# Three- and Four-point correlators of excited bosonic twist fields 

## Pascal Anastasopoulos*

Technische Univ. Wien Inst. für Theoretische Physik, A-1040 Vienna, Austria
E-mail: pascal@hep.itp.tuwien.ac.at

## Mark D. Goodsell

1- Sorbonne Universités, UPMC Univ Paris 06, UMR 7589, LPTHE, F-75005, Paris, France
2- CNRS, UMR 7589, LPTHE, F-75005, Paris, France
E-mail: goodsell@lpthe.jussieu.fr

## Robert Richter

II. Institut für Theoretische Physik, Hamburg University, Germany

E-mail: robert.richter@desy.de

This proceeding is based on [1], where we computed correlation functions containing excited bosonic twist fields. Our results can be used for the phenomenological study of massive string excitations localised at D-brane intersections. Such massive states may be observable at the LHC provided that we live a universe with a low string scale.

Proceedings of the Corfu Summer Institute 2014 "School and Workshops on Elementary Particle Physics and Gravity",
3-21 September 2014
Corfu, Greece

[^0]
## 1. Introduction

D-branes provide a very rich framework for realistic model building. One of the most exciting properties of this paradigm is that it allows for a low string scale [2-4] even at a few TeV range ${ }^{1}$. Therefore, phenomenological studies of these models are particularly interesting and several directions have been extensively analysed, such as anomalous $Z^{\prime}$ physics (see, e.g. [8-21]), Kaluza Klein states (see, e.g. [22-28]), and purely stringy signatures (see, e.g. [29-42]) ${ }^{2}$.

In this proceeding, we will take a different direction which has been less studied and focus on so-called light stringy states. More precisely, in semirealistic constructions which consist of intersecting D-branes, there is a whole tower of states living on each intersection, sharing the same quantum numbers but with masses are proportional to the intersection angle $\theta$

$$
\begin{equation*}
M^{2} \sim 0, \theta M_{s}^{2}, 2 \theta M_{s}^{2}, 3 \theta M_{s}^{2}, \ldots \tag{1.1}
\end{equation*}
$$

In this framework, the Standard Model matter content is described by the first (massless) modes (for reviews see [45-48] and references therein). Such towers look like the Kaluza-Klein (KK) towers with the significant difference that in D-brane models each tower has a unique mass gap (depending on the specific angle of the intersection where each state lives) in contrast to the KK models where all particle towers share the same mass gaps (depending on the global properties of the internal space). That makes D-brane and KK models easily distinguishable.


Figure 1: A D-brane realisation of the Standard Model. Gauge fields live on D-branes, matter content stretches between different D-branes. On each intersection we have a whole tower of massive copies of the corresponding Standard Model field.

If the string scale is low and some of these angles are very small, some of these excited stringy copies of the Standard Model particles become very light and can be produced at LHC. The study of decay rates and life times of these light stringy states is very interesting and it requires the knowledge of their vertex operators [49-51] as well as the relevant correlation functions ${ }^{3}$.

[^1]In [1], we evaluated various useful correlation functions involving (higher) excited bosonic twist fields, which are a crucial ingredient in the vertex operators massive stringy excitations localized at the intersections of two D-brane stacks. For simplicity and reasons of calculability we assumed the compactification manifold to be a factorizable six-torus $T^{6}=T^{2} \times T^{2} \times T^{2}$ wrapped by D6-branes (for previous related work on similar backgrounds see e.g. [59-78], for recent work, see e.g. $[79,80])$. In that case the internal part of the world sheet integrand of the scattering amplitudes splits into three separate factorizable parts for each two-tori $T^{2}$, for which one can apply the developed CFT techniques.

Summarising, we determine various correlators containing (higher) excited bosonic twist fields, where the latter live on one of three two-tori. More precisely, we derive correlators of the type

$$
\begin{equation*}
\left\langle\tau_{\alpha_{1}}^{+}\left(x_{1}\right) \tilde{\tau}_{\alpha_{2}}^{+}\left(x_{2}\right) \sigma_{\alpha_{3}}^{+}\left(x_{3}\right) \ldots\right\rangle,\left\langle\omega_{\alpha_{1}}^{+}\left(x_{1}\right) \sigma_{\alpha_{2}}^{+}\left(x_{2}\right) \sigma_{\alpha_{3}}^{+}\left(x_{3}\right) \ldots\right\rangle, \text { etc } \tag{1.2}
\end{equation*}
$$

where the $\sigma$ fields are the usual bosonic twist fields while $\tau$ and $\omega$ denote the excited and double excited bosonic twist fields. The subscript denotes the intersection angle which is measured in units of $\pi$ and ranges in the interval $[0,1)$. The " + " upper-index denotes that all angles are positive.

Note that the excited twist fields $\tau$ and $\omega$ are not primary conformal fields, therefore, we first need to evaluate some higher point correlators with only primary fields, namely solely bosonic twist fields $\sigma$ as well as the conformal fields $\partial Z$ and $\partial \bar{Z}$, and next apply various operator product expansions OPE's by performing appropriate limits in order to obtain (1.2). This method is called the energy momentum tensor method and originally used in [81-83] in the context of closed string theory on orbifolds and more recently it applied to open string theory [53, 84-89] ${ }^{4}$.

