

# Matching NLO QCD Corrections in WHIZARD with the POWHEG scheme

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Building on the new automatic subtraction of NLO amplitudes in WHIZARD, we present our implementation of the POWHEG scheme to match radiative corrections consistently with the parton shower. We apply this general framework to two linear collider processes,  $e^+e^- \rightarrow t\bar{t}$  and  $e^+e^- \rightarrow t\bar{t}H$ .

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## 1. Introduction

Polarized lepton colliders (LC) operated at high energies are indispensable tools to increase the precision of various SM parameters and often complement the measurements that are possible at hadron colliders like the LHC. For numerous LC studies, the multi-purpose event generator WHIZARD [1, 2] is a commonly used simulation tool, as it allows to study beamstrahlung as well as initial-state radiation (ISR) effects. Moreover, high-multiplicity final states can be automatically and efficiently generated using O'MEGA [3]. So far, these predictions have been based on tree-level matrix elements combined with conventional parton showers to describe the effects of QCD radiation. To systematically improve this description, it is mandatory to include the next-to-leading order (NLO) and avoid double counting with the parton shower.

The rigorous matching of NLO computations with parton showers has been pioneered with MC@NLO [4]. Its main principle is the subtraction of the expansion of the parton shower from the cross section. P. Nason proposed a similar method that avoids the inherent problem of MC@NLO of producing negative weight events, in the sense that negative weights can only occur in regions where fixed-order perturbation theory fails [5]. Following the first implementation [6], the algorithm has been worked out in detail [7] and dubbed the POWHEG method (Positive Weight Hardest Emission Generator). As the hardest, i.e. highest relative  $p_T$ , emission is not generated by the attached parton shower but by the algorithm itself, it is guaranteed to maintain the NLO accuracy of the sample, irrespective of the used shower. This requires, though, that the shower respects the hardest emission, which is easily satisfied with a *veto* of higher  $p_T$  on subsequent emissions. In case the ordering variable of the shower is not  $p_T$ , soft radiation before the hardest emission has to be added as well in terms of a *truncated* shower.

Following the work of Ref. [7], the semi-automated NLO+PS event generator called POWHEG-BOX [8] has been developed. In this framework a multitude of LHC processes has been made publicly available. The drawback of the POWHEG-BOX is that it only automates parts of the algorithm meaning that adding a new process requires considerable theoretical effort from the construction of the phase space to the implementation of the matrix elements. With the advent of automated One-Loop Providers (OLPs) like GOSAM [9] or OPENLOOPS [10], it has become feasible to build a fully automated event generator using the POWHEG method. First steps in this direction were made in SHERPA [11] as well as HERWIG [12]. In this work, we sketch the automation of the POWHEG matching in WHIZARD. There also was earlier work on QED resummation matched to QED NLO calculations within WHIZARD leading to strictly positive-weight events [13, 14].

WHIZARD has recently been augmented by an automation [15] of the FKS subtraction [16, 17], which will be a key ingredient in our discussion. The impact of POWHEG matching on event shapes at a lepton collider like  $e^+e^- \rightarrow$  hadrons has first been discussed in Ref. [18], where a significant improvement in the description of the measured data from LEP has been found in almost all observables, compared to leading order (LO) with a matrix element correction. In Ref. [19], this work has been extended to consider on-shell top-pair production with semi-leptonic decays at the ILC. Although WHIZARD spear-headed many beyond the SM phenomenological studies [20–25], we will focus here on SM QCD effects.

## 2. POWHEG matching

For completeness, we briefly sketch here how POWHEG events are generated. The corresponding proofs and more detailed information can be found in Ref. [5, 7]. Contrary to the subtractive approach of MC@NLO, POWHEG is a unitary method to generate  $n$ - and  $n + 1$ -parton event samples. Hereby, we distribute the Born kinematics  $d\Phi_n$  according to the inclusive NLO cross section

$$\bar{B} = B + V + \int d\Phi_{\text{rad}} (R - C), \quad (2.1)$$

where  $V = V_0 + \int C$  is the virtual part including the analytically integrated subtraction terms  $\int C$  that are subtracted again in differential form from the real emission part  $R$ . The integral over the radiation phase space  $d\Phi_{\text{rad}}$  in Eq. (2.1) is evaluated numerically in a Monte Carlo (MC) sampling using WHIZARD's standard phase space integrator VAMP [26] together with the sampling over  $d\Phi_n$ . To obtain at least leading log (LL) correctness in  $p_T$ , we have to attach the corresponding Sudakov form factors  $\Delta(p_T)$ , yielding the probability that no emission occurs between a high scale  $p_{T\text{max}}$  and  $p_T$

$$\Delta(p_T) = \exp \left\{ - \int d\Phi_{\text{rad}} \frac{R(\Phi_{\text{rad}})}{B} \theta(k_T^2(\Phi_{\text{rad}}) - p_T^2) \right\}. \quad (2.2)$$

