and leave away the radiative generation of Yukawa couplings. By virtue of the SUSY one-loop corrections, the flavor transitions enter via flavor changing self-energies Σ^{ψ}_{fi} for the fermion ψ as shown in Fig. 1. We have drawn the quark self-energy in the flavor basis, where the quark–squark–gluino interaction is diagonal—the flavor change sits in the mass insertion $\Delta^{\tilde{q}}_{if}$ related to the squark

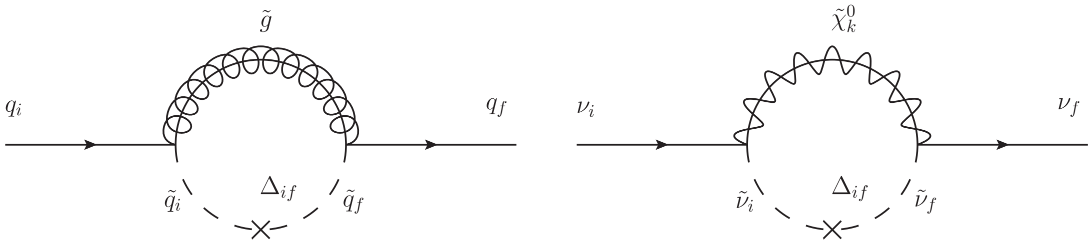

Figure 1: Flavor changing self-energies for quarks (left, squark–gluino loop) and neutrinos (right, sneutrino–neutralino loop) with the flavor changing mass insertion Δ_{if}. A similar diagram exist with charged sleptons and charginos in the loop. The same contributions in the quark–squark sector are subdominant compared to the gluino loop diagram.

mass matrix. A similar diagram can be drawn for the bino/wino-like neutralino and also exists for quarks, which is subdominant there, but the only contributing self-energy for neutrinos (including the other neutralinos and chargino–slepton loops). The fermion self-energies can be decomposed as

$$\Sigma^{\psi}_{fi}(p) = \Sigma^{\psi,RL}_{fi}(p^2)\, P_{\mathrm{L}} + \Sigma^{\psi,LR}_{fi}(p^2)\, P_{\mathrm{R}} + \not{p}\left[\Sigma^{\psi,LL}_{fi}(p^2)\, P_{\mathrm{L}} + \Sigma^{\psi,RR}_{fi}(p^2)\, P_{\mathrm{R}}\right], \tag{2.4}$$

with the left- and right-handed projectors $P_{\mathrm{L,R}}$. Renormalization of the mixing matrix follows the prescription of [21] as done in [5]. The renormalized CKM matrix is found to be

$$V = \left(\mathbb{1} + \Delta U^{u\dagger}_L\right) V^{(0)} \left(\mathbb{1} + \Delta U^{d}_L\right), \tag{2.5}$$

where $V^{(0)}$ is the unrenormalized, "bare" mixing matrix (the quark mixing matrix at tree-level) and the ΔU^q_L are defined via their contribution to the weak charged current vertex

$$\mathrm{i}\,\frac{g_2}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}} V^{(0)} \quad \to \quad \mathrm{i}\,\frac{g_2}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}\left(V^{(0)} + D_L + D_R\right)$$

with

$$D_{L,fi} = \sum_{j=1}^{3} V^{(0)}_{fj}\left[\Delta U^d_L\right]_{ji} \qquad \text{and} \qquad D_{R,fi} = \sum_{j=1}^{3}\left[\Delta U^{u\dagger}_L\right]_{fj} V^{(0)}_{ji}. \tag{2.6}$$

The contributions $D_{L,R}$ are calculated in terms of the components in Eq. (2.4)

$$D_{L,fi} = \sum_{j\neq i} V^{(0)} \frac{m_{d_j}\left(\Sigma^{d,RL}_{ji} + m_{d_i}\Sigma^{d,RR}_{ji}\right) + m_{d_i}\left(\Sigma^{d,LR}_{ji} + m_{d_i}\Sigma^{d,LL}\right)}{m^2_{d_i} - m^2_{d_j}}. \tag{2.7}$$