This proceeding is organised as follows: In section 2 we illustrate the method which we use to evaluate correlators containing the excited bosonic twist fields. In section 3, we apply that method to compute four-point correlators with one independent angle. The appendix A contains various OPE's.

## 2. The method

Lets assume the compactification manifold to be a factorizable six-torus $T^{6}=T^{2} \times T^{2} \times T^{2}$. In terms of the compactified coordinates $Z^{i}, \bar{Z}^{i}$ we have

$$
\begin{equation*}
Z^{i}=X^{2 i+2}+i X^{2 i+3}, \quad \bar{Z}^{i}=X^{2 i+2}-i X^{2 i+3} \tag{2.1}
\end{equation*}
$$

with the index $i$ denoting the three different two-tori. In the following, we will focus on a single two-torus $T^{2}$. The analysis for the other two two-tori is analogous.

In order to derive the displayed correlators (1.2) we will use the energy momentum tensor method [86-89]

- First, evaluate some extended correlators with only primary fields:

$$
\begin{equation*}
\left\langle\prod_{i} \partial Z_{c l}\left(z_{i}\right) \prod_{j} \partial \bar{Z}_{c l}\left(w_{j}\right) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\beta}^{+}\left(x_{2}\right) \ldots\right\rangle \tag{2.2}
\end{equation*}
$$

where it contains

[^2]1. as many $\sigma$ 's as the total number of states in the desired correlators (1.2),
2. as many $\partial Z, \partial \bar{Z}$ we need to transform some of the $\sigma$ to $\tau$ or $\omega$ 's etc using the following diagrammatic figure

where $\nearrow, \nwarrow$ denote the OPE action of $\partial Z, \partial \bar{Z}$ on various twisted fields $\sigma, \tau, \omega$ and can be easily extended to higher excited bosonic twist fields (see appendx A).

Since $\partial Z, \partial \bar{Z}$ split to classical and quantum parts

$$
\begin{equation*}
\partial Z=\partial Z_{c l}+\partial Z_{q u} \quad \partial \bar{Z}=\partial \bar{Z}_{c l}+\partial \bar{Z}_{q u} \tag{2.4}
\end{equation*}
$$

we also split the correlators to a pure quantum and some mixed parts and we evaluate them separately

- The quantum part looks like

$$
\begin{equation*}
\left\langle\prod_{i} \partial Z_{q u}\left(z_{i}\right) \prod_{j} \partial \bar{Z}_{q u}\left(w_{j}\right) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\beta}^{+}\left(x_{2}\right) \ldots\right\rangle \tag{2.5}
\end{equation*}
$$

we use the local behaviour
$\partial Z_{q u}(z) \partial \bar{Z}_{q u}(w) \sim \frac{1}{(z-w)^{2}}, \quad \partial Z_{q u}(z) \partial Z_{q u}(w) \sim$ regular,$\quad \partial \bar{Z}_{q u}(z) \partial \bar{Z}_{q u}(w) \sim$ regular
and the OPEs (see appendx A) in order to make an ansatz about the relevant correlators. The above generically does not uniquely specify the correlators but we also need to impose the monodromy conditions extracted by the behaviour of $\partial Z$ and $\partial \bar{Z}$ at the specific configurations.

- The mixed parts on the other hand have the form

$$
\begin{equation*}
\prod_{i^{\prime}} \partial Z_{c l}\left(\widetilde{z}_{i^{\prime}}\right) \prod_{j^{\prime}} \partial \bar{Z}_{c l}\left(\widetilde{w}_{j^{\prime}}\right)\left\langle\prod_{i^{\prime \prime}} \partial Z_{q u}\left(z_{i^{\prime \prime}}\right) \prod_{j^{\prime \prime}} \partial \bar{Z}_{q u}\left(w_{j^{\prime \prime}}\right) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\beta}^{+}\left(x_{2}\right) \ldots\right\rangle \tag{2.7}
\end{equation*}
$$

with $\sum_{i^{\prime}} 1+\sum_{i^{\prime \prime}} 1=\sum_{i} 1$ and $\sum_{j^{\prime}} 1+\sum_{j^{\prime \prime}} 1=\sum_{j} 1$. Therefore, it splits into a product of $\partial Z_{c l}$ and $\partial \bar{Z}_{c l}$ which are known for the specific configurations [87,88] and a pure quantum part, similar to (2.5) which can be independently evaluated by the energy momentum tensor method described above.

Finally, any correlator is suppressed by the world-sheet instanton, which is given by $e^{-S_{c l}}$ where the whole world-sheet instanton contribution ${ }^{5}$

$$
\begin{equation*}
S_{c l}=\frac{1}{4 \pi \alpha^{\prime}} \int d^{2} z\left\{\partial Z_{c l}(z) \partial Z_{c l}(z)+\partial \bar{Z}_{c l}(z) \partial \bar{Z}_{c l}(z)\right\} \tag{2.8}
\end{equation*}
$$

[^3]is a sum over all possible closed triangles connecting the three intersection points [86-89] (See also [32]) ${ }^{6}$.

