

Spin structure of the "forward" nucleon charge-exchange reaction $n + p \rightarrow p + n$ and the deuteron charge-exchange breakup

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The structure of the nucleon charge-exchange process $n + p \rightarrow p + n$ is theoretically investigated on the basis of the isotopic invariance of the nucleon-nucleon scattering. By using the operator of permutation of the spin projections of the neutron and proton, the connection between the spin matrices, which describe the amplitude of the "forward" nucleon charge-exchange process and the amplitude of neutron elastic scattering on the proton in the "backward" direction, has been obtained. Due to the optical theorem, the spin-independent part of the differential cross-section of the process $n + p \rightarrow p + n$ at zero angle for unpolarized particles is expressed through the difference of total cross-sections of unpolarized proton-proton and neutron-proton scattering. Meantime, the spin-dependent part of this cross-section is proportional to the differential cross-section of the "forward" deuteron charge-exchange breakup $d + p \rightarrow (pp) + n$ at the deuteron momentum $\mathbf{k}_d = 2\mathbf{k}_n$ (\mathbf{k}_n is the initial neutron momentum). As shown by the performed analysis, it is just the spin-dependent term that provides the main contribution into the differential cross-section of the "forward" process $n + p \rightarrow p + n$ in the wide range of neutron laboratory momenta $k_n > 700 \text{ MeV}/c$.

*The European Physical Society Conference on High Energy Physics
22-29 July 2015
Vienna, Austria*

* Speaker.

1. Isotopic structure of NN -scattering

Taking into account the isotopic invariance, the nucleon-nucleon scattering is described by the following operator:

$$\hat{f}(\mathbf{p}, \mathbf{p}') = \hat{a}(\mathbf{p}, \mathbf{p}') + \hat{b}(\mathbf{p}, \mathbf{p}') \hat{\tau}^{(1)} \hat{\tau}^{(2)}. \quad (1.1)$$

Here $\hat{\tau}^{(1)}$ and $\hat{\tau}^{(2)}$ are vector Pauli operators in the isotopic space, $\hat{a}(\mathbf{p}, \mathbf{p}')$ and $\hat{b}(\mathbf{p}, \mathbf{p}')$ are 4-row matrices in the spin space of two nucleons; \mathbf{p} and \mathbf{p}' are the initial and final momenta in the c.m. frame, the directions of \mathbf{p}' are defined within the solid angle in the c.m. frame, corresponding to the front hemisphere.

One should note that the process of elastic neutron-proton scattering into the back hemisphere is interpreted as the charge-exchange process $n + p \rightarrow p + n$.

According to Eq. (1.1), the matrices of amplitudes of proton-proton, neutron-neutron and neutron-proton scattering take the form:

$$\begin{aligned} \hat{f}_{pp \rightarrow pp}(\mathbf{p}, \mathbf{p}') &= \hat{f}_{nn \rightarrow nn}(\mathbf{p}, \mathbf{p}') = \hat{a}(\mathbf{p}, \mathbf{p}') + \hat{b}(\mathbf{p}, \mathbf{p}'); \\ \hat{f}_{np \rightarrow np}(\mathbf{p}, \mathbf{p}') &= \hat{a}(\mathbf{p}, \mathbf{p}') - \hat{b}(\mathbf{p}, \mathbf{p}'); \end{aligned} \quad (1.2)$$

meantime, the matrix of amplitudes of the charge-exchange process is as follows:

$$\hat{f}_{np \rightarrow pn}(\mathbf{p}, \mathbf{p}') = 2\hat{b}(\mathbf{p}, \mathbf{p}') = \hat{f}_{pp \rightarrow pp}(\mathbf{p}, \mathbf{p}') - \hat{f}_{np \rightarrow np}(\mathbf{p}, \mathbf{p}'). \quad (1.3)$$

