PROCEEDINGS OF SCIENCE

On Bimixing and GUT's

Davide Meloni*
Dipartimento di Matematica e Fisica,
Università degli studi Roma Tre,
INFN, Sezione di Roma Tre
via della Vasca Navale 84, 00146 Roma (Italy)
E-mail: meloni@fis.uniroma3.it

In this talk I briefly discuss the present status of models of neutrino mixing. In particular I review the concept of Bimaximal Mixing corrected by terms arising from the charged lepton mass matrix diagonalization. Two versions of particular GUT models, one based on $SU(5) \times S_4$ and one on $SO(10)$, will be analysed in details.

The European Physical Society Conference on High Energy Physics
22–29 July 2015
Vienna, Austria

*Speaker.

http://pos.sissa.it/

PoS(EPS-HEP2015)051

1. Introduction

With the continuous improvement of the data, most of the models for neutrino masses and mixing have been discarded by experiment. However, the surviving ones still span a wide range of possibilities ranging from a maximum of symmetry, as those with discrete non-abelian flavour groups (for reviews, see, for example, Refs. [1, 2, 3]), to the opposite extreme of Anarchy [4, 5, 6, 7].

Models based on discrete flavour groups were motivated by the fact that the data suggest some special mixing patterns as good first approximations (like Tri-Bimaximal (TB) or Golden Ratio (GR) or Bi-Maximal (BM) mixing), all having $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 0$ and differing by the value of the solar angle $\sin^2\theta_{12}$. The relatively large measured value of $\sin\theta_{13}$ has disfavoured TB and GR models because they in general predict too small corrections for $\sin\theta_{13}$. Instead in most models of BM the measured value of $\theta_{13} \sim 9^\circ$ [8] is more natural [1]. Here I focus on BM mixing [14]. The mixing matrix has the form

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1.1}$$

corresponding to the following mass matrix:

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}, \tag{1.2}$$

where x,y and z are three complex numbers. One can consider the possibility that BM is the mixing in the neutrino sector and that the rather large corrective terms to θ_{12} and θ_{13} arise from the diagonalization of the charged lepton mass matrix, as obtained in models based on the discrete symmetry S_4 [15, 16]. This idea is in agreement with the well-known empirical quark-lepton complementarity relation [17]-[18], $\theta_{12} + \theta_C \sim \pi/4$, where θ_C is the Cabibbo angle or, to be less optimistic, with the "weak" complementarity relation $\theta_{12} + \mathcal{O}(\theta_C) \sim \pi/4$. In addition, the measured value of θ_{13} is itself of order θ_C: $\theta_{13} \sim \theta_C/\sqrt{2}$.

In this talk I discuss two examples of GUT models of BM. One is based on $SU(5)$ [16] and realizes the program of imposing the BM structure in the neutrino sector and then correcting it by terms arising from the diagonalization of charged lepton masses. The other is an $SO(10)$ model based on Type-II see-saw [19], where the origin of BM before diagonalization of charged leptons is left unspecified.

[1] An intense work to interpret the new data span a wide range of possibilities: suitable modifications of the minimal TB models [9, 10], larger symmetries that already at LO lead to non vanishing θ_{13} and non maximal θ_{23} [11], models where the flavour group and a generalised CP transformation are combined in a non trivial way [12, 13].

2. A SUSY $SU(5)$ model with S_4 discrete symmetry

This is a variant of the SUSY $SU(5)$ model in 4+1 dimensions with a flavour symmetry $S_4 \otimes Z_3 \otimes U(1)_R \otimes U(1)_{FN}$ [15, 16], where $U(1)_R$ implements the R-symmetry while $U(1)_{FN}$ is a Froggatt-Nielsen (FN) symmetry [20] that induces the hierarchies of fermion masses and mixings. The particle assignments are displayed in Tab.1.

`Field`	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_ν	ξ_ν	φ_ℓ	χ_ℓ	θ	θ'	φ_ν^0	ξ_ν^0	ψ_ℓ^0	χ_ℓ^0
SU(5)	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1	1	1
S_4	3_1	1	1	1	1	1	3_1	1	3_1	3_2	1	1	3_1	1	2	3_2
Z_3	ω	ω	1	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	1	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	2	2	2	2
$U(1)_{FN}$	0	2	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0
	`br`	`bu`	`bu`	`br`	`bu`	`bu`	`br`	`br`	`br`	`br`	`br`	`br`	`br`	`br`	`br`	`br`

Table 1: *Matter assignment of the model. The symbol* `br`(`bu`) *indicates that the corresponding fields live on the brane (bulk).*

