

# The low-lying spectrum of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

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**Pietro Giudice\***, Gernot Münster, Stefano Piemonte

*Universität Münster, Institut für Theoretische Physik,  
Wilhelm-Klemm-Str. 9, D-48149 Münster, Germany*

*E-mail: [p.giudice@uni-muenster.de](mailto:p.giudice@uni-muenster.de), [munsteg@uni-muenster.de](mailto:munsteg@uni-muenster.de),  
[spiemonte@uni-muenster.de](mailto:spiemonte@uni-muenster.de)*

**Georg Bergner**

*Universität Bern, Institut für Theoretische Physik,  
Sidlerstr. 5, CH-3012 Bern, Switzerland*

*E-mail: [bergner@itp.unibe.ch](mailto:bergner@itp.unibe.ch)*

**Istvan Montvay**

*Deutsches Elektronen-Synchrotron DESY,  
Notkestr. 85, D-22603 Hamburg, Germany*

*E-mail: [montvay@mail.desy.de](mailto:montvay@mail.desy.de)*

The spectrum of the lightest bound states in  $\mathcal{N} = 1$  supersymmetric Yang-Mills theory with  $SU(2)$  gauge group, calculated on the lattice, is presented. The masses have first been extrapolated towards vanishing gluino mass and then to the continuum limit. The final picture is consistent with the formation of degenerate supermultiplets.

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\*Speaker.

## 1. Introduction

Supersymmetry (SUSY) was first studied in the early seventies and since then it has been a central topic in particle physics. Fermions, the constituents of matter, have previously been considered fundamentally different from bosons, the particles that transmit the forces between them. In SUSY, fermions and bosons are unified.

The only way we have to study SUSY non-perturbatively from first principles is by lattice field theory approach: the theory is studied on a space-time that has been discretised onto a lattice.

The simplest non-abelian supersymmetric gauge theory, which is the subject of our study, is the  $\mathcal{N} = 1$  supersymmetric Yang-Mills (SYM) theory with gauge group  $SU(2)$ . It describes the interaction between gluons and gluinos. The theory is asymptotically free at high energies and becomes strongly coupled in the infrared limit. The mass spectrum is expected to consist of colourless bound states. If supersymmetry is unbroken, particles should belong to mass degenerate SUSY multiplets.

The investigation of the mass spectrum of the theory is intrinsically a non-perturbative problem. First predictions [1, 2] were only possible constraining the form of the low-energy effective actions by means of the symmetries of the theory. Verifying these predictions is the central task of our work. Some important results have already been obtained by our collaboration in previous studies, see [3, 4, 5]. The theory has also been studied at non-zero temperatures in [6].

## 2. $\mathcal{N} = 1$ SYM theory

The Euclidean on-shell action for  $\mathcal{N} = 1$  SYM theory in the continuum is given by:

$$S(g, m_g) = \int d^4x \left\{ \frac{1}{4} (F_{\mu\nu}^a F_{\mu\nu}^a) + \frac{1}{2} \bar{\lambda}_a (\gamma^\mu D_\mu^{ab} + m) \lambda_b - \frac{\Theta}{16\pi} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right\}. \quad (2.1)$$

The theory contains gluons ( $A_\mu(x)$  gauge fields) as bosonic particles, and gluinos ( $\lambda(x)$  fields) as their fermionic superpartners. It is an extension of pure gauge theory:  $SU(N_c)$  gauge symmetry is imposed together with a single conserved supercharge, obeying the algebra:

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu C)_{\alpha\beta} P_\mu. \quad (2.2)$$

The Majorana spinors  $Q_\alpha$  are the generators of the supersymmetry,  $C$  is the charge conjugation matrix and  $P_\mu$  the momentum operator. In our present investigations the gauge group is  $SU(2)$ , *i. e.*  $N_c = 2$ . Supersymmetry relates gluons and gluinos:

$$A_\mu(x) \rightarrow A_\mu(x) - 2i \bar{\lambda}(x) \gamma_\mu \varepsilon \quad (2.3)$$

$$\lambda^a(x) \rightarrow \lambda^a(x) - \sigma_{\mu\nu} F_{\mu\nu}^a(x) \varepsilon, \quad (2.4)$$

where  $\varepsilon$  is a global fermionic parameter (consistent with the Majorana condition), parametrising the transformation.

In Eq. 2.1 the  $\Theta$ -term has been added to the action as can also be done for QCD. It does not violate the underlying symmetries of the model. In our study only the case  $\Theta = 0$  has been considered. The role of this term to study the topological properties of the theory can be seen in [7].

