

Composite resonances and their impact on the electroweak chiral Lagrangian

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In this talk we study the low-energy effective couplings generated by strongly-coupled electroweak models that contain heavy composite resonances. Invariance under $SU(2)_L \times SU(2)_R$ is a key ingredient in the construction of the resonance action. For simplicity, in these proceedings we focus our attention on the impact of a heavy colourless vector V , which transforms as a triplet under the custodial group. More precisely, we study the couplings that are relevant for the vector form-factors of the $L + R$ current into two electroweak Goldstones and into two Standard Model fermions, which contribute to the oblique parameters S and T and the anomalous $Z \rightarrow f\bar{f}$ couplings, respectively. Our predictions are compatible with bounds from direct and indirect searches for $M_V \gtrsim 1.5$ TeV. Finally, although we consider an antisymmetric tensor formalism to describe the vector resonance, we derive the equivalent action in the Proca four-vector representation and show that the predictions for low-energy couplings and form-factors are identical, as expected.

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1. Impact of heavy resonances on the low-energy electroweak effective theory

So far the Large Hadron Collider (LHC) has not found any trace of beyond the Standard Model (BSM) states with masses below 1 TeV. Likewise, no significant deviation has been observed in the low-energy interactions between Standard Model (SM) particles. Effective field theories are then the natural approach. In this talk [1, 2] we discuss the possibility of strongly-coupled BSM scenarios with the approximate custodial symmetry invariance of the SM, exact in the SM scalar sector. We develop an invariant Lagrangian under $\mathcal{G} = SU(2)_L \times SU(2)_R$, which spontaneously breaks down to the custodial subgroup $\mathcal{H} = SU(2)_{L+R}$ and generates the electroweak (EW) would-be Goldstone bosons φ^a , described a unitary 2×2 matrix $U(\varphi)$. In these (non-linear) EW chiral Lagrangian with a light Higgs (ECLh), the low-energy amplitude \mathcal{M} has an expansion in powers of infrared scales p (external momenta and SM masses) of the form (e.g., for $2 \rightarrow 2$ processes) [2, 3, 4, 5, 6],

$$\mathcal{M} \sim \underbrace{\frac{p^2}{v^2}}_{\text{LO (tree)}} + \left(\underbrace{a_k^r}_{\text{NLO (tree)}} - \underbrace{\frac{\Gamma_k}{16\pi^2} \ln \frac{p}{\mu}}_{\text{NLO (1-loop)}} + \dots \right) \frac{p^4}{v^4} + \mathcal{O}(p^6). \quad (1.1)$$

The EW effective theory (EWET) Lagrangian operators can be sorted out based on their chiral dimension:

$$\mathcal{L}_{\text{EWET}} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \quad (1.2)$$

where the operators in \mathcal{L}_i are of $\mathcal{O}(p^i)$ [2, 3, 4, 5]. Covariant derivatives and masses are $\mathcal{O}(p)$ [7] and each fermion field scales like $\mathcal{O}(p^{1/2})$ in naive dimensional analysis (NDA) [2, 4, 5, 8]. The \mathcal{G} -invariant operators in $\mathcal{L}_{\text{EWET}}$ are built with the Goldstone tensors $U(\varphi)$, functions \mathcal{F}_k of the Higgs singlet h , its derivatives $\partial_{\mu_1} \dots \partial_{\mu_n} h$, the gauge fields and the SM fermions ψ [8, 9, 10, 11, 12, 13]. From the chiral counting point of view \mathcal{L}^{SM} would be $\mathcal{O}(p^2)$ but its underlying renormalizable structure makes all $\Gamma_k = 0$ and ensures the absences of higher-dimension divergences [6, 14]. The most important contributions to a given process are given by the operators of lowest chiral dimension. The leading order (LO) contribution is $\mathcal{O}(p^2)$ and is given by tree-level diagrams with only \mathcal{L}_2 vertices. Likewise, the one-loop contribution with only \mathcal{L}_2 vertices is $\mathcal{O}(p^4)$; it is suppressed in (1.1) with respect to the LO by a factor $p^2/\Lambda_{\text{NL}}^2$, with $\Lambda_{\text{NL}}^2 \sim 16\pi^2 v^2 \Gamma_k^{-1} \gtrsim 3 \text{ TeV}$ (with $v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$). This suppression factor is related to the non-linearity of the ECLh and $\Lambda_{\text{NL}} \rightarrow \infty$ when the Higgs can be embedded in a complex doublet Φ [6].¹