The first two generation tenplets T_1 and T_2 and the Higgs H_5 and $H_{\bar{5}}$ are in the bulk while all the other ones are on the brane at $y = 0$; this introduces some extra hierarchy for some of the couplings [21]-[24]. At leading order (LO) the S_4 symmetry is broken down to suitable different subgroups in the charged lepton sector and in the neutrino sector by the VEV's of the flavons φ_ν, ξ_ν, φ_ℓ and χ_ℓ (whose proper alignment is implemented in a natural way by the driving fields φ_ν^0, ξ_ν^0, ψ_ℓ^0, χ_ℓ^0). The VEVs of the θ and θ' fields break the FN symmetry. As a result, at LO the charged lepton masses are diagonal and exact BM is realized for neutrinos. Corrections to diagonal charged leptons and to exact BM are induced by vertices of higher dimension in the Lagrangian, suppressed by powers of a large scale Λ. We adopt the definitions:

$$\frac{v_{\varphi_\ell}}{\Lambda} \sim \frac{v_\chi}{\Lambda} \sim \frac{v_{\varphi_\nu}}{\Lambda} \sim \frac{v_\xi}{\Lambda} \sim \frac{\langle\theta\rangle}{\Lambda} \sim \frac{\langle\theta'\rangle}{\Lambda} \sim s \equiv \lambda_C\,, \tag{2.1}$$

where $s = \dfrac{1}{\sqrt{\pi R \Lambda}}$ is the volume suppression factor. It turns out that this simple choice leads to a good description of masses and mixings.

2.1 Charged lepton mass matrices and neutrinos

For the charged lepton masses we have the following mass matrix:

$$m_e \sim \begin{pmatrix} a_{11}\lambda^5 & a_{21}\lambda^4 & a_{31}\lambda^2 \\ a_{12}\lambda^4 & -c\lambda^3 & \ldots\ldots \\ a_{13}\lambda^4 & c\lambda^3 & a_{33}\lambda \end{pmatrix} \lambda, \tag{2.2}$$

where all matrix elements are multiplied by generic coefficients of $\mathcal{O}(1)$. The corresponding lepton rotation is given by:

$$U_\ell \sim \begin{pmatrix} 1 & u_{12}\lambda & u_{13}\lambda \\ -u_{12}^*\lambda & 1 & 0 \\ -u_{13}^*\lambda & -u_{12}^* u_{13}^*\lambda^2 & 1 \end{pmatrix}, \tag{2.3}$$

(u_{ij} again of $\mathcal{O}(1)$) so that $\theta^{\ell}_{23}=0$ in this approximation.

The neutrino sector of the model is unchanged with respect to Ref.[16]. At LO, the mass matrix of eq.(1.2) is obtained from the Weinberg operator, so the results for the mixing angles are easily derived:

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}|u_{12}-u_{13}|\lambda \qquad \sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\,Re(u_{12}+u_{13})\lambda \qquad \sin^2\theta_{23} = \frac{1}{2}+\mathcal{O}(\lambda^2)\,.$$

We see that, with $\lambda \sim \lambda_C$, the model realizes the "weak" complementarity relation and the experimental fact that $\sin\theta_{13}$ is of the same order than the shift of $\sin^2\theta_{12}$ from the BM value of 1/2, both of order λ_C.

3. Bimaximal mixing in a $SO(10)$ GUT model

In $SO(10)$ the main added difficulty with respect to $SU(5)$ is that on generation of fermion belongs to the 16-dimensional representation, so that one cannot take advantage of the properties of the $SU(5)$-singlet right-handed neutrinos. A possible strategy to separate charged fermions and neutrinos is to assume the dominance of type-II see-saw with respect to the more usual type-I see-saw. In models of this type, the neutrino mass formula becomes

$$\mathcal{M}_\nu \sim f v_L\,, \tag{3.1}$$

where v_L is the vev of the $B-L=2$ triplet in the $\overline{\mathbf{126}}$ Higgs field and f is the Yukawa coupling matrix of the $\mathbf{16}$ with the same $\overline{\mathbf{126}}$.

For generic eigenvalues m_i, the most general matrix that is diagonalized by the BM unitary transformation is given by:

$$f = U^*_{BM}\mathrm{diag}(m_1,m_2,m_3)U^\dagger_{BM} \quad , \tag{3.2}$$

where U_{BM} is the BM mixing matrix given in eq.(1.1). However, a similar transformation can also be used with U_{BM} replaced by U_{TB}; as a result the matrices f obtained with this two different approaches are related by a change of the charged lepton basis induced by a unitary matrix. As one could decide to work in a basis where the matrix f is diagonalised by the TB matrix or by BM matrix, the result of a fit performed in one basis should lead to the same χ^2 than the fit in other basis, so the χ^2 cannot decide whether TB or BM is a better starting point. Then we need another "variable" to compare whether the data prefer to start from TB or BM. One possibility is to measure the amount of fine-tuning needed to fit a set of data; to this aim, a parameter d_{FT} was introduced in Ref. [19]:

$$d_{FT} = \sum |\frac{par_i}{err_i}| \tag{3.3}$$

where err_i is the "error" of a given parameter par_i defined as the shift from the best fit value that changes the χ^2 by one unit, with all other parameters fixed at their best fit values.