A similar expression holds for $D_{R,fi}$ with $d \to u$. The large hierarchy in quark masses allows to expand in small ratios as m_s/m_b, in general m_{q_i}/m_{q_j} with $i<j$, and one has (neglecting $\Sigma^{q,LL/RR}$ which are relatively suppressed by the heavy masses in the loop, i. e. the overall SUSY mass scale)

$$\Delta U^q_L = \begin{pmatrix} 0 & \frac{1}{m_{q_2}}\Sigma^{q,LR}_{12} & \frac{1}{m_{q_3}}\Sigma^{q,LR}_{13} \\ \frac{-1}{m_{q_2}}\Sigma^{q,RL}_{21} & 0 & \frac{1}{m_{q_3}}\Sigma^{q,LR}_{23} \\ \frac{-1}{m_{q_3}}\Sigma^{q,RL}_{31} & \frac{-1}{m_{q_3}}\Sigma^{q,RL}_{32} & 0 \end{pmatrix}. \tag{2.8}$$

This is not true in the neutrino case as we shall see in the following section.

3. Radiative Lepton Flavor Violation in the νMSSM

The description of Radiative Lepton Flavor Violation (RLFV) follows basically the setup of Sec. 2. If the neutrino sector were just a copy of the up-type quark sector, we would be done and could discuss the phenomenological output of RLFV. However, things are likely to be different.

Though nothing is odd with mirroring of what we have reviewed to the neutrino sector, there is one puzzle connected to neutrinos: in the SM they are exactly massless. Experiment nevertheless tells us that they at least have tiny masses [22, and references therein]. This can be accomplished on the one hand by setting the neutrino Yukawa couplings to a small value. On the other hand, this smells artificial. A tree-level solution to that puzzle was proposed via an effective operator [23]

$$\mathscr{L}_{\dim 5} = \frac{\lambda_{ij}}{\Lambda} (L_i \cdot H) C \left(H \cdot L_j\right), \tag{3.1}$$

where the couplings λ_{ij} are dimensionless but $\mathscr{O}(1)$ couplings. Neutrino masses are generated after spontaneous symmetry breaking as $m_\nu = v^2 \lambda/(2\Lambda)$ and are suppressed by the scale Λ which can be much larger than the electroweak scale, $\Lambda \gg v$. The matrix C in Eq. (3.1) is the charge conjugation matrix: neutrino masses generated via this mechanism are Majorana masses. UV complete models leading to Eq. (3.1) have been elaborated with additional fermions and scalars [24–29]. We shall extend the MSSM with three singlet chiral superfields that act as right-handed neutrinos and get a Majorana mass term in the superpotential

$$\mathscr{W}_{\nu\text{MSSM}} = \mathscr{W}_{\text{MSSM}} + \frac{1}{2} M^{\text{R}}_{ij} \bar{N}_{\text{R},i} \bar{N}_{\text{R},j}. \tag{3.2}$$

We refer to this model as νMSSM. The effective neutrino mass matrix is given by the combination

$$m_\nu = -\frac{v^2}{2}\, Y_\nu\, M_{\text{R}}^{-1}\, Y_\nu^{\text{T}}. \tag{3.3}$$

Without loss of generality, we can choose both the charged lepton Yukawa couplings Y_{e} and the right-handed Majorana mass matrix M_{R} diagonal. The weak mixing matrix is then determined by the diagonalization of m_ν:

$$\hat{m}_\nu = U^*_{\text{PMNS}}\, m_\nu\, U^\dagger_{\text{PMNS}} = \text{diagonal}. \tag{3.4}$$

We do not know exactly the masses of the light neutrinos. However, out of neutrino oscillations, mass squared differences $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$ can be obtained and therewith in principle the spectrum calculated, here in case of a "normal" ordering ("inverted" ordering has $|m_{\nu_3}| = m_\nu^{(0)}$):

$$|m_{\nu_1}| = m_\nu^{(0)}, \quad |m_{\nu_2}| = \sqrt{\left(m_\nu^{(0)}\right)^2 + \Delta m^2_{21}}, \quad |m_{\nu_3}| = \sqrt{\left(m_\nu^{(0)}\right)^2 + \Delta m^2_{31}}. \tag{3.5}$$

What we do not know is the mass of the lightest neutrino $m_\nu^{(0)}$. The Δm^2_{ij} are known and

$$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\text{eV}^2, \qquad \Delta m^2_{31} = 2.457 \pm 0.047 \times 10^{-3}\,\text{eV}^2, \tag{3.6}$$

as follows from a global fit on neutrino data [30]. We keep $m_\nu^{(0)}$ as free parameter.

