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Dynamically Induced Planck Scale and Inflation

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We present a minimal model of inflation where the Planck scale is dynamically generated from dimensionless interactions. The inflaton field ϕ gets a vacuum expectation value via dimensional transmutation. The Planck scale is generated via its non-minimal coupling to gravity $\xi\phi^2 R$. To generate the minimum of the inflaton potential, one needs a singlet scalar and new fermion(s) which can provide a dark matter candidate. The spectral index is predicted to be $n_s \approx 0.96$. The tensor-to-scalar ratio can vary from $r \approx 0.13$ down to $r \approx 0.04$ in presence of large couplings or $r \approx 0.003$ if the Lagrangian contains an R^2 term, interpolating between the quadratic and Starobinsky inflation. These theories relate the smallness of the weak scale to the smallness of inflationary perturbations: both arise naturally because of small couplings, implying a reheating temperature of $10^{7\cdots 9}$ GeV. A measurement of r by Keck/BICEP3 would give us information on quantum gravity in the dimensionless scenario.

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1. Classical Scale Invariance

We introduce a classically scale invariant model [1] with no explicit mass scales at tree level in the Jordan frame. Quantum corrections break classical scale invariance and induce a vacuum expectation value (VEV) for the inflaton via the Coleman-Weinberg mechanism. We perform a conformal transformation of the potential into the Einstein frame, where we have usual general relativity + inflation.

Classical scale invariance may help solve the naturalness problem [2], physical naturalness [3], finite naturalness [4], and many others (for the full list of references please see [1]).

Classical scale invariance strongly restricts the terms that can be included in the Lagrangian. In particular, there are no mass terms, no soft (cubic) terms, no Planck mass $M_{\rm P}$ and no cosmological constant Λ. Dimensionless terms such as Yukawa couplings and scalar quartic couplings are allowed, whereas all dimensionful terms will be generated by quantum corrections.

The small amplitude of scalar perturbations $P_R = (2.14 \pm 0.05) \times 10^{-9}$ [5] means that $E_{\text{inflation}} \ll M_{\rm P}$ and the fact that the Planck scale does not seem to produce appreciable quantum corrections to the Higgs mass, despite the ratio $M_h/M_{\rm P} \sim 10^{-16}$ suggests that quantum gravity should be weakly coupled.

2. Induced Gravity and Inflation

in the Jordan frame, physics is described by

$$\sqrt{-g^J}\mathscr{L}^J = \sqrt{-g^J}\left[-\frac{\xi_S}{2}s^2 R + \frac{(\partial s)^2}{2} - \frac{1}{4}\lambda_S(s)s^4 + \mathscr{L}_{\text{extra matter}}\right], \tag{2.1}$$

where s is the inflaton. Extra matter is needed to induce the correct renormalisation group running of the inflaton quartic λ_S.

As usual, we take $\mu = s$ to resum log-enhanced loop corrections. Thus the potential in the Jordan frame

$$V(s) = \frac{1}{4}\lambda_S(s)s^4 \tag{2.2}$$

gets a non-zero minimum due to running λ_S. The Jordan frame potential is sketched in Figure 1. The minimum that induces the Planck mass allows two kinds of inflation, small-field (hilltop) and large-field inflation.

We have three conditions on the minimum:

- Planck mass generation $\frac{\xi_S}{2}s^2 R \to \frac{M_{\rm P}^2}{2}R$ implies

$$v_s^2 = \frac{M_{\rm P}^2}{\xi_S},$$

- Dark energy $\Lambda \simeq 0$ and *no* eternal inflation imply

$$V(v_s) \simeq 0, \text{ thus } \lambda_S(v_s) \simeq 0, \tag{2.3}$$

- of course, the minimum condition

$$\frac{d\lambda_S}{dt}(v_s) = \beta_{\lambda_S}(v_s) = 0. \tag{2.4}$$

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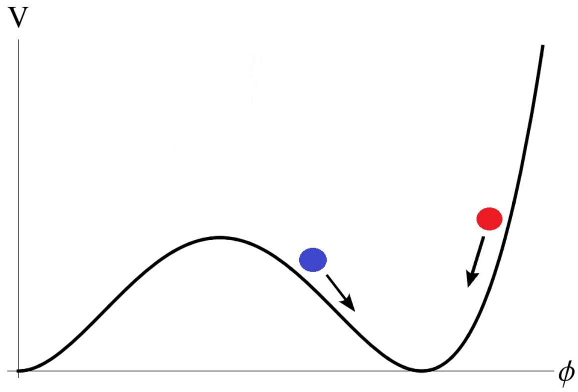

Figure 1: A sketch of the Jordan frame potential. Blue means small-field and red means large-field inflation.