### 3. $\mathcal{N} = 1$ SYM theory on the lattice

In our simulations, the gauge part of the action is discretised with a tree-level Symanzik improved action, while the gluino part is represented using a discretised version of the Dirac operator, which depends on the gauge fields in the adjoint representation, which is known as Wilson-Dirac operator. The latter depends on the hopping parameter  $\kappa$  which is inversely proportional to the gluino mass. To reduce the lattice artifacts also in the fermionic part of the action, we apply one level of stout smearing to the gauge fields in the Wilson-Dirac operator [4]. The parameter  $m$ , which introduces a bare mass for the gluino, breaks supersymmetry softly, *i. e.* the main features of the supersymmetric theory, concerning the ultraviolet renormalisability, remain intact.

There are two other sources of supersymmetric breaking: the finite volume effects [4] and the discretisation of the theory on the space-time lattice [8]. The former have been already clarified and it is totally under control in our simulations. Above a box size of about 1.2 fm (in QCD units) the statistical errors and the systematic errors of the finite size effects, with our current numerical precision, are of the same order and hence the finite size effects can be neglected. The latter is instead a fundamental problem related to the breaking of translational invariance on the lattice. Consequently, the relation 2.2 cannot be satisfied on the lattice.

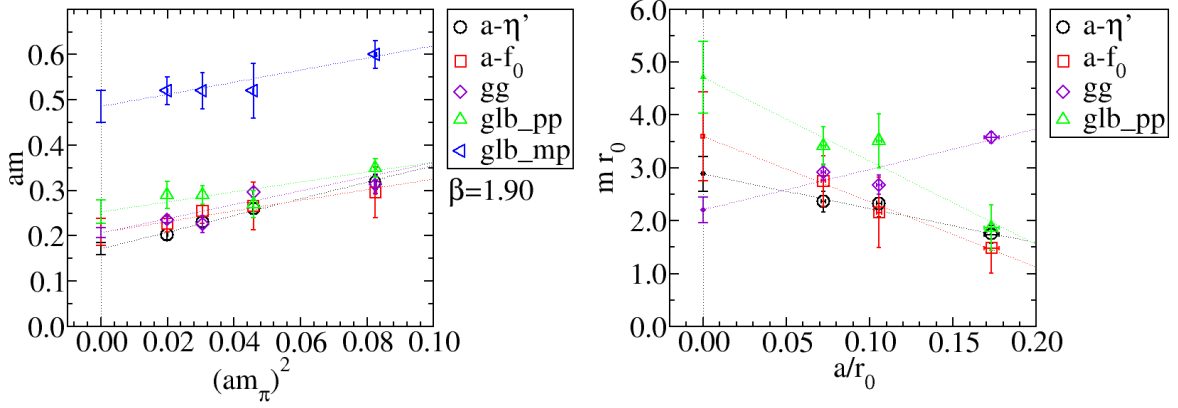
In [8] a No-Go theorem is discussed: a realisation of a complete supersymmetry on the lattice can only be achieved with nonlocal interactions and a nonlocal derivative. This statement is circumvented by fine tuning of the bare parameters of the theory towards the supersymmetric continuum limit. In SYM the fine-tuning of a single parameter is enough to approach not only supersymmetry but also the (spontaneously broken) chiral symmetry [9, 10]: the hopping parameter  $\kappa$  is tuned to the point where the renormalised gluino mass vanishes. In practice, this is achieved by monitoring the mass of the unphysical adjoint pion  $a-\pi$ , which is defined by the connected contribution of the  $a-\eta'$  correlator. A formal definition can be given in a partially quenched setup [11]. An important result, which was also established by means of the OZI approximation [1], is that the adjoint pion mass vanishes for a massless gluino with the law:  $m_{a-\pi}^2 \propto m_g$ .

### 4. Setting the scale

Monte Carlo simulations are able to extract only dimensionless quantities. The lattice spacing, which characterises the system under simulation, is not an input parameter. To make contact with the measurements done in a laboratory, the value of the lattice spacing  $a$  has to be determined. This value is implicitly defined once a dimensionful observable, for instance a distance in physical units  $d$ , is chosen as a reference to set the scale. Matching the result of a simulation  $\hat{d} = d/a$ , with the previous value, one determines the desired quantity:  $a = d/\hat{d}$ .

In the last years mainly two different observables have been chosen to set the scale, both of which have the dimension of a length: the Sommer parameter  $r_0$  [12] and the Wilson flow scale  $w_0$  [13].

$r_0$  is defined as the distance  $r$  such that the strong force between a static quark-antiquark pair, multiplied by the squared distance,  $r^2 F(r)$ , is equal to a specific value, typically 1.65. Systematic errors arise because of different smoothing procedures that can be used to improve the signal of  $F(r)$  and because of different fitting procedures employed to extract the parameters.