In these proceedings [1, 2] we focus our attention on the tree-level next-to-leading order (NLO) contributions. They are $\mathcal{O}(p^4)$ and are provided by tree-level diagrams with one \mathcal{L}_4 vertex with low-energy coupling a_k (LEC) and an arbitrary number of \mathcal{L}_2 vertices. They get contributions from tree-level heavy resonance exchanges. At low energies, these $\mathcal{O}(p^4)$ terms in (1.1) are typically suppressed with respect to the LO amplitude, $\mathcal{O}(p^2)$, by a factor $a_k p^2/v^2 \sim p^2/M_R^2$ [1, 2, 15, 16].

¹Ref. [14] provides a geometrical interpretation in terms of the curvature of metric of the internal weak space of the Higgs. In the flat-space limit one has $\Lambda_{\text{NL}} \rightarrow \infty$. Linear-Higgs scenarios with a complex Higgs doublet Φ correspond to this case. True “non-linear models” are defined by a non-zero curvature, not by their (non-linear) representation.

At high energies, one must include both the light dof (SM particles) and the possible composite resonances as active degrees of freedom (dof) in the Lagrangian [1, 2, 17]:

$$\mathcal{L} = \mathcal{L}_{\text{non-res}} + \mathcal{L}_R, \quad (1.3)$$

where $\mathcal{L}_{\text{non-res}}$ contains only SM fields and \mathcal{L}_R is the part of the Lagrangian that also contains resonances [1]. The part of the interaction Lagrangian \mathcal{L}_R relevant for our analysis of the \mathcal{L}_4 LECs is given by the terms linear in the resonance fields, $\Delta\mathcal{L}_R = R \mathbb{O}_{p^2}[\chi, \psi]$ [1, 2, 15, 16, 17], with χ (ψ) referring to the light bosonic (fermionic) fields. The tensor $\mathbb{O}_{p^2}[\chi, \psi]$ that couples the heavy resonance R to the light dof is going to provide the first correction to the low-energy ECLh by means of diagrams where one has a heavy resonance propagator $\sim 1/M_R^2$ exchanged between two vertices with $\mathbb{O}_{p^2}[\chi, \psi]$. This gives an EWET operator of $\mathcal{O}(p^4)$. At low energies, resonance operators with tensors $\mathbb{O}[\chi, \psi]$ of a higher order in p or containing two or more R fields contribute only to $\mathcal{L}_{\hat{d}}$ with $\hat{d} > 4$.

The tree-level contribution to $\mathcal{L}_{\text{EWET}}[\chi, \psi]$ is given by the underlying high-energy action $S[\chi, \psi, R]$ with the resonance fields R evaluated at the classical solution $R_{\text{cl}}(\chi, \psi)$ of their equations of motion (EoM). Solving the resonance EoM and expanding their solutions in powers of momenta for $p \ll M_R$, one can write the heavy fields as local operators of the EWET dof [15]. This prediction for the contribution to the low-energy ECLh can be complemented through the consideration of ultraviolet-completion hypotheses (sum-rules [18, 19], unitarity [16], asymptotic form-factor counting rules [20]...). This imposes constraints on the resonance couplings that then turn into predictions for the low-energy theory.