- Having at hand the extended correlator (2.2), we take various limits $z_{i} \rightarrow x_{k}, w_{j} \rightarrow x_{m}$ (and also $\left.\widetilde{z}_{i} \rightarrow x_{l}, \widetilde{w}_{j} \rightarrow x_{n}\right)$ which brings $\partial Z\left(z_{i}\right)$ and $\partial \bar{Z}\left(w_{j}\right)$ close to the position of $\sigma\left(x_{k}\right)$ and we can use the OPEs given in appendix A in order to determine the correlators (1.2).

In the next chapter we will provide one example of the method above and we will evaluate the correlator function of four twisted fields with two excited $\tau, \tilde{\tau}$ (aka $\langle\tau \tilde{\tau} \sigma \sigma\rangle$ ) or one double excited field $\omega$ (aka $\langle\omega \sigma \sigma \sigma\rangle$ ) with one independent angle. The complete list of our results is provided in [1].

## 3. Four-point correlators containing two excited twist fields

As mentioned above, in this section we will evaluate four-point correlation function containing two exited twist fields $\tau, \tilde{\tau}$ or one double-excited twist field $\omega$

$$
\begin{align*}
& \left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \tilde{\tau}_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.1}\\
& \left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \tilde{\tau}_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.2}\\
& \left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \tau_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.3}\\
& \left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \tau_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.4}\\
& \left\langle\omega_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \tag{3.5}
\end{align*}
$$

for one independant angle $\alpha$, where the boundary conditions on the world-sheet are

$$
\begin{align*}
\partial Z-\partial \bar{Z} & =0 \text { for }\left(-\infty, x_{1}\right) \cup\left(x_{2}, x_{3}\right) \cup\left(x_{4}, \infty\right) \\
e^{i \pi \alpha} \partial Z-e^{-i \pi \alpha} \partial \bar{Z} & =0 \text { for }\left(x_{1}, x_{2}\right) \cup\left(x_{3}, x_{4}\right) . \tag{3.6}
\end{align*}
$$

Following the steps which were presented above

- first, we derive the extended six-point correlators

$$
\begin{align*}
& \left\langle\partial Z(z) \partial \bar{Z}(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
& \left\langle\partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.7}\\
& \left\langle\partial Z(z) \partial Z(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle
\end{align*}
$$

which split to a pure quantum and a mixed part (where $\partial Z(z), \partial \bar{Z}(w)$ are classical) and we will deal with them separately.

- the quantum part

[^4]Let us define the following auxiliary functions

$$
\begin{align*}
g(z, w) & =\frac{\left\langle\partial Z_{q u}(z) \partial \bar{Z}_{q u}(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle}{\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle}  \tag{3.8}\\
k(z, w) & =\frac{\left\langle\partial \bar{Z}_{q u}(z) \partial \bar{Z}_{q u}(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle}{\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle}  \tag{3.9}\\
m(z, w) & =\frac{\left\langle\partial Z_{q u}(z) \partial Z_{q u}(w) \sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle}{\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle} \tag{3.10}
\end{align*}
$$

where the four-point correlator is given by [53, 84-88,92]

$$
\begin{align*}
\left\langle\sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x)\right. & \left.\sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle \\
& =\frac{\sin ^{\frac{1}{2}}(\pi \alpha)}{{ }_{2} F_{1}\left[\alpha, 1-\alpha, 1, \frac{x_{12} x_{33}}{x_{13} x_{24}}\right]^{\frac{1}{2}}{ }_{2} F_{1}\left[\alpha, 1-\alpha, 1, \frac{x_{14} x_{23}}{x_{13} x_{24}}\right]^{\frac{1}{2}}}\left[\frac{x_{12} x_{23} x_{14} x_{34}}{x_{13} x_{24}}\right] \tag{3.11}
\end{align*}
$$

with the hypergeometric function

$$
\begin{equation*}
{ }_{2} F_{1}[\alpha, \beta, \gamma, x]=\frac{\Gamma(\gamma)}{\Gamma(\alpha) \Gamma(\beta)} \int_{0}^{1} d z z^{\beta-1}(1-z)^{\gamma-\beta-1}(1-x z)^{-\alpha} \tag{3.12}
\end{equation*}
$$

and $\Gamma(\alpha)$ being $\Gamma(\alpha)=\int_{0}^{\infty} d z z^{\alpha-1} e^{-z}$.
Using the local behaviour of the bosonic twist fields (A.1) as well as (2.6) we obtain for the functions $g(z, w), k(z, w)$ and $m(z, w)$