To go to the Einstein frame, we perform the conformal transformation

$$g^E_{\mu\nu} = \Omega(s)^2 g^J_{\mu\nu}, \qquad \text{where} \qquad \Omega(s)^2 = \frac{\xi_S s^2}{M_{\rm P}^2} = \frac{s^2}{v_s^2}. \tag{2.5}$$

In the Einstein frame, the potential of the inflaton consists of only a cosmological constant term:

$$V_E(s) = \frac{V(s)}{\Omega(s)^4} = \frac{1}{4}\lambda_S(s)\frac{M_{\rm P}^4}{\xi_S^2}. \tag{2.6}$$

However, because λ_S is running, the cosmological constant depends on s and can accommodate inflation.

What does the potential look like in the Einstein frame?

The Taylor expansion of λ_S around v_s in the Jordan frame is

$$\lambda_S(s) = \lambda_S(v_s) + \beta_{\lambda_S}(v_s)\ln\frac{s}{v_s} + \frac{1}{2}\beta'_{\lambda_S}(v_s)\ln^2\frac{s}{v_s} + \frac{1}{6}\beta''_{\lambda_S}(v_s)\ln^3\frac{s}{v_s} + \dots. \tag{2.7}$$

By conditions (2.3) and (2.4), the first two terms are zero.

Now in the Einstein frame, the field s is not canonically normalised. We define a canonically normalised field s_E by

$$s = v_s e^{\sqrt{\frac{\xi_S}{1+6\xi_S}}\frac{s_E}{M_{\rm P}}}. \tag{2.8}$$

Inserting (2.8) into the expansion (2.7), we obtain in the Einstein frame

$$V_E(s) = \frac{1}{4}\lambda_S(s)\frac{M_{\rm P}^4}{\xi_S^2} = \frac{1}{4}\left[\frac{1}{2}\beta'_{\lambda_S}(v_s)\frac{\xi_S}{1+6\xi_S}\frac{s_E^2}{M_{\rm P}^2} + \frac{1}{3!}\beta''_{\lambda_S}(v_s)\left(\frac{\xi_S}{1+6\xi_S}\right)^{\frac{3}{2}}\frac{s_E^3}{M_{\rm P}^3} + \dots\right]\frac{M_{\rm P}^4}{\xi_S^2}. \tag{2.9}$$

The potential in the Einstein frame (with the VEV shifted to zero for convenience) for the particular set of parameters $\xi_S(v_s) = 300$, $\beta'_{\lambda_S}(v_s) = 6\times 10^{-5}$ and $\beta''_{\lambda_S}(v_s) = 9\times 10^{-6}$ is shown on Figure 2. The gray dashed line shows the quadratic approximation that is quite bad for all but small values of couplings.

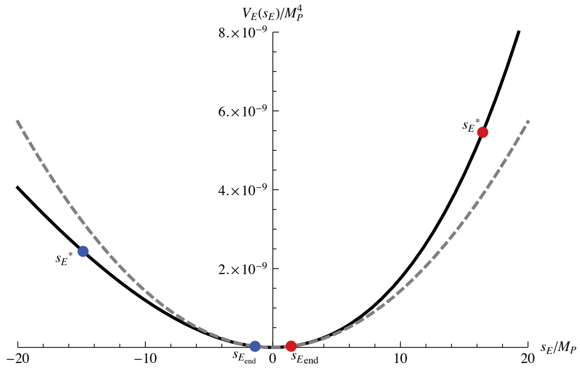

Figure 2: An example of the Einstein frame potential. Gray dashed line shows the quadratic approximation. Blue means negative-field and red means positive-field inflation.

3. The Minimal Model

In order to fulfill the minimum condition (2.4), the one-loop β-function for λ_S in the approximation $\lambda_S \simeq 0$ must contain at least one negative and one positive term that can cancel each other. Therefore the minimum set of extra matter is a Majorana fermion ψ and a real scalar σ, and the full Lagrangian in the Jordan frame is given by

$$\begin{aligned}\sqrt{-g^J}\mathscr{L}^J &= \sqrt{-g^J}\bigg[\mathscr{L}_{\rm SM} - \frac{\xi_S}{2}s^2 R \\ &\quad + \frac{(\partial s)^2}{2} + \frac{(\partial\sigma)^2}{2} + \frac{i}{2}\bar{\psi}^c \not{D}\psi + \mathscr{L}_Y - V\bigg], \\ \mathscr{L}_Y &= \frac{1}{2}y_S s\bar{\psi}^c\psi + \frac{1}{2}y_\sigma\sigma\bar{\psi}^c\psi, \\ V &= \frac{1}{4}\lambda_S s^4 + \frac{1}{4}\lambda_{S\sigma}s^2\sigma^2 + \frac{1}{4}\lambda_\sigma\sigma^4.\end{aligned}$$