**Figure 1:** (Left) Mass spectrum (gg: gluino-gluon; glb\_pp: glueball  $0^{++}$ ; glb\_mp: glueball  $0^{-+}$ ) at  $\beta = 1.90$  for four values of the  $a\text{-}\pi$  mass. The extrapolated chiral limit is shown as well. (Right) Mass spectrum extrapolated to the continuum limit, using  $r_0$ .

The scale  $w_0$  is based on the gradient flow generated by the Wilson or the Symanzik gauge field action. The procedure is specified by a partial differential equation, similar to a diffusion equation, which evolves the gauge field along a fictitious time.  $w_0$ , as we show also in this work, is less effected by systematic errors and provides a more reliable method to set the scale.

## 5. Mass spectrum from effective Lagrangians

An important similarity between Yang-Mills theory and *supersymmetric* Yang-Mills theory is confinement. In the supersymmetric theory the glueballs are completed with the mesonic states, the gluinoballs, and the fermionic gluino-gluonballs. The low-lying spectrum of particles has been predicted by means of effective Lagrangians. In [1] a first supermultiplet was described: it consists of a scalar ( $0^+$  gluinoball:  $a\text{-}f_0 \sim \bar{\lambda}\lambda$ ), a pseudoscalar ( $0^-$  gluinoball:  $a\text{-}\eta' \sim \bar{\lambda}\gamma_5\lambda$ ), and a Majorana fermion (spin 1/2 gluino-gluonball:  $\chi \sim \sigma^{\mu\nu}\text{Tr}[F_{\mu\nu}\lambda]$ ). This first work was generalised in [2] introducing pure gluonic states in the effective Lagrangian. In addition to the first chiral supermultiplet a new one was found: a  $0^-$  glueball, a  $0^+$  glueball, and again a gluino-gluonball.

It is worth noting that neither of these supermultiplets contains a pure gluino-gluino, a gluino-gluon or a gluon-gluon bound state: the physical excitations are mixed states of them.

In [2] it has been argued that the glueball states, in the supersymmetric limit, are lighter than the gluinoball states. Later, other authors [14], using different arguments, and information about ordinary QCD, deduce that the lighter states are gluinoballs. This point has not been totally clarified so far.

## 6. Mass spectrum from Monte Carlo simulations

The most important signature for the existence of a continuum limit with unbroken supersymmetry is the low-lying mass spectrum of bound states: we expect in fact degeneracy inside the same supermultiplet.

The investigation of the spectrum of the theory represents a non-perturbative challenge, which requires numerical simulations of the theory, discretised on a space-time lattice, with the help of supercomputers. One of the tasks of our project is to show that the results of the numerical simulations are consistent with restoration of supersymmetry in the continuum limit. We have already published results on the spectrum of the theory for two values of the lattice spacing, which is characterised by the value of the parameter  $\beta$ : in [3] we presented our results at  $\beta = 1.60$ ; in [5, 15] we decreased the value of the lattice spacing by  $\sim 40\%$  using  $\beta = 1.75$ . In the latter, for the first time we had indication of restoration of SUSY. In [16] we presented our preliminary results at  $\beta = 1.90$ , where the lattice spacing was further reduced by  $\sim 30\%$ . In this conference proceedings we present our final results at  $\beta = 1.90$  for the first time, with the extrapolation to the continuum limit of the spectrum. In Figure 1 (Left) we plot five bound states for different values of the adjoint pion mass squared; they are linearly extrapolated to the chiral limit, according to Section 3. This numerical extrapolation shows a degeneracy of the masses in the lowest supermultiplet within two standard deviations. Note that the glueball  $0^{++}$  is particularly noisy: it is well known that this channel requires a statistic  $\sim 20$  times larger than the others to have comparable errors. In our case all channels are measured on the same sample of field configurations.

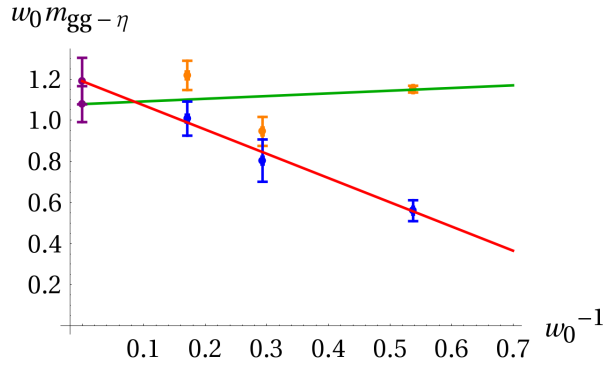
The lattice methods used in this work allow to extract directly only of the lowest mass in each channel. The operator used to extract the mass of the glueball  $0^{++}$  has a strong mixing with  $a\text{-}f_0$ , indeed they have the same quantum numbers, and therefore the value of its mass fall in the lower supermultiplet. The operator used to describe the glueball  $0^{-+}$  instead has a weak mixing with the  $a\text{-}\eta'$ . As a consequence in Figure 1 (Left) we see that only the glueball  $0^{-+}$  is present in the higher supermultiplet. These facts seem to suggest that the glueball states are higher than the gluinoball as argued in [14].