It should be stressed that the differential cross-section of the charge-exchange reaction, defined in the front hemisphere $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi \leq 2\pi$ (here θ is the angle between the momenta of initial neutron and final proton, ϕ is the azimuthal angle), should coincide with the differential cross-section of the elastic neutron-proton scattering into the back hemisphere by the angle $\tilde{\theta} = \pi - \theta$ at the azimuthal angle $\tilde{\phi} = \pi + \phi$ in the c.m. frame. Due to the antisymmetry of the state of two fermions with respect to the total permutation, including the permutation of momenta ($\mathbf{p}' \rightarrow -\mathbf{p}'$), permutation of spin projections and permutation of isotopic projections ($p \leftrightarrow n$), the following relation between the amplitudes $\hat{f}_{np \rightarrow pn}(\mathbf{p}, \mathbf{p}')$ and $\hat{f}_{np \rightarrow np}(\mathbf{p}, -\mathbf{p}')$ holds [1–4]:

$$\hat{f}_{np \rightarrow pn}(\mathbf{p}, \mathbf{p}') = -\hat{P}^{(1,2)} \hat{f}_{np \rightarrow np}(\mathbf{p}, -\mathbf{p}'), \quad (1.4)$$

where $\hat{P}^{(1,2)}$ is the operator of permutation of spin projections of two particles with equal spins; the matrix elements of this operator are [5]: $\langle m'_1 m'_2 | \hat{P}^{(1,2)} | m_1 m_2 \rangle = \delta_{m'_1 m_2} \delta_{m'_2 m_1}$. For particles with spin 1/2 [1–5]

$$\hat{P}^{(1,2)} = \frac{1}{2} (\hat{I}^{(1,2)} + \hat{\sigma}^{(1)} \hat{\sigma}^{(2)}), \quad (1.5)$$

where $\hat{I}^{(1,2)}$ is the four-row unit matrix, $\hat{\sigma}^{(1)}$, $\hat{\sigma}^{(2)}$ – vector Pauli operators. It is evident that $\hat{P}^{(1,2)}$ is the unitary and Hermitian operator:

$$\hat{P}^{(1,2)} = \hat{P}^{(1,2)+}, \quad \hat{P}^{(1,2)}\hat{P}^{(1,2)+} = \hat{I}^{(1,2)}. \quad (1.6)$$

Taking into account the relations (1.5) and (1.6), the following matrix equality holds:

$$\hat{f}_{np \rightarrow pn}^+(\mathbf{p}, \mathbf{p}') \hat{f}_{np \rightarrow pn}(\mathbf{p}, \mathbf{p}') = \hat{f}_{np \rightarrow np}^+(\mathbf{p}, -\mathbf{p}') \hat{f}_{np \rightarrow np}(\mathbf{p}, -\mathbf{p}'). \quad (1.7)$$

As a result, the differential cross-sections of the charge-exchange process $n + p \rightarrow p + n$ and the elastic np -scattering in the corresponding back hemisphere coincide at any polarizations of initial nucleons:

$$\frac{d\sigma_{np \rightarrow pn}}{d\Omega}(\mathbf{p}, \mathbf{p}') = \frac{d\sigma_{np \rightarrow np}}{d\Omega}(\mathbf{p}, -\mathbf{p}'). \quad (1.8)$$

However, the separation into the spin-dependent and spin-independent parts is different for the amplitudes $\hat{f}_{np \rightarrow pn}(\mathbf{p}, \mathbf{p}')$ and $\hat{f}_{np \rightarrow np}(\mathbf{p}, -\mathbf{p}')$!

2. Nucleon charge-exchange process at zero angle

Now let us investigate in detail the nucleon charge transfer reaction $n + p \rightarrow p + n$ at zero angle. In the c.m. frame of the (np) -system, the amplitude of the nucleon charge transfer in the "forward" direction $\hat{f}_{np \rightarrow pn}(0)$ has the following spin structure:

$$\hat{f}_{np \rightarrow pn}(0) = c_1 \hat{I}^{(1,2)} + c_2 [\hat{\sigma}^{(1)}\hat{\sigma}^{(2)} - (\hat{\sigma}^{(1)}\mathbf{I})(\hat{\sigma}^{(2)}\mathbf{I})] + c_3 (\hat{\sigma}^{(1)}\mathbf{I})(\hat{\sigma}^{(2)}\mathbf{I}), \quad (2.1)$$

where \mathbf{I} is the unit vector directed along the incident neutron momentum. In so doing, the second term in Eq. (2.1) describes the spin-flip effect, and the third term characterizes the difference between the amplitudes with the parallel and antiparallel orientations of the neutron and proton spins.