2. Phenomenological example: vector form-factors

Let us illustrate this with a basic example. We consider a colourless triplet vector resonance V in a composite theory with the same symmetries of the scalar sector of the SM –invariance under parity and charge conjugation–, with its high energy interaction provided by the Lagrangian [1, 2],

$$\Delta\mathcal{L}_V^{(A)} = \langle V_{\mu\nu} \mathbb{O}_V^{\mu\nu} \rangle, \quad \mathbb{O}_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}} [u^\mu, u^\nu] + \frac{c_1^V}{2} (\nabla^\mu J_V^\nu - \nabla^\nu J_V^\mu) / v^2, \quad (2.1)$$

with $\langle \dots \rangle$ for the matrix trace, $u_\mu = iu(D_\mu U)^\dagger u$, the combinations $f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u \hat{B}^{\mu\nu} u^\dagger$ of the left and right field-strength tensors $\hat{W}^{\mu\nu}$ and $\hat{B}^{\mu\nu}$, respectively, and $U = u^2 = \exp\{i\varphi^a \sigma^a / v\}$ [21, 22]. The precise definition of the covariant derivatives D_μ and ∇_μ can be found in [21, 22]. The tensor $J_V^\mu = -\text{Tr}_D\{\xi \bar{\xi} \gamma^\mu\}$ introduces the fermionic vector current in a covariant way, with $\xi = u\psi_R + u^\dagger\psi_L$ given by the $SU(2)_{R,L}$ doublets $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$, with $\psi = (t, b)^T$ (other SM doublets can be also added [6]) and the Dirac trace Tr_D . The superscript (A) refers to the antisymmetric tensor formulation employed for the spin-1 resonance [15]. The full Lagrangian may contain additional operators not relevant for the form-factors analyzed in this talk [2]. Integrating out V one gets a contribution to the EWET, which at lowest order is given by

$$\Delta\mathcal{L}_{\text{EWET}}^{\text{from } V} = \frac{\langle \mathbb{O}_V^{\mu\nu} \rangle^2}{2M_V^2} - \frac{\langle \mathbb{O}_V^{\mu\nu} \mathbb{O}_{V\mu\nu} \rangle}{M_V^2} = \underbrace{-i \frac{F_V G_V}{4M_V^2} \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle}_{=i\mathcal{F}_3/2} - \underbrace{\frac{F_V c_1^V}{\sqrt{2}M_V^2} \langle f_+^{\mu\nu} \nabla_\mu J_{V\nu} / v^2 \rangle}_{=\mathcal{F}^X \psi^2} + \dots \quad (2.2)$$

with the dots standing for other effective operators not relevant in these proceedings. For the Higgsless part, one has $\mathcal{F}_3 = a_2 - a_3$ in Longhitano's notation of [9, 10]. In what follows, we will focus on the Higgsless sector and $\mathcal{F}_3, \mathcal{F}^{X\psi^2}, F_V, G_V$ and c_1^V simply represent coupling constants.

The resonance Lagrangian (2.1) provides the vector form-factors of the $L + R$ current into two-Goldstones and into two-fermions [2, 23, 21, 22]:

$$\mathbb{F}_{\varphi\varphi}^v(q^2) = 1 + \frac{F_V G_V}{v^2} \frac{q^2}{M_V^2 - q^2}, \quad \mathbb{F}_{f\bar{f}}^v(q^2) = 1 - \frac{\sqrt{2} F_V c_1^V}{v^2} \frac{q^2}{M_V^2 - q^2}, \quad (2.3)$$

with momentum transfer q^μ . The square form-factors $|\mathbb{F}_{ii}^v(s)|^2$ contribute to the S -parameter at one-loop through the Peskin-Takeuchi sum-rule on the left-right correlator Π_{W^3B} [19]. If one requires that these form-factors give a ultraviolet-convergent contribution to the sum-rule, they must vanish at $q^2 \rightarrow \infty$ and one obtains short-distance (SD) constraints [16, 23, 21, 22] and predictions for the LECs [1, 2, 16]:

$$F_V G_V = v^2 \quad \longrightarrow \quad \mathcal{F}_3 = (a_2 - a_3) = -\frac{F_V G_V}{2M_V^2} \stackrel{\text{SD constr.}}{=} -\frac{v^2}{2M_V^2}. \quad (2.4)$$