Before we calculate the SUSY one-loop contribution to the mixing matrix renormalization, we have to keep in mind, that our neutrinos are Majorana fermions which is a property that relates

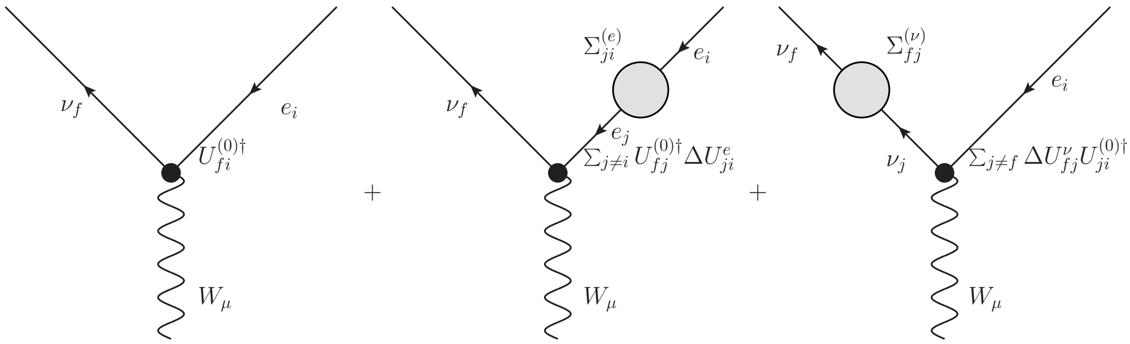

Figure 2: Renormalization of the mixing matrix by means of flavor changing self-energies at external legs according to [21]. Shown are diagrams for the lepton mixing matrix. The inclusion of finite mixing matrix counterterms $\Delta U_{ij}^{\nu,e}$ may significantly alter the tree-level mixing matrix $U_{ij}^{(0)}$. Similar diagrams with obvious changes in the labels can be obtained for quarks.

the components of Eq. (2.4) in a way that $\Sigma_{LR}^{\nu} = {\Sigma_{RL}^{\nu}}^*$ and $\Sigma_{RR}^{\nu} = {\Sigma_{LL}^{\nu}}^*$. Majorana masses and self-energies are symmetric, so $\Sigma_{fi}^{\nu} = \Sigma_{if}^{\nu}$. We decompose the neutrino self-energy in a "scalar" (S) and "vectorial" (V) part

$$\Sigma_{fi}^{\nu}(p) = \Sigma_{fi}^{\nu,S}(p^2)\, P_{\mathrm{L}} + \Sigma_{fi}^{\nu,S*}(p^2)\, P_{\mathrm{R}} + \not{p}\left[\Sigma_{fi}^{\nu,V}(p^2)\, P_{\mathrm{L}} + \Sigma_{fi}^{\nu,V*}(p^2)\, P_{\mathrm{R}}\right]. \tag{3.7}$$

Since in the definition of the lepton mixing matrix up and down are interchanged, we have

$$U_{\mathrm{PMNS}} = (\mathbb{1} + \Delta U_L^{\mathrm{e}})\, U^{(0)} \left(\mathbb{1} + \Delta U_L^{\nu}\right)^{\dagger} \approx\; U^{(0)} + (\Delta U_L^{\mathrm{e}})\, U^{(0)} + U^{(0)} \left(\Delta U_L^{\nu}\right)^{\dagger}, \tag{3.8}$$