The spin structure of the amplitude of the elastic neutron-proton scattering in the "backward" direction $\hat{f}_{np \rightarrow np}(\pi)$ is analogous:

$$\hat{f}_{np \rightarrow np}(\pi) = \tilde{c}_1 \hat{I}^{(1,2)} + \tilde{c}_2 [\hat{\sigma}^{(1)}\hat{\sigma}^{(2)} - (\hat{\sigma}^{(1)}\mathbf{I})(\hat{\sigma}^{(2)}\mathbf{I})] + \tilde{c}_3 (\hat{\sigma}^{(1)}\mathbf{I})(\hat{\sigma}^{(2)}\mathbf{I}). \quad (2.2)$$

However, the coefficients \tilde{c} in Eq. (2.2) do not coincide with the coefficients c in Eq. (2.1). According to Eq. (1.4), the connection between the amplitudes $\hat{f}_{np \rightarrow pn}(0)$ and $\hat{f}_{np \rightarrow np}(\pi)$ is the following:

$$\hat{f}_{np \rightarrow pn}(0) = -\hat{P}^{(1,2)} \hat{f}_{np \rightarrow np}(\pi), \quad (2.3)$$

where the unitary operator $\hat{P}^{(1,2)}$ is determined by Eq. (1.5).

As a result of calculations with Pauli matrices, we obtain:

$$c_1 = -\frac{1}{2}(\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3); \quad c_2 = -\frac{1}{2}(\tilde{c}_1 - \tilde{c}_3); \quad c_3 = -\frac{1}{2}(\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3). \quad (2.4)$$

Hence, it follows from here that the "forward" differential cross-section of the nucleon charge-exchange reaction $n + p \rightarrow p + n$ for unpolarized initial nucleons is described by the expression:

$$\begin{aligned} \frac{d\sigma_{np \rightarrow pn}}{d\Omega}(0) &= |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \\ &= \frac{1}{4}|\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3|^2 + \frac{1}{2}|\tilde{c}_1 - \tilde{c}_3|^2 + \frac{1}{4}|\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3|^2 = |\tilde{c}_1|^2 + 2|\tilde{c}_2|^2 + |\tilde{c}_3|^2. \end{aligned} \quad (2.5)$$

Thus,

$$\frac{d\sigma_{np \rightarrow pn}}{d\Omega}(0) = \frac{d\sigma_{np \rightarrow np}}{d\Omega}(\pi),$$

just as it should hold in accordance with the relation (1.8).

3. Spin-independent and spin-dependent parts of the cross-section of the reaction $n + p \rightarrow p + n$ at zero angle

It is clear that the amplitudes of the proton-proton and neutron-proton elastic scattering at zero angle have the structure (2.1) with the replacements $c_1, c_2, c_3 \rightarrow c_1^{(pp)}, c_2^{(pp)}, c_3^{(pp)}$, $c_1, c_2, c_3 \rightarrow c_1^{(np)}, c_2^{(np)}, c_3^{(np)}$, respectively. It follows from the isotopic invariance (see Eq. (1.3)) that

$$c_1 = c_1^{(pp)} - c_1^{(np)}, \quad c_2 = c_2^{(pp)} - c_2^{(np)}, \quad c_3 = c_3^{(pp)} - c_3^{(np)}. \quad (3.1)$$

In accordance with the optical theorem, the following relation holds, taking into account Eq. (3.1):

$$\frac{4\pi}{k} \text{Im } c_1 = \frac{4\pi}{k} (\text{Im } c_1^{(pp)} - \text{Im } c_1^{(np)}) = \sigma_{pp} - \sigma_{np}, \quad (3.2)$$

where σ_{pp} and σ_{np} are the total cross-sections of interaction of two unpolarized protons and of the unpolarized neutron and proton, respectively (due to the isotopic invariance, $\sigma_{pp} = \sigma_{nn}$); $k = |\mathbf{p}| = |\mathbf{p}'|$ is the modulus of neutron momentum in the c.m. frame of the colliding nucleons ¹⁾.

Taking into account Eqs. (2.1), (2.5) and (3.2), the differential cross-section of the process $n + p \rightarrow p + n$ in the "forward" direction for unpolarized nucleons can be presented in the following form, distinguishing the spin-independent and spin-dependent parts:

$$\frac{d\sigma_{np \rightarrow pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \frac{d\sigma_{np \rightarrow pn}^{(si)}}{d\Omega}(0) + \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{d\Omega}(0). \quad (3.3)$$