For $M_V > 1.5$ TeV one finds the bound

$$-1.3 \cdot 10^{-2} < \mathcal{F}_3 = (a_2 - a_3) < 0. \quad (2.5)$$

One can obtain analogous bounds for the LEC $\mathcal{F}^{X\psi^2} = v^2/(2M_V^2)$ by demanding a similar SD behaviour $\mathbb{F}_{f\bar{f}}^v(q^2) \xrightarrow{q^2 \rightarrow \infty} 0$ to the fermion form-factor, which would give $\sqrt{2} F_V c_1^V = -v^2$.

2.1 $\mathbb{F}_{\varphi\varphi}^v$ form-factor: S-parameter

The impact of the bosonic form-factor $F_{\varphi\varphi}^v$ on the oblique parameters S and T was studied in a dispersive one-loop resonance analysis [23, 21, 22], where the lightest triplet vector (V) and axial-vector (A) resonances were taken into account. Therein, the contribution from the Goldstone and Higgs absorptive channels was incorporated. In particular the $F_{\varphi\varphi}^v(q^2)$ determined the contribution from the $\varphi\varphi$ and $B\varphi$ cuts to the S and T parameter, respectively [22]. We studied asymptotically-free strongly coupled theories, where Π_{W^3B} satisfies the two Weinberg Sum Rules (WSRs), and scenarios with weaker ultraviolet (UV) conditions (only the 1st WSR applies) such as Conformal [24] or Walking [25] Technicolour, obtaining the 68% confidence level determinations [22]:

$$\begin{aligned} 0.97 < \kappa_W = M_V^2/M_A^2 < 1, \quad M_V > 5 \text{ TeV} \quad (\text{1st \& 2nd WSR}), \\ 0.84 < \kappa_W < 1.30, \quad M_V > 1.5 \text{ TeV} \quad (\text{only 1st WSR, for } 0.5 < M_V/M_A < 1), \end{aligned} \quad (2.6)$$

where κ_W denotes the hWW (and $h\varphi\varphi$) coupling in SM units ($\kappa_W^{\text{SM}} = 1$).

2.2 $\mathbb{F}_{f\bar{f}}^v$ form-factor: $Z \rightarrow f\bar{f}$ anomalous couplings

The v_f and a_f constants that parametrize the $Z \rightarrow f\bar{f}$ decay have the form [26],

$$v_f = T_3^f - 2Q_f \sin^2 \theta_W + (\delta g_R^{Zf} + \delta g_L^{Zf}), \quad a_f = T_3^f + (\delta g_R^{Zf} - \delta g_L^{Zf}), \quad (2.7)$$

with $T_3^f = +1/2$, $T_3^b = -1/2$, the electric charge Q_f , the weak angle θ_W and the new physics parametrized through the $\delta g_{R,L}^{Zf}$, given in our low-energy description by

$$|\delta g_{R,L}^{Zf}| = |\mathcal{F}^{X\psi^2}| \cos(2\theta_W) m_Z^2/v^2, \quad (2.8)$$

in agreement with current bounds of $\mathcal{O}(10^{-3})$ [27] for the fermion coupling $\mathcal{F}^{X\psi^2} \sim v^2/(2M_V^2) < 1.3 \cdot 10^{-2}$ that one gets from the previous resonance coupling estimate $\sqrt{2}F_V c_1^V = -v^2$, the bound $M_V > 1.5$ TeV [22] and the experimental value $\cos(2\theta_W) m_Z^2/v^2 = 0.07$.