$$
\begin{align*}
& g(z, w)=\alpha^{\prime} \omega_{1-\alpha, 1-\alpha}(z) \omega_{\alpha, \alpha}(w)\left\{\frac{P}{(z-w)^{2}}+A\left(\left\{x_{i}\right\}\right)\right\}  \tag{3.12}\\
& k(z, w)=\alpha^{\prime} \omega_{\alpha, \alpha}(z) \omega_{\alpha, \alpha}(w) B\left(\left\{x_{i}\right\}\right)  \tag{3.14}\\
& m(z, w)=\alpha^{\prime} \omega_{1-\alpha, 1-\alpha}(z) \omega_{1-\alpha, 1-\alpha}(w) C\left(\left\{x_{i}\right\}\right) \tag{3.15}
\end{align*}
$$

where we use various symmetries of the functions under the exchanges of the $x_{i}$ 's. Here

$$
\begin{equation*}
\omega_{\alpha, \beta}(z)=\left(z-x_{1}\right)^{-\alpha}\left(z-x_{2}\right)^{-1+\alpha}\left(z-x_{3}\right)^{-\beta}\left(z-x_{4}\right)^{-1+\beta} . \tag{3.16}
\end{equation*}
$$

and $P$ takes the form

$$
\begin{align*}
2 P= & (1-\xi)\left(z-x_{1}\right)\left(z-x_{2}\right)\left(w-x_{3}\right)\left(w-x_{4}\right) \\
& +2(-\alpha+\xi)\left(z-x_{1}\right)\left(w-x_{2}\right)\left(z-x_{3}\right)\left(w-x_{4}\right) \\
& +(1-\xi)\left(z-x_{1}\right)\left(w-x_{2}\right)\left(w-x_{3}\right)\left(z-x_{4}\right) \\
& +(1-\xi)\left(w-x_{1}\right)\left(z-x_{2}\right)\left(z-x_{3}\right)\left(w-x_{4}\right) \\
& +2(-1+\alpha+\xi)\left(w-x_{1}\right)\left(z-x_{2}\right)\left(w-x_{3}\right)\left(z-x_{4}\right) \\
& +(1-\xi)\left(w-x_{1}\right)\left(w-x_{2}\right)\left(z-x_{3}\right)\left(z-x_{4}\right) . \tag{3.17}
\end{align*}
$$

The $\xi$ is a free parameter that is neither fixed by the local behavior nor by any symmetry of the functions. The $A\left(\left\{x_{i}\right\}\right), B\left(\left\{x_{i}\right\}\right)$ and $C\left(\left\{x_{i}\right\}\right)$ are functions on positions of the bosonic twist fields only and thus independent of $z$ and $w$. Those functions will be
determined using constraints arising from global monodromies of the boundary conditions (3.6) ${ }^{7}$

$$
\begin{align*}
& \int_{x}^{1} d z(g(z, w)-k(z, w))=0, \quad \int_{0}^{x} d z\left(e^{i \pi \alpha} g(z, w)-e^{-i \pi \alpha} k(z, w)\right)=0  \tag{3.18}\\
& \int_{x}^{1} d w(m(z, w)-g(z, w))=0, \quad \int_{0}^{x} d w\left(e^{i \pi \alpha} m(z, w)-e^{-i \pi \alpha} g(z, w)\right)=0 \tag{3.19}
\end{align*}
$$

After using $S L(2, \mathbf{R})$ symmetry to fix $x_{1}=0, x_{2}=x, x_{3}=1$ and $x_{4}=x_{\infty}$ and solving the above conditions we get

$$
\begin{align*}
\left\langle\partial Z_{q u}(z)\right. & \left.\partial \bar{Z}_{q u}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle  \tag{3.20}\\
=\alpha^{\prime} & \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(w-x)(z-1)]^{\alpha-1}[w(z-x)(w-1)]^{-\alpha}}{\sqrt{\Gamma(\alpha) \Gamma(1-\alpha)_{2} F_{1}[1-\alpha, \alpha, 1, x]_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}} \\
& \times\left\{(1-\alpha) \frac{z(w-x)(z-1)}{(z-w)^{2}}+\alpha \frac{w(z-x)(w-1)}{(z-w)^{2}}\right. \\
& \left.\quad+\frac{1}{2} \alpha(1-\alpha)\left(x \frac{2 F_{1}[1-\alpha, \alpha, 2, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}-(1-x) \frac{2 F_{1}[1-\alpha, \alpha, 2,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}\right)\right\} .
\end{align*}
$$

$$
\begin{equation*}
\left\langle\partial \bar{Z}_{q u}(z) \partial \bar{Z}_{q u}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle \tag{3.21}
\end{equation*}
$$

$$
\times\left\{1-\frac{{ }_{2} F_{1}[-\alpha, \alpha, 1,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}-\frac{{ }_{2} F_{1}[-\alpha, \alpha, 1, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}\right\}
$$

$$
\begin{aligned}
& \left\langle\partial Z_{q u}(z) \partial Z_{q u}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle \\
& =\alpha^{\prime}(1-\alpha) e^{-2 \pi i \alpha} \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(z-1) w(w-1)]^{-\alpha}[(z-x)(w-x)]^{\alpha-1}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha){ }_{2} F_{1}[1-\alpha, \alpha, 1, x]_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}} \\
& \quad \times\left\{1-\frac{{ }_{2} F_{1}[1-\alpha, \alpha-1,1,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}-\frac{2 F_{1}[1-\alpha, \alpha-1,1, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}\right\} .
\end{aligned}
$$

$$
=\alpha^{\prime} \alpha e^{2 i \pi \alpha} \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(z-1) w(w-1)]^{-\alpha}[(z-x)(w-x)]^{\alpha-1}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha)_{2} F_{1}[1-\alpha, \alpha, 1, x]_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}}
$$

Note that all results are as expected completely independent of $\xi$.