With these quantities, we can write down the differential cross section as

$$d\sigma = \bar{B} d\Phi_n \left( \Delta(p_T^{\text{min}}) + d\Phi_{\text{rad}} \Delta(k_T(\Phi_{\text{rad}})) \frac{R(\Phi_{\text{rad}})}{B} \right). \quad (2.3)$$

Note that the expression in parentheses in Eq. (2.3) integrates to one by the unitary construction as can be easily verified. The first term corresponds to no emission down to  $p_{T\text{min}}$  and the second to an emission at the scale  $k_T$ . This ensures that the NLO cross section is conserved, implying that the POWHEG matching only changes the spectrum. Especially, it damps the emission of soft and collinear radiation of the pure NLO prediction since  $\lim_{p_T \rightarrow 0} \Delta(p_T) = 0$ .

The ratio  $R/B$  in Eq. (2.2) is the differential splitting probability and is approximated in parton showers by universal splitting kernels. Using a process dependent ratio instead makes it significantly harder to generate  $p_T$  distributions according to this form factor. There are two ways to circumvent this problem: Obviously, one can use the universal properties of this ratio, i.e. the known soft and collinear divergence structure, to construct an overestimator  $U$  weighted with a constant factor  $N$ . Emissions are then accepted according to the probability  $R/B/(NU)$ . A different approach is the fully numerical evaluation of the exponent in Eq. (2.2) as it is done in EXSAMPLE [27]. In our implementation, we decided to use a hybrid version, where  $N$  is a grid that depends on the radiation variables multiplied with the general  $U$  functions, similar to the approach in the POWHEG-BOX. A dedicated performance and validation comparison with EXSAMPLE featuring multiple processes would be an interesting future project.

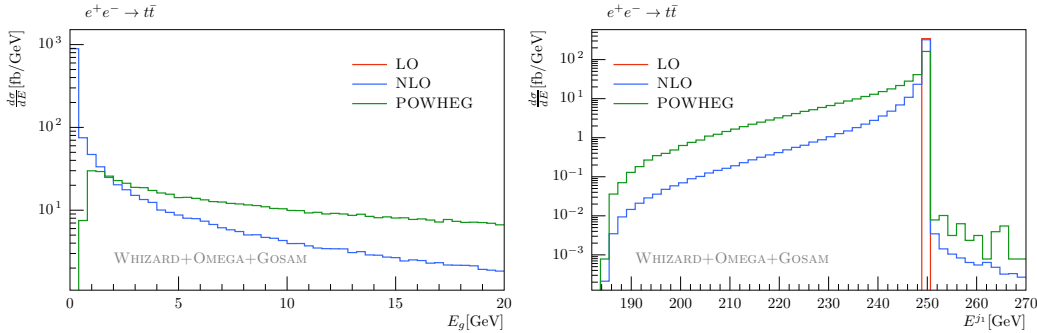
At this point, we want to stress the nice interplay between the FKS subtraction and POWHEG event generation. While we have written Eq. (2.2) in a general way, there are of course different possible singular regions  $\alpha$ , each having a different emission probability  $R_\alpha/B$  with  $R = \sum_\alpha R_\alpha$ . By having FKS at hand, we can directly retrieve these  $R_\alpha$ , which are used to divide the real part into regions with at most one soft and one collinear singularity. Analogously, the overall Sudakov

form factor will be a product of the  $\Delta_\alpha$  of the different regions. In the implementation, this results to a generation of either one or no emission in each region. The region with the largest  $p_T$  is kept in the event, also known as highest bid method, cf. Appendix B in Ref. [7].

### 3. Effects of QCD radiation on top and electroweak physics at a future linear collider

In the following, jets are possible combinations of all occurring quarks and gluons, clustered with FASTJET [28] according to an anti- $k_T$  algorithm that uses energies and spherical coordinates instead of transverse momentum and rapidities as distance measure with  $R = 1.0$ . The shown events have been simulated only up to the first emission, leaving out the subsequent simulation chain, in order to focus only on the POWHEG implementation. It has been checked, though, that processing the POWHEG events produced by WHIZARD with PYTHIA8's  $p_T$ -ordered shower [29] in the corresponding veto mode delivers reasonable physical results.