reflecting the decomposition of the mixing matrix in contributions from charged and neutral leptons, $U_{\mathrm{PMNS}} = S_{\mathrm{L}}^{\mathrm{e}} \left(S_{\mathrm{L}}^{\nu}\right)^{\dagger}$. The contribution from the neutrino leg (the "left" leg in Fig. 2) is given analogously to Eq. (2.7) by

$$D_{L,fi} = \sum_{j=1}^{n} \left[\Delta U_L^{\nu}\right]_{fj} U_{ji}^{(0)\dagger} = \sum_{j\neq f} \frac{m_{\nu_f}\left(\Sigma_{fj}^{\nu,S} + m_{\nu_f}\Sigma_{fj}^{\nu,V}\right) + m_{\nu_j}\left(\Sigma_{fj}^{\nu,S*} + m_{\nu_f}\Sigma_{fj}^{\nu,V*}\right)}{m_{\nu_j}^2 - m_{\nu_f}^2} U_{ji}^{(0)\dagger}. \tag{3.9}$$

Neglecting $\Sigma^{\nu,V}$ (which again is relatively suppressed with $1/M_{\mathrm{SUSY}}$), we have

$$\left[\Delta U_L^{\nu}\right]_{fi} = \frac{m_{\nu_f}\Sigma_{fi}^{\nu,S} + m_{\nu_i}\Sigma_{fi}^{\nu,S*}}{m_{\nu_i}^2 - m_{\nu_f}^2}. \tag{3.10}$$

In view of this result, we find an enhancement of the contribution ΔU_L^{ν} in case of quasi-degenerate neutrino masses [31]. The neutrino self-energies $\Sigma^{\nu,S}$ are of the same order as the neutrino masses, so $\left[\Delta U_L^{\nu}\right]_{fi} \sim m_{\nu_f} m_{\nu_i}/\Delta m_{fi}^2$ which can be as large as 5×10^3 for $m_{\nu}^{(0)} = 0.35\,\mathrm{eV}$ and $f,i = 1,2$.

The charged lepton contribution, in contrast, obeys a hierarchical structure like Eq. (2.8) which give at most CKM-like mixing and can be neglected in the further discussion while the neutrino-leg contribution is the dominating one in case of quasi-degenerate neutrinos. For hierarchical neutrino masses, flavor changing threshold corrections to the mixing matrix as described above are subdominant. However, hierarchical masses may ask for a special treatment which can be combined

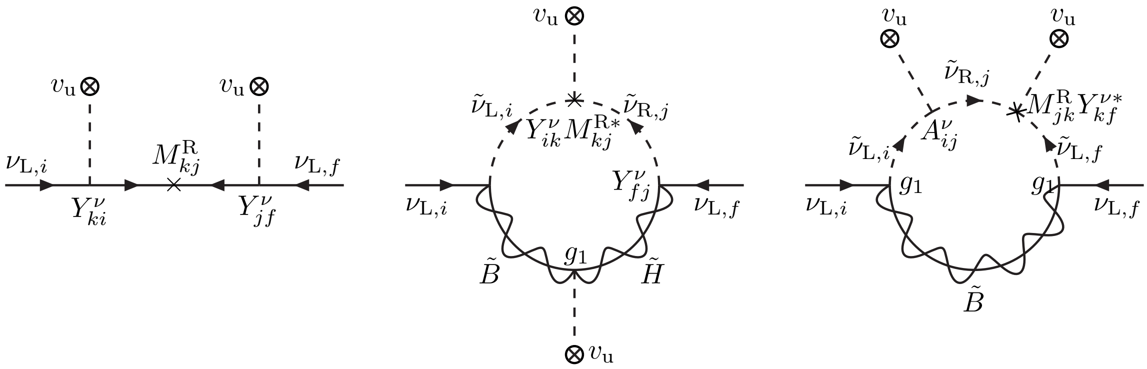

Figure 3: Tree-level and one-loop contributions to the seesaw neutrino mass. Insertions of the right-handed Majorana mass M_R provide the lepton number violating fermion flip.

with the radiative Yukawa couplings discussed in the introduction: a successive breaking of an initial $[\mathrm{U}(3)]^6$ flavor symmetry leads to a description of fermion mixing in terms of mass ratios only, that can be related to the symmetry breaking parameters [32].