- The mixed parts are given by

$$
\begin{align*}
& \partial Z_{c l}(z) \partial \bar{Z}_{c l}(w)\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
& \partial \bar{Z}_{c l}(z) \partial \bar{Z}_{c l}(w)\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.23}\\
& \partial Z_{c l}(z) \partial Z_{c l}(w)\left\langle\sigma_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle .
\end{align*}
$$

since mixed parts with one quantum $\partial Z_{q u}$ or $\partial \bar{Z}_{q u}$ vanish. Now, it is very simple to evaluate (3.23) since all pieces are known. The four point correlator is given in (3.11).

[^5]The asymptotic behaviour close to the insertion of the bosonic twist fields specify the classical solutions [86, 88] ${ }^{8}$

$$
\begin{align*}
& \partial Z_{c l}(z)=-\sqrt{\alpha^{\prime}} \frac{\sin (\pi \alpha)}{2 \pi} \frac{v_{b}[m]+e^{i \pi \alpha} v_{a}[n] \tau(x)}{{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]} \omega_{1-\alpha, 1-\alpha}(z) \\
& \partial \bar{Z}_{c l}(z)=-\sqrt{\alpha^{\prime}} \frac{\sin (\pi \alpha)}{2 \pi} \frac{v_{b}[m]-e^{i \pi \alpha} v_{a}[n] \tau(x)}{{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]} \omega_{\alpha, \alpha}(z) \tag{3.24}
\end{align*}
$$

where $\tau(x)$ is given by the ratio

$$
\begin{equation*}
\tau(x)=\frac{{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]}{{ }_{2} F_{1}[\alpha, 1-\alpha, 1, x]} \tag{3.25}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{a}[n]=f_{3}-f_{1}+n \widetilde{L}_{a}, \quad v_{b}[m]=f_{3}-f_{2}+m \widetilde{L}_{b} \tag{3.26}
\end{equation*}
$$

with $n, m$ being integers and $\widetilde{L}_{x}$ given by

$$
\begin{equation*}
\widetilde{L}_{a}=\frac{\left|I_{b c}\right|}{\operatorname{gcd}\left(\left|I_{a b}\right|,\left|I_{b c}\right|, \mid I_{c a}\right) \mid} L_{a} . \tag{3.27}
\end{equation*}
$$

In eq. (3.27) $I_{x y}=n_{x} m_{y}-n_{y} m_{x}$ is the intersection number between two D-brane stacks $x$ and $y, L_{x}=R_{1} \sqrt{n_{x}^{2}+\left(m_{x} \mathscr{T}\right)^{2}}$ the length of the D-brane stack $x$, with $\left(n_{x}, m_{x}\right)$ being the wrapping numbers and $\mathscr{T}$ denoting the complex structure of the two-torus. In the simple setup in which all three D -brane stacks intersect each other exactly once $\widetilde{L}_{x}$ coincides with the length $L_{x}$ of the respective D -brane.


Figure 2: Let us consider the three intersecting D-brane stacks wrapping the one-cycles $a=(1,0), b=(3,1)$, $c=(0,1)$ on a rectangular two-torus.

Let us examplify the notion of the $v_{c}[n]$ with the explicit example depicted in figure 2. Here we have three stacks of D-branes, $a, b$ and $c$, wrapping one-cycles on a $T^{2}$. Their

[^6]intersection numbers are given by $I_{a b}=1, I_{b c}=3, I_{a c}=1$, where $f_{1}$ denotes the intersection between D-brane stack $c$ and $a$ while $f_{2}, \widetilde{f}_{2}$ and $\widetilde{\widetilde{f}}_{2}$ denote the three intersection points among the D-brane stacks $b$ and $c$. The primed $f$ 's are the corresponding lattice shifted intersection points. For this explicit example the vectors $v_{c}[0]$ and $v_{c}[1]$ take the form $v_{c}[0]=f_{1}-f_{2}$ and $v_{c}[1]=f_{1}^{\prime \prime \prime}-f_{2}^{\prime}=f_{1}-f_{2}+\widetilde{L}_{c}$, respectively.