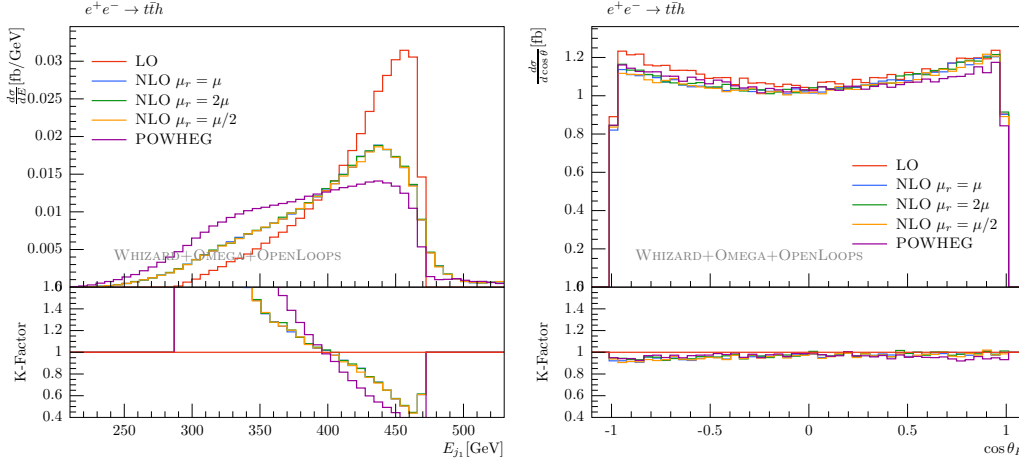
In our setup, the top mass is set to  $m_t = 172 \text{ GeV}$ . We chose  $\mu_r = m_t$  as renormalization scale. The coupling constants are  $\alpha^{-1} = 132.160$  with no running and  $\alpha_s(M_Z) = 0.118$  with a NLL running with 5 active flavors. LO and POWHEG events are unweighted during generation. In the NLO event samples, we associate Born kinematics with a weight of  $\mathcal{B} + \mathcal{V} - \sum_\alpha \mathcal{C}_\alpha$ . Together with this Born event, we generate for each singular region  $\alpha$ , a real-emission event with weight  $\mathcal{R}_\alpha$ . The histograms are generated with RIVET [30], using 500 K LO and POWHEG as well as 1500 K NLO events.



**Figure 1:** Energy distributions of the emitted gluon and of the hardest jet.

Figure 1 shows onshell  $t\bar{t}$  production at a lepton collider with  $\sqrt{s} = 500 \text{ GeV}$ . Polarization and beamstrahlung effects as well as lepton ISR are neglected. The soft gluon divergence can be seen in the NLO event samples either directly in the (unphysical) energy distribution of the gluon or indirectly in the distribution of the hardest jet, which peaks around the Born value due to mostly soft gluons. By applying the Sudakov form factor, the POWHEG events have the expected suppression of this divergence. Due to the unitary construction, this leads to an increase of the differential cross section in the remaining part of the spectrum, a well known feature of pure POWHEG distributions. As one might wish to restrict the effect of the Sudakov suppression to the area where  $p_T$  is small, the real radiation can be divided in a hard and a soft part by means of *damping factors*, bringing POWHEG formally and phenomenologically closer to MC@NLO, while maintaining the benefits

discussed in Section 2. The associated freedom in the division between hard and soft part has to be regarded as a theoretical uncertainty and will be discussed in a future work.



**Figure 2:** The energy distribution of the hardest jet and the angular distribution of the Higgs boson.

Let us now address  $e^+e^- \rightarrow t\bar{t}H$  with the same setup as above but at  $\sqrt{s} = 1000$  GeV. Figure 2 shows distributions of two observables: In the energy distribution of the hardest jet, we can see again the effect of Sudakov suppression at the high energy peak. For comparison, we also show the effect of scale variation, which, as expected, does not cover the difference between NLO and POWHEG. On the other hand, we observe that in inclusive quantities like the angular distribution of the Higgs boson, the POWHEG matching has no significant effect. This is of course only a cross check that inclusive quantities remain correct to NLO. We find that although the total K-factor at this value of  $\sqrt{s}$  is close to 1, distributions of observables that are sensitive to QCD radiation can change drastically.

#### 4. Summary & Outlook

We have presented an independent implementation of the POWHEG matching scheme that builds on the recent automation of QCD NLO corrections in WHIZARD. The key feature of the POWHEG matching, namely the suppression of the differential cross section for small relative  $p_T$ , has been reproduced and we have shown for the first time the impact of the POWHEG matching on distributions for  $e^+e^- \rightarrow t\bar{t}H$ . A more detailed analysis that focuses on the impact of damping factors and the general assessment of the associated uncertainties in various processes will follow. Our implementation is process-independent but still subject to extensive validation. It can be tested for any lepton-collider process in the current publicly available release of WHIZARD 2.2.7 but should be regarded as experimental feature and results analyzed with care. Hadron-collider processes are currently not supported but planned for the near future.

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