The spectrum of the theory extrapolated to the continuum limit using the inverse of the Sommer scale  $r_0$  is shown in Figure 1 (Right). According to this plot we finally can state that we see the restoration, within two standard deviation, of supersymmetry in the continuum limit.

In Figure 2 we present for comparison the extrapolation to the continuum limit of two particles which belong to the same supermultiplet, the gluino-glueball  $\chi$  and the gluinoball  $a\text{-}\eta'$ . In this case, using the scale parameter  $w_0$ , we clearly see that in the continuum limit the two particles are degenerate. In general, we have verified that the extrapolations based on the parameter  $w_0$  give better results.

## 7. Conclusion and Outlooks

We have studied the spectrum of  $\mathcal{N} = 1$  supersymmetric Yang-Mills theory non-perturbatively employing lattice calculations. We have shown that, within two standard deviations, the theory is



**Figure 2:** Extrapolation to the continuum limit of the gluino-glueball  $\chi$  and the gluinoball  $a\text{-}\eta'$ , using  $w_0$ .

characterised by degeneracy of the mass spectrum inside the same supermultiplet. In other words, it has been possible to see restoration of supersymmetry in the continuum limit.

Our next step is to clarify the role of the mixing, studying the excited states of the theory. Moreover we are planning to move from SU(2) to the more realistic SU(3) gauge theory, which is part of the supersymmetric extension of the Standard Model.

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## References

- [1] G. Veneziano and S. Yankielowicz, Phys. Lett. B **113** (1982) 231.
- [2] G. R. Farrar, G. Gabadadze and M. Schwetz, Phys. Rev. D **58** (1998) 015009 [hep-th/9711166].
- [3] K. Demmouche, F. Farchioni, A. Ferling, I. Montvay, G. Münster, E. E. Scholz and J. Wuilloud, Eur. Phys. J. C **69** (2010) 147 [arXiv:1003.2073 [hep-lat]].
- [4] G. Bergner, T. Berheide, G. Münster, U. D. Özugurel, D. Sandbrink and I. Montvay, JHEP **1209** (2012) 108 [arXiv:1206.2341 [hep-lat]].
- [5] G. Bergner, I. Montvay, G. Münster, U. D. Özugurel and D. Sandbrink, JHEP **1311** (2013) 061 [arXiv:1304.2168 [hep-lat]].
- [6] G. Bergner, P. Giudice, G. Münster, S. Piemonte and D. Sandbrink, JHEP **1411** (2014) 049 [arXiv:1405.3180 [hep-lat]].
- [7] G. Bergner, P. Giudice, I. Montvay, G. Münster and S. Piemonte, arXiv:1411.6995 [hep-lat].
- [8] G. Bergner, JHEP **1001** (2010) 024 [arXiv:0909.4791 [hep-lat]].
- [9] G. Curci and G. Veneziano, Nucl. Phys. B **292** (1987) 555.
- [10] H. Suzuki, Nucl. Phys. B **861** (2012) 290 [arXiv:1202.2598 [hep-lat]].
- [11] G. Münster and H. Stüwe, JHEP **1405** (2014) 034 [arXiv:1402.6616 [hep-th]].
- [12] R. Sommer, Nucl. Phys. B **411** (1994) 839 [hep-lat/9310022].
- [13] S. Borsanyi *et al.*, JHEP **1209** (2012) 010 [arXiv:1203.4469 [hep-lat]].
- [14] A. Feo, P. Merlati and F. Sannino, Phys. Rev. D **70** (2004) 096004 [hep-th/0408214].
- [15] G. Bergner, I. Montvay, G. Münster, D. Sandbrink and U. D. Özugurel, PoS LATTICE **2013** (2014) 483 [arXiv:1311.1681 [hep-lat]].
- [16] G. Bergner, P. Giudice, I. Montvay, G. Münster, U. D. Özugurel, S. Piemonte and D. Sandbrink, PoS LATTICE **2014** (2014) 273 [arXiv:1411.1746 [hep-lat]].