Combining the quantum and mix parts we obtain the extended six-point correlators:

$$
\begin{align*}
& \left\langle\partial Z(z) \partial \bar{Z}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle  \tag{3.28}\\
& =\alpha^{\prime} \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(w-x)(z-1)]^{\alpha-1}[w(z-x)(w-1)]^{-\alpha}}{\sqrt{\Gamma(\alpha) \Gamma(1-\alpha){ }_{2} F_{1}[1-\alpha, \alpha, 1, x]_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}} \\
& \times \sum_{n, m}\left\{(1-\alpha) \frac{z(w-x)(z-1)}{(z-w)^{2}}+\alpha \frac{w(z-x)(w-1)}{(z-w)^{2}}-\frac{\sin ^{2}(\pi \alpha)}{4 \pi^{2}} \frac{v_{b}^{2}[m]-e^{2 i \pi \alpha} v_{a}^{2}[n] \tau^{2}(x)}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]^{2}}\right. \\
& \left.+\frac{1}{2} \alpha(1-\alpha)\left(x \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}-(1-x) \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}\right)\right\} e^{-S_{c l}[n, m]} \\
& \left\langle\partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle  \tag{3.29}\\
& =\alpha^{\prime} \alpha e^{2 i \pi \alpha} \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(z-1) w(w-1)]^{-\alpha}[(z-x)(w-x)]^{\alpha-1}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha){ }_{2} F_{1}[1-\alpha, \alpha, 1, x]_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}} \\
& \times \sum_{n, m}\left\{1-\frac{{ }_{2} F_{1}[-\alpha, \alpha, 1,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}-\frac{{ }_{2} F_{1}[-\alpha, \alpha, 1, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}\right. \\
& \left.+\frac{\sin ^{2}(\pi \alpha)}{2 \pi^{2} \alpha} \frac{\left(v_{b}[m]-e^{i \pi \alpha} v_{a}[n] \tau(x)\right)^{2}}{{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]^{2}}\right\} e^{-S_{c l}[n, m]}
\end{align*}
$$

$$
\begin{align*}
& \left\langle\partial Z(z) \partial Z(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle  \tag{3.30}\\
& =\alpha^{\prime}(1-\alpha) e^{-2 \pi i \alpha} \frac{\sqrt{\pi}[x(1-x)]^{-\alpha(1-\alpha)}[z(z-1) w(w-1)]^{\alpha-1}[(z-x)(w-x)]^{-\alpha}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha){ }_{2} F_{1}[1-\alpha, \alpha, 1, x]{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}} \\
& \quad \times \sum_{n, m}\left\{1-\frac{{ }_{2} F_{1}[1-\alpha, \alpha-1,1,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}-\frac{{ }_{2} F_{1}[1-\alpha, \alpha-1,1, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}\right. \\
& \left.\quad+\frac{\sin ^{2}(\pi \alpha)}{2 \pi^{2}(1-\alpha)} \frac{\left(v_{b}[m]+e^{i \pi \alpha} v_{a}[n] \tau(x)\right)^{2}}{{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]^{2}}\right\} e^{-S_{c l}[n, m]}
\end{align*}
$$

Note that all correlators are suppressed by world-sheet instanton contributions which can be easily computed applying (2.8) as well as using the classical solutions (3.24). One obtains

$$
\begin{equation*}
S_{c l}^{T^{2}}[n, m]=\pi \sin (\pi a)\left\{\left|v_{a}[n]\right|^{2} \tau(x)+\left|v_{b}[m]\right|^{2} \tau(1-x)\right\} \tag{3.31}
\end{equation*}
$$

where $\tau(x)$ is given by (3.25).

- With the extended six-point correlators at hand, we can derive the various four-point correlators containing higher excited twist fields by taking the appropriate limits.

Taking for example the limit $z \rightarrow 0$ and $w \rightarrow x$ of $\left\langle\partial Z \partial \bar{Z} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-}\right\rangle$(3.28) and using the OPE's (A.1) one finds

$$
\begin{align*}
& \left\langle\tau_{\alpha}^{+}(0) \tilde{\tau}_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty)\right\rangle  \tag{3.32}\\
& \quad=\alpha^{\prime} \frac{\sqrt{\pi} x^{-\alpha(3-\alpha)}(1-x)^{-\alpha(2-\alpha)}}{\sqrt{\Gamma(\alpha) \Gamma(1-\alpha)_{2} F_{1}[\alpha, 1-\alpha, 1, x]{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]}} \\
& \quad \times \sum_{n, m}\left\{\alpha(1-x)+\frac{1}{2} \alpha(1-\alpha)\left(x \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}-(1-x) \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}\right)\right. \\
& \left.\quad-\frac{\sin ^{2}(\pi \alpha)}{4 \pi^{2}} \frac{v_{b}^{2}[m]-e^{2 i \pi \alpha}}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]^{2}}\right\} e^{-S_{c l}[n, m]} \tau^{2}(x)
\end{align*}
$$