3. Equivalent Proca four-vector representation

Through an appropriate duality transformation in the generating functional it is possible to rewrite the underlying resonance Lagrangian $\mathcal{L}^{(A)}$ in (2.1) as a Proca Lagrangian $\mathcal{L}^{(P)}$ in terms of four-vector field \hat{V}_μ and its field strength tensor $\hat{V}_{\mu\nu} = \nabla_\mu \hat{V}_\nu - \nabla_\nu \hat{V}_\mu$. A similar procedure [2, 16, 28] can be applied to models where the resonances are introduced as gauge fields [29]. In the process, additional non-resonant operators with only light dof are generated, which guarantee a proper UV behaviour. [16, 23, 28]. On-shell, this duality can be read as $V^{\alpha\beta} = \hat{V}^{\alpha\beta}/M_V$ and $\nabla_\rho V^{\rho\mu} = -M_V \hat{V}^\mu$. In our particular case, the duality transformation [2, 28] changes the antisymmetric tensor Lagrangian (2.1) into

$$\begin{aligned} \mathcal{L}^{(A)} \longrightarrow \mathcal{L}^{(P)} = & \langle \hat{V}_{\mu\nu} \left(\frac{f_{\hat{V}}}{2\sqrt{2}} f_+^{\mu\nu} + \frac{ig_{\hat{V}}}{2\sqrt{2}} [u^\mu, u^\nu] \right) + \hat{V}_\mu \left(\zeta_{\hat{V}} J_V^\mu/v^2 \right) \rangle \\ & - \left\langle \left(\frac{f_{\hat{V}}}{2\sqrt{2}} f_+^{\mu\nu} + \frac{ig_{\hat{V}}}{2\sqrt{2}} [u^\mu, u^\nu] \right)^2 \right\rangle, \end{aligned} \quad (3.1)$$

with $f_{\hat{V}} = F_V/M_V$, $g_{\hat{V}} = G_V/M_V$ and $\zeta_{\hat{V}} = c_1^V M_V$. In the low-energy limit $p \ll M_V$, Eq. (3.1) leads to the same EWET,

$$\mathcal{L}_{\text{EWET}} = -i \frac{f_{\hat{V}} g_{\hat{V}}}{4} \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle - \frac{f_{\hat{V}} \zeta_{\hat{V}}}{\sqrt{2} M_V^2} \langle f_+^{\mu\nu} \nabla_\mu J_{V\nu}/v^2 \rangle + \dots \quad (3.2)$$

The same agreement is found for the two form-factors previously obtained in (2.3):

$$F_{\varphi\varphi}^v(q^2) = 1 + \frac{f_{\hat{V}} g_{\hat{V}}}{v^2} q^2 + \frac{f_{\hat{V}} g_{\hat{V}}}{v^2} \frac{q^4}{M_V^2 - q^2}, \quad F_{f\bar{f}}^v(q^2) = 1 - \frac{\sqrt{2} f_{\hat{V}} \zeta_{\hat{V}}}{v^2} \frac{q^2}{M_V^2 - q^2}. \quad (3.3)$$

4. Conclusions

The EWET couplings can be predicted in terms of resonance parameters; different resonance quantum numbers lead to different patterns for the LECs [1, 15, 17]. Further assumptions about the UV structure of the underlying theory can be used to refine the predictions [1, 22]. In this talk we have provided a couple of examples (oblique parameters S and T and the anomalous $Zf\bar{f}$ couplings) to show that composite resonances with masses of a few TeV ($M_R \sim 4\pi v \approx 3$ TeV) are compatible with present direct and indirect searches. The $SU(2)_L \times SU(2)_R$ chiral invariance of the ECLh leads to an appropriate low-energy suppression of tree-level NLO corrections by factors $a_k p^2/v^2 \sim p^2/M_R^2$ with respect to the LO prediction, $\mathcal{O}(p^2)$ [1, 15, 16]. Finally, we have shown the equivalence between the antisymmetric tensors $V^{\mu\nu}$ and Proca four-vectors \hat{V}^α representations for spin-1 fields [16, 28].

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