We do not give analytic expressions for self-energies in this article since such results can be found in [33] and [34]. Instead, we shall discuss the flavor structure of the contributing one-loop diagrams and show that we observe a *non-decoupling* effect with respect to the SUSY scale: if all SUSY-scale parameters are uniformly shifted to higher values, the results do not change.

Diagrams contributing to the renormalization of the PMNS matrix and the one-loop neutrino masses are given in Fig. 3, where the first one shows the tree-level contributions and the second and third diagram (from left to right) two phenomenologically different one-loop self-energies. The second diagram leaves the mixing matrix invariant in case of degenerate right-handed neutrinos as can be seen from the structure

$$m_\nu^{\text{tree + 1-loop}} = v_\mathrm{u}^2 Y_\nu \,\mathrm{diag}\left(\frac{1}{M_{\mathrm{R},k}} + \frac{g_1^2}{64\pi^2} \frac{\log\left(\frac{M^2_\mathrm{SUSY}}{M^2_{\mathrm{R},k}}\right)}{M_{\mathrm{R},k}} \right) Y_\nu^\mathrm{T}, \tag{3.11}$$

in the limit of degenerate SUSY masses and with the approximation $m_{\tilde\nu_{\mathrm{R},k}} = M_{\mathrm{R},k}$ where the difference is of $\mathscr{O}(M_\mathrm{SUSY})$. With degenerate right-handed masses, the diagonal matrix in Eq. (3.11) is proportional to the unit matrix and therefore does not alter the diagonalization of $Y_\nu Y_\nu^\mathrm{T}$.

The third (outer right) diagram of Fig. 3 obviously scales as $\sim A^\nu_{ij} \frac{1}{M^2_\mathrm{R}} M^\mathrm{R}_{jk} Y^\nu_{kf} / M_\mathrm{SUSY}$, taking degenerate right-handed neutrinos and (for simplicity and to see the scaling behavior) $Y^\nu_{ij} = y_\nu \delta_{ij}$. The suppression with M_SUSY is for common SUSY masses. So actually, the A-term contribution to the neutrino self-energy is

$$\Sigma^{\nu,(A)}_{fi} \sim v_\mathrm{u}^2 \frac{y_\nu}{M_\mathrm{R}} \frac{A^\nu_{fi}}{M_\mathrm{SUSY}}, \tag{3.12}$$

which is of the same order of magnitude as the tree-level neutrino mass, $\sim v_\mathrm{u}^2/M_\mathrm{R}$ and is seen to be suppressed by a typical loop factor $g_1^2/(16\pi^2)$. The ratio $A^\nu_{fi}/M_\mathrm{SUSY}$ stays the same when A^ν and M_SUSY are scaled uniformly. This behavior (using the full analytic expressions with mixing matrices evaluated numerically) is shown in Fig. 4 where we plot several projections of the same dataset. The off-diagonal values A^ν_{ij} are determined such that the renormalized PMNS matrix of Eq. (3.8) with the full expression of ΔU^ν_L given in Eq. (3.9) and ΔU^e_L equal the physical mixing matrix. Furthermore, we have assumed $U^{(0)}_{ij} = \delta_{ij}$ to show that it is indeed possible to generate the full mixing radiatively. The generic soft breaking masses, especially $\tilde{m}^2_\ell$ give significant contributions to charged lepton flavor violation as soon as off-diagonal elements are considered. Strong constraints on such observables let us arrange $\tilde{m}^2_{\ell,e,\nu} = m_{\text{soft}}\mathbb{1}$. A complete analysis also has to include the proper renormalization of the masses and constrains also the diagonal terms A^ν_{ii}.

The mixing matrix renormalization of Fig. 2 relies on *non-degenerate* neutrino masses. For degenerate masses, Eq. (3.9) is ill-defined and nevertheless there is no need for a renormalization of the mixing matrix [21]. Degenerate neutrino masses, however, allow for a special treatment and also cause the trivial mixing matrix after inclusion of threshold corrections to be non-trivial [34,35].