which after reinstating all $x_{i}$ dependence gives ${ }^{9}$

$$
\begin{align*}
& \left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \widetilde{\tau}_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle  \tag{3.34}\\
& \quad=\alpha^{\prime} \frac{\sqrt{\pi} x_{12}^{-\alpha(3-\alpha)} x_{34}^{-\alpha(1-\alpha)}\left(\frac{x_{13} x_{24}}{x_{14} x_{23}}\right)^{\alpha(2-\alpha)}}{\sqrt{\Gamma(\alpha) \Gamma(1-\alpha){ }_{2} F_{1}[\alpha, 1-\alpha, 1, x]{ }_{2} F_{1}[\alpha, 1-\alpha, 1,1-x]}} \\
& \quad \times \sum_{n, m}\left\{\alpha(1-x)+\frac{1}{2} \alpha(1-\alpha)\left(x \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2, x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1, x]}-(1-x) \frac{{ }_{2} F_{1}[1-\alpha, \alpha, 2,1-x]}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]}\right)\right. \\
& \left.\quad-\frac{\sin ^{2}(\pi \alpha)}{4 \pi^{2}} \frac{v_{b}^{2}[m]-e^{2 i \pi \alpha} v_{a}^{2}[n] \tau^{2}(x)}{{ }_{2} F_{1}[1-\alpha, \alpha, 1,1-x]^{2}}\right\} e^{-S_{c l}[n, m]} .
\end{align*}
$$

The above technique can be applied to any of the six-point correlators generating various four-point correlators.

$$
\begin{align*}
&\left\langle\partial Z \partial \bar{Z} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-}\right\rangle \rightarrow\left\{\begin{array}{l}
\left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \widetilde{\tau}_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
\left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \tilde{\tau}_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle
\end{array}\right.  \tag{3.35}\\
&\left\langle\partial Z \partial Z \sigma_{\alpha}^{+} \sigma_{\alpha}^{-} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-}\right\rangle \rightarrow\left\{\begin{array}{l}
\left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \tau_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
\left\langle\tau_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \tau_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
\left\langle\omega_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle
\end{array}\right.  \tag{3.36}\\
&\left\langle\partial \bar{Z} \partial \bar{Z} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-}\right\rangle \rightarrow\left\{\begin{array}{l}
\left\langle\tilde{\tau}_{\alpha}^{+}\left(x_{1}\right) \widetilde{\tau}_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
\left\langle\tilde{\tau}_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \tau_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle \\
\left\langle\widetilde{\omega}_{\alpha}^{+}\left(x_{1}\right) \sigma_{\alpha}^{-}\left(x_{2}\right) \sigma_{\alpha}^{+}\left(x_{3}\right) \sigma_{\alpha}^{-}\left(x_{4}\right)\right\rangle
\end{array}\right. \tag{3.37}
\end{align*}
$$

Note that all other correlators (like $\left\langle\tilde{\tau}_{\alpha}^{+} \tau_{\alpha}^{-} \sigma_{\alpha}^{+} \sigma_{\alpha}^{-}\right\rangle$) can be obtained by the ones presented above by simply applying the rules displayed in (A.2)

$$
\begin{array}{lll}
\sigma_{\alpha}^{-}=\sigma_{1-\alpha}^{+} & \tau_{\alpha}^{-}=\tau_{1-\alpha}^{+} & \omega_{\alpha}^{-}=\omega_{1-\alpha}^{+} \\
& \widetilde{\tau}_{\alpha}^{-}=\widetilde{\tau}_{1-\alpha}^{+} & \widetilde{\omega}_{\alpha}^{-}=\widetilde{\omega}_{1-\alpha}^{+} \tag{3.39}
\end{array}
$$

${ }^{9}$ Here we use the fact that an open string four-point correlator is given by

$$
\begin{equation*}
\left\langle\phi_{h_{1}}\left(x_{1}\right) \phi_{h_{2}}\left(x_{2}\right) \phi_{h_{3}}\left(x_{3}\right) \phi_{h_{4}}\left(x_{4}\right)\right\rangle=\mathscr{F}(x) \prod_{i<j} x_{i j}^{-\left(h_{i}+h_{j}\right)+\frac{\Delta}{3}}, \tag{3.33}
\end{equation*}
$$

where $\Delta=\sum_{i=1}^{4} h_{i}$ and $x=\frac{x_{12} x_{34}}{x_{13} x_{2}}$. This expression allows us to reinstate the $x_{i}$ dependence in all the four-point correlators.

All our results for one and two independent angles can be found in [1].

## 4. Conclusions

In this proceeding we have presented a method for the evaluation of three- and four-point correlators containing excited bosonic twist fields. The knowledge of these correlators is required for the evaluation of lifetime and decay rates of massive string excitations that arise at intersections of D-branes.

In order to evaluate these correlators, we took a detour and determined higher-point correlators containing the conformal fields $\partial Z$ and $\partial \bar{Z}$ as well as the regular bosonic twist fields $\sigma$. Given those correlators we performed various limits to derive the three- and four-point correlators containing higher excited bosonic twist fields.

Our results can be used in order to evaluate decay rates and lifetimes for the massive excited stringy states of the Standard Model matter fields [93]. If the string scale is low (at a few TeV range) and the intersection angles between the D -branes very small, then such light stringy states can be the first stringy effect to be observed at LHC.