Then the one-loop β-function at the minimum is

$$16\pi^2\beta_{\lambda_S}(v_s) = \frac{1}{2}\lambda_{S\sigma}^2(v_s) - 4y_S^4(v_s) = 0.$$

In the Einstein frame, the Lagrangian is

$$\sqrt{-g^E}\mathscr{L}^E = \sqrt{-g^E}\left[\frac{\mathscr{L}_{\rm SM}}{\Omega(s)^4} - \frac{1}{2}M_{\rm P}^2 R + \frac{(\partial s_E)^2}{2} + \frac{(\partial\sigma_E)^2}{2} + \frac{i}{2}\bar{\psi}_E^c\not{D}\psi_E + \mathscr{L}_{Y_E} - V_E\right], \tag{3.1}$$

$$\mathscr{L}_{Y_E} = \frac{1}{2}y_S v_s\bar{\psi}_E^c\psi_E + \frac{1}{2}y_\sigma\sigma_E\bar{\psi}_E^c\psi_E \equiv \frac{1}{2}m_\psi\bar{\psi}_E^c\psi_E + \frac{1}{2}y_\sigma\sigma_E\bar{\psi}_E^c\psi, \tag{3.2}$$

$$V_E = \frac{1}{4}\lambda_S v_s^4 + \frac{1}{4}\lambda_{S\sigma}v_s^2\sigma_E^2 + \frac{1}{4}\lambda_\sigma\sigma_E^4 \equiv \Lambda + \frac{1}{2}m_\sigma^2\sigma_E^2 + \frac{1}{4}\lambda_\sigma\sigma_E^4, \tag{3.3}$$

where the inflaton self-coupling has become the (running) cosmological constant, the Yukawa term of the inflaton has become a fermion mass term and the portal coupling to the scalar singlet σ the mass of that scalar.

Matching the renormalisation group equations in the two frames [1] gives us that

$$\begin{array}{l}\text{Frame equivalence (at loop level)}\\ \text{in the weak/soft gravity limit}\end{array} \Rightarrow \begin{cases}\lambda_S \ll \lambda_{S\sigma} \ll \lambda_\sigma \\ \beta_{y_S} \ll y_S,\ \beta_{\xi_S} \ll \xi_S\end{cases}.$$

The predictions for the tensor to scalar ratio r and the spectral tilt n_s together with the 1 and 2σ fit of the latest combination of the Planck/BICEP2/Keck [6, 7, 8, 5] are given in the left side panel of Figure 3.

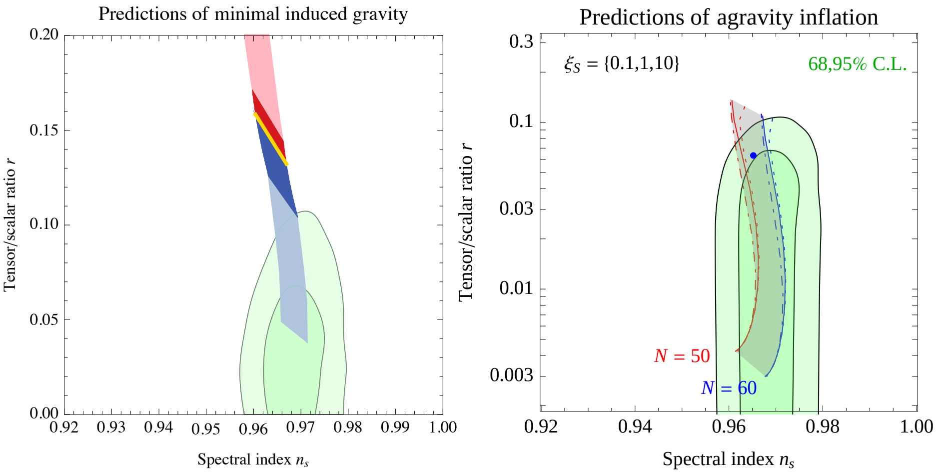

Figure 3: r vs. n_s in the effective theory and in agravity.

Adding an R^2 term to general relativity gives the Starobinsky inflation with $r = 0.003$ [9]. With both the inflaton s and an R^2 term, we can interpolate between quadratic and Starobinsky inflation as shown on the right side panel of Figure 3 in a concrete realisation of such a model is inflation in agravity (an ultraviolet completion of general relativity) [10].

4. Conclusions

In conclusion, we have a classically scale invariant model with Planck mass generated by the inflaton. The minimal field content required comprises the inflaton s, an extra scalar σ and a Majorana fermion ψ. The Einstein frame potential is generated by the running of the inflaton self-coupling λ_S. With large couplings, large deviations from quadratic inflation can be obtained. Another way to deviate from quadratic inflation is to add an additional R^2 term, which allows to interpolate the tensor to scalar ratio between $0.003 \leqslant r \leqslant 0.13$.

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