A study of the fine tuning parameter when the fit is repeated with the same data except for $\sin^2\theta_{13}$, which is moved from small to large, shows that the fine tuning increases (decreases) with $\sin\theta_{13}$ for TB (BM), as shown in Fig.(1). A closer look at the figure reveals that both BM and TB scenarios are compatible with the data for similar values of the fine tuning parameter, especially for relatively large θ_{13}. We have also observed that high d_{FT} values are predominantly driven by the smallness of the electron mass combined with its extraordinary measurement precision.

PoS(EPS-HEP2015)051

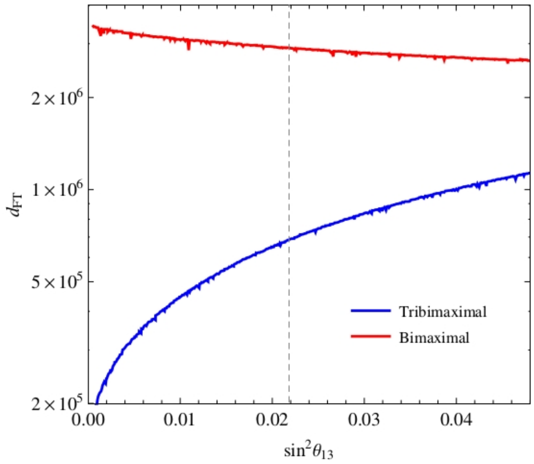

Figure 1: *In the* $SO(10)$ *model the fine tuning parameter* d_{FT} *increases (decreases) with* $\sin^2\theta_{13}$ *in the TB (BM) cases. For the physical value* $\sin^2\theta_{13} \sim 0.022$ *it is about 4 times larger in the BM case.*

4. Summary and Conclusion

I have discussed two examples of GUT models of BM, one based on $SU(5) \times S_4$ and one on $SO(10)$. In the $SU(5)$ model the broken flavour symmetry imposes the BM structure in the neutrino sector (then corrected by the diagonalization of charged lepton masses) and is useful to implement the weak form of complementarity. In the $SO(10)$ model based on Type-II see-saw no clear preference for BM or TB as good LO approximation of the data emerged.

References

[1] G. Altarelli and F. Feruglio, , Rev. Mod. Phys. **82** 2701 (2010) , arXiv:1002.0211.

[2] H. Ishimori *et. al.*, Prog. Theor. Phys. Suppl. 183, 1 (2010) , arXiv:1003.3552; S. F. King and C. Luhn, arXiv:1301.1340; S. F. King *et. al.*, arXiv:1402.4271.

[3] W. Grimus and P. O. Ludl, J. Phys. A 45, 233001 (2012) , arXiv:1110.6376.

[4] L. J. Hall, H. Murayama, and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000) , arXiv:hep-ph/9911341.

[5] A. de Gouvea and H. Murayama, Phys. Lett. **B573**, 94 (2003) , arXiv:hep-ph/0301050.

[6] A. de Gouvea and H. Murayama, arXiv:1204.1249.

[7] J. Bergstrom, D. Meloni and L. Merlo, Phys. Rev. D **89** (2014) 9, 093021 [arXiv:1403.4528 [hep-ph]].

[8] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP **1411**, 052 (2014) [arXiv:1409.5439 [hep-ph]].

[9] E. Ma and D. Wegman, Phys. Rev. Lett. 107, 061803 (2011) , arXiv:1106.4269; S. F. King and C. Luhn, JHEP 09, 042 (2011) , arXiv:1107.5332; F. Bazzocchi, arXiv:1108.2497; S. Antusch, S. F. King, C. Luhn and M. Spinrath, Nucl. Phys. B **856** (2012) 328 [arXiv:1108.4278 [hep-ph]]; I. de Medeiros Varzielas and L. Merlo, JHEP **02**, 062 (2011) , arXiv:1011.6662; F. Bazzocchi and L. Merlo, Fortsch. Phys. **61**, 571 (2013) [arXiv:1205.5135 [hep-ph]].