## Acknowledgements

We would like to thank the organizers of Corfu 2013 for giving us the opportunity to present this work. P. A. is supported by the Austrian Science Fund (FWF) program P 26731-N27. M. D. G. was supported during the time the work upon which this proceeding is based was originally performed by a Marie Curie Intra European Fellowship within the 7th European Community Framework Programme and by ERC advanced grant 226371. R. R. acknowledges the support from the FP7 Marie Curie Actions of the European Commission, via the Intra-European Fellowships (project number: 328170).

## A. OPE's of twisted states

Here, we provide the OPEs of $\partial Z(x)$ and $\partial \bar{Z}(x)$ on twisted fields:

$$
\begin{array}{ll}
\partial Z(z) \sigma_{\alpha}^{+}(w) \sim(z-w)^{\alpha-1} \tau_{\alpha}^{+}(w) & \partial \bar{Z}(z) \sigma_{\alpha}^{+}(w) \sim(z-w)^{-\alpha} \widetilde{\tau}_{\alpha}^{+}(w) \\
\partial Z(z) \tau_{\alpha}^{+}(w) \sim(z-w)^{\alpha-1} \omega_{\alpha}^{+}(w) & \partial \bar{Z}(z) \tau_{\alpha}^{+}(w) \sim(z-w)^{-\alpha-1} \sigma_{\alpha}^{+}(w) \\
\partial Z(z) \tilde{\tau}_{\alpha}^{+}(w) \sim(z-w)^{-2+\alpha} \sigma_{\alpha}^{+}(w) & \partial \bar{Z}(z) \omega_{\alpha}^{+}(w) \sim(z-w)^{-\alpha-1} \tau_{\alpha}^{+}(w) \\
\partial Z(z) \sigma_{\alpha}^{-}(w) \sim(z-w)^{-\alpha} \tau_{\alpha}^{-}(w) & \partial \bar{Z}(z) \tilde{\tau}_{\alpha}^{+}(w) \sim(z-w)^{-\alpha} \widetilde{\omega}_{\alpha}^{+}(w)  \tag{A.1}\\
\partial Z(z) \tau_{\alpha}^{-}(w) \sim(z-w)^{-\alpha} \omega_{\alpha}^{-}(w) & \partial \bar{Z}(z) \sigma_{\alpha}^{-}(w) \sim(z-w)^{\alpha-1} \widetilde{\tau}_{\alpha}^{-}(w) \\
\partial Z(z) \widetilde{\tau}_{\alpha}^{-}(w) \sim(z-w)^{-1+\alpha} \sigma_{\alpha}^{-}(w) & \partial \bar{Z}(z) \tau_{\alpha}^{-}(w) \sim(z-w)^{-2+\alpha} \sigma_{\alpha}^{-}(w) \\
\partial Z(z) \widetilde{\omega}_{\alpha}^{-}(w) \sim(z-w)^{-1+\alpha} \tilde{\tau}_{\alpha}^{-}(w) & \partial \bar{Z}(z) \tilde{\tau}_{\alpha}^{-}(w) \sim(z-w)^{-1-\alpha} \widetilde{\omega}_{\alpha}^{-}(w)
\end{array}
$$

The above OPEs suggest the following identifications among twist- and anti-twist fields

$$
\begin{equation*}
\sigma_{\alpha}^{-}(z)=\sigma_{1-\alpha}^{+}(z) \quad \tau_{\alpha}^{-}(z)=\tau_{1-\alpha}^{+}(z) \quad \tilde{\tau}_{\alpha}^{-}(z)=\tilde{\tau}_{1-\alpha}^{+}(z) \tag{A.2}
\end{equation*}
$$

which can be easily generalised to higher excited twist fields.

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[^0]:    *Speaker.

[^1]:    ${ }^{1}$ At the moment, the lower bound of the string scale is at 4.5 TeV [5-7].
    ${ }^{2}$ For recent reviews, see [43,44].
    ${ }^{3}$ For some preliminary work on excited bosonic twist field correlators, see [52-58].

[^2]:    ${ }^{4}$ For similar work on the T-dual side with magnetized branes, see [90].

[^3]:    ${ }^{5}$ Note that we applied the doubling trick that extends the world-sheet from the upper half complex plane to the whole complex plane.

[^4]:    ${ }^{6}$ For analogous results in the T-dual IIB framework with magnetized D-branes, see [90, 91$]$.

[^5]:    ${ }^{7}$ The setup with four bosonic twist field insertions has two independent world-sheet contours which we choose without loss of generality to be 0 to $x$ and $x$ to 1 .

[^6]:    ${ }^{8}$ Here we used $S L(2, \mathbf{R})$ invariance to fix the twist field insertions to $x_{1}=0, x_{2}=x, x_{3}=1$ and $x_{4}=x_{\infty}=\infty$. Furthermore we suppress all $x_{\infty}$ dependence.