Non-degenerate masses generically come along with a non-trivial mixing at tree-level. We have artificially switched it off by a special choice of the fundamental parameters (i. e. both M_{R} and $Y^\nu \sim \mathbb{1}$). The scale of right-handed neutrinos was chosen high as suggested by the seesaw mechanism: with couplings $\lambda_{ij} \sim \mathscr{O}(1)$ in Eq. (3.1) and the electroweak scale $v \sim \mathscr{O}(100\,\text{GeV})$, we impose $M_{\text{R}} \sim \mathscr{O}(10^{13}\,\text{GeV})$ to have sub-eV neutrinos $m^{(0)}_\nu \sim \mathscr{O}(0.1\,\text{eV})$.

4. Conclusions

We have pursued the idea of RFV in the MSSM and applied radiative techniques to generate lepton mixing in an extension of the MSSM with right-handed Majorana neutrinos and a seesaw mechanism of type I. Though the lepton mixing angles are largish compared to the small quark mixing, the SUSY threshold corrections to the lepton mixing matrix described in this article are enhanced compared to the corrections to the CKM matrix. Differently from quarks and charged leptons, the neutrino mass spectrum is less hierarchic and, depending on the lightest neutrino mass, can be rather quasi-degenerate. In this case, the large contributions can be resummed which stabilizes the corrections with respect to the neutrino mass.[1]

The corrections do not decouple with the SUSY scale and persist if the SUSY mass spectrum is shifted to higher values. We constrain the off-diagonal neutrino A-terms with the requirement that the renormalized mixing matrix equals the experimentally determined PMNS matrix. In that description, there is a linear correlation $U_{ij} \sim \Sigma^\nu_{ij} \sim A^\nu_{ij}$ between the renormalized mixing matrix elements, the flavor changing self-energy and the soft SUSY breaking trilinear coupling. We have shown that this corrections scales like $A^\nu_{ij}/M_{\text{SUSY}}$ resulting in the non-decoupling behavior with the SUSY scale.

In general, the application of Eqs. (3.8) and (3.9) to a radiative generation of PMNS elements is *not* restricted to SUSY theories, however needs a flavor changing self-energy. Any theory beyond the SM which brings new flavor structures may lead to such self-energies. The realization within the MSSM (though extended with right-handed neutrinos) is in line with earlier studies of RFV and proposes a combined solution of the flavor puzzle together with SUSY breaking. The origin of flavor may lie in the origin of SUSY breaking and although the problem is only shifted into a

[1] The resummation has not been covered here and was performed after the Corfu Summer Institute 2013 where the results of this work date back.

different sector, the large number of unknown parameters in the general MSSM can be drastically reduced from radiative flavor physics. Especially, this formulation does not rely on tree-level flavor symmetries in the neutrino sector and is not restricted to specific textures in the mass matrices or Yukawa couplings. What we have not shown here is the analogous treatment with any non-trivial flavor mixing at the tree-level. Imposing tribimaximal mixing, $U^{(0)} = U_{\text{TBM}}$, we can equally well generate a non-vanishing 1-3 element and adjust the other mixing angles to their measured values. This holds for arbitrary tree-level mixing and emphasizes the importance of flavor non-universal threshold corrections in the presence of quasi-degenerate neutrinos.