[10] Y. Lin, Nucl. Phys. **B824**, 95 (2010) , arXiv:0905.3534.

[11] R. d. A. Toorop, F. Feruglio and C. Hagedorn, Phys. Lett. B **703** (2011) 447 [arXiv:1107.3486 [hep-ph]]; Nucl. Phys. B**858**, 437 (2012) , arXiv:1112.1340; S. F. King, C. Luhn and A. J. Stuart, Nucl. Phys. B **867** (2013) 203 [arXiv:1207.5741 [hep-ph]]; C. Hagedorn and D. Meloni, Nucl. Phys. B **862** (2012) 691 [arXiv:1204.0715 [hep-ph]]; S. F. King, T. Neder and A. J. Stuart, Phys. Lett. B **726** (2013) 312 [arXiv:1305.3200 [hep-ph]];

[12] G. Ecker, W. Grimus and H. Neufeld, J. Phys. A **20** (1987) L807; Int. J. Mod. Phys. A **3** (1988) 603; W. Grimus and M. N. Rebelo, Phys. Rept. **281** (1997) 239 [hep-ph/9506272]; W. Grimus and L. Lavoura, Phys. Lett. B **579** (2004) 113 [hep-ph/0305309]; M. Holthausen, M. Lindner and M. A. Schmidt, JHEP **1304** (2013) 122 [arXiv:1211.6953 [hep-ph]]; F. Feruglio, C. Hagedorn and R. Ziegler, JHEP **1307** (2013) 027 [arXiv:1211.5560 [hep-ph]]; Eur. Phys. J. C **74** (2014) 2753 [arXiv:1303.7178 [hep-ph]].

[13] I. de Medeiros Varzielas and D. Emmanuel-Costa, Phys. Rev. D **84** (2011) 117901 [arXiv:1106.5477 [hep-ph]]; G. Bhattacharyya, I. de Medeiros Varzielas and P. Leser, Phys. Rev. Lett. **109** (2012) 241603 [arXiv:1210.0545 [hep-ph]]; M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **681** (2009) 444 [arXiv:0904.1721 [hep-ph]]; G. J. Ding and S. F. King, Phys. Rev. D **89** (2014) 9, 093020 [arXiv:1403.5846 [hep-ph]]; L. L. Everett, T. Garon and A. J. Stuart, arXiv:1501.04336 [hep-ph]; C. C. Li and G. J. Ding,arXiv:1503.03711 [hep-ph]; A. Di Iura, C. Hagedorn and D. Meloni, arXiv:1503.04140 [hep-ph].

[14] G. Altarelli, P. A. N. Machado and D. Meloni, PoS CORFU **2014** (2015) 012 [arXiv:1504.05514 [hep-ph]].

[15] G. Altarelli, F. Feruglio and L. Merlo, JHEP **0905**, 020 (2009) [arXiv:0903.1940 [hep-ph]].

[16] D. Meloni, JHEP **1110** (2011) 010 [arXiv:1107.0221 [hep-ph]].

[17] M. Raidal, Phys. Rev. Lett. **93**, 161801 (2004) [hep-ph/0404046].

[18] S. Antusch, S. F. King and R. N. Mohapatra, Phys. Lett. B **618**, 150 (2005) [hep-ph/0504007].

[19] G. Altarelli and G. Blankenburg, JHEP **1103** (2011) 133 [arXiv:1012.2697 [hep-ph]].

[20] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277.

[21] E. Witten, Nucl. Phys. B **258** (1985) 75; Y. Kawamura, Prog. Theor. Phys. **105** (2001) 999 [arXiv:hep-ph/0012125]; A. E. Faraggi, Phys. Lett. B **520** (2001) 337 [arXiv:hep-ph/0107094] and references therein.

[22] L. J. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003 [arXiv:hep-ph/0103125]; Y. Nomura, Phys. Rev. D **65** (2002) 085036 [arXiv:hep-ph/0108170]; L. J. Hall and Y. Nomura, Phys. Rev. D **66** (2002) 075004 [arXiv:hep-ph/0205067].

[23] G. Altarelli and F. Feruglio, Phys. Lett. B **511** (2001) 257 [arXiv:hep-ph/0102301]; A. Hebecker and J. March-Russell, Nucl. Phys. B **613** (2001) 3 [arXiv:hep-ph/0106166]; A. Hebecker and J. March-Russell, Phys. Lett. B **541** (2002) 338 [arXiv:hep-ph/0205143].

[24] G. Altarelli, F. Feruglio, C. Hagedorn, JHEP **0803**, 052-052 (2008). [arXiv:0802.0090 [hep-ph]].

PoS(EPS-HEP2015)051