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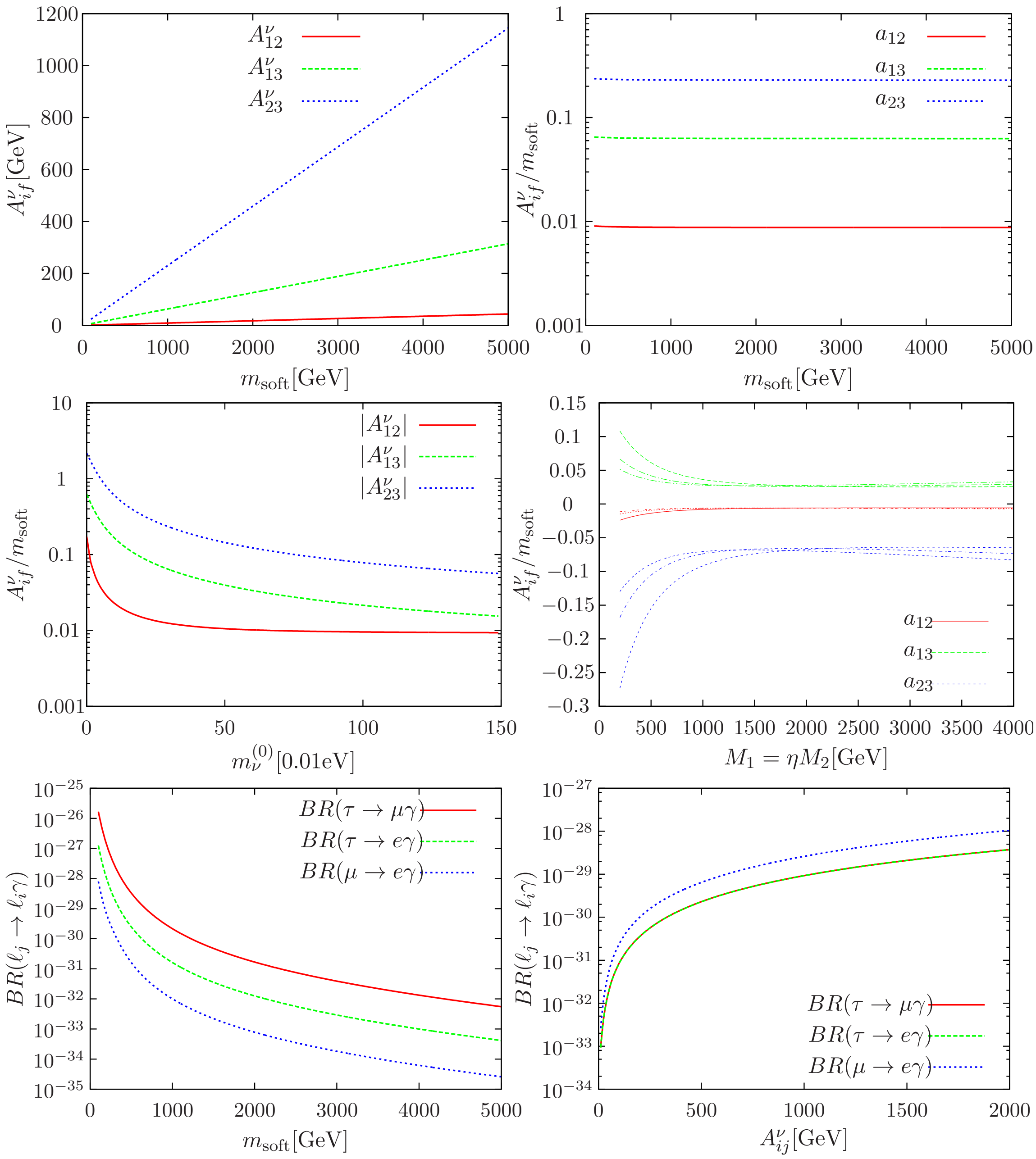

Figure 4: We display the results for a sample data point. In the first row shows the values of off-diagonal A^ν_{ij} in order to reproduce the lepton mixing matrix, which is a non-decoupling effect as demonstrated by the ratio $A^\nu_{ij}/m_{\text{soft}}$. We have set a common SUSY mass, $m_{\text{soft}} = M_{\text{SUSY}}$ for scalar and gaugino masses. The gaugino masses itself have no influence on the result as long as they are not much smaller than m_{soft} as shown below. In case of hierarchical neutrinos (i. e. with small $m_\nu^{(0)}$), $A^\nu_{ij}/m_{\text{soft}}$ has to be much larger to give the same mixing whereas the same ratio may be much smaller the more degenerate the neutrino mass spectrum is. In the last row, we finally give the contributions to radiative lepton decays $\ell_j \to \ell_i\gamma$ with $j > i$. Because we only introduce off-diagonal in A^ν not A^e, the effect on the branching ratios is small arising only in the (s)neutrino sector and additionally being suppressed with M_{R}.