In doing so, the spin-independent part $\frac{d\sigma_{np \rightarrow pn}^{(si)}}{d\Omega}(0)$ in Eq. (3.3) is determined by the difference of total cross-sections of the unpolarized proton-proton and neutron-proton interaction:

$$\frac{d\sigma_{np \rightarrow pn}^{(si)}}{d\Omega}(0) = |c_1|^2 = \frac{k^2}{16\pi^2} (\sigma_{pp} - \sigma_{np})^2 (1 + \alpha^2), \quad (3.4)$$

¹⁾ We use the unit system with $\hbar = c = 1$.

where $\alpha = \text{Re } c_1 / \text{Im } c_1$. The spin-dependent part of the cross-section of the "forward" charge-exchange process is

$$\frac{d\sigma_{np \rightarrow pn}^{(sd)}}{d\Omega}(0) = 2|c_2|^2 + |c_3|^2. \quad (3.5)$$

Meantime, according to Eqs. (2.2), (2.4) and (2.5), the spin-dependent part of the cross-section of the "backward" elastic np -scattering is

$$\frac{d\sigma_{np \rightarrow np}^{(sd)}}{d\Omega}(\pi) = 2|\tilde{c}_2|^2 + |\tilde{c}_3|^2. \quad (3.6)$$

We see that $\frac{d\sigma_{np \rightarrow pn}^{(sd)}}{d\Omega}(0) \neq \frac{d\sigma_{np \rightarrow np}^{(sd)}}{d\Omega}(\pi)$.

Further it is advisable to deal with the differential cross-section $\left. \frac{d\sigma}{dt} \right|_{t=0}$, being a relativistic invariant ($t = -(p_1 - p_2)^2 = (\mathbf{p} - \mathbf{p}')^2 - (E - E')^2$ is the square of the 4-dimensional transferred momentum). In the c.m. frame we have: $t = 2k^2(1 - \cos\theta)$ and $\frac{d\sigma}{dt} = (\pi/k^2) \frac{d\sigma}{d\Omega}$. So, in this representation, the spin-independent and spin-dependent parts of the differential cross-section of the "forward" charge transfer process $\left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0}$ are as follows: $\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} = (\pi/k^2) |c_1|^2$, $\left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0} = (\pi/k^2) (2|c_2|^2 + |c_3|^2)$, and we may write, instead of Eq. (3.3):

$$\left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} = \left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0} + \frac{1}{16\pi} (\sigma_{pp} - \sigma_{np})^2 (1 + \alpha^2). \quad (3.7)$$

Now it should be noted that, in the framework of the impulse approach, there exists a simple connection between the spin-dependent part of the differential cross-section of the charge-exchange reaction $n + p \rightarrow p + n$ at zero angle $\left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0}$ (not the "backward" elastic neutron-proton scattering, that would be an error !) and the differential cross-section of the deuteron charge-exchange breakup $d + p \rightarrow (pp) + n$ in the "forward" direction $\left. \frac{d\sigma_{dp \rightarrow (pp)n}}{dt} \right|_{t=0}$ at the deuteron momentum $\mathbf{k}_d = 2\mathbf{k}_n$ (\mathbf{k}_n is the the initial neutron momentum). In the case of unpolarized particles we have [6–8]:

$$\left. \frac{d\sigma_{dp \rightarrow (pp)n}}{dt} \right|_{t=0} = \frac{2}{3} \left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0}. \quad (3.8)$$

In doing so, this formula remains still valid if one takes into account the deuteron D -wave state [8].

It is easy to understand also that, due to the isotopic invariance, the same relation (similar to Eq. (3.8)) takes place also for the process $p + d \rightarrow n + (pp)$ at the proton laboratory momentum $\mathbf{k}_p = \mathbf{k}_n$ and for the process $n + d \rightarrow p + (nn)$ at the neutron laboratory momentum \mathbf{k}_n .

Thus, in principle, taking into account Eqs. (3.7) and (3.8), the modulus of the ratio of the real and imaginary parts of the spin-independent charge transfer amplitude at zero angle ($|\alpha|$) may

be determined using the experimental data on the total cross-sections of interaction of unpolarized nucleons and on the differential cross-sections of the "forward" nucleon charge transfer process and the charge-exchange breakup of an unpolarized deuteron $d + p \rightarrow (pp) + n$ in the "forward" direction.

The analysis shows: if we suppose that the real part of the spin-independent amplitude of charge transfer $n + p \rightarrow p + n$ at zero angle is smaller or of the same order as compared with the imaginary part ($\alpha^2 \leq 1$), then it follows from the available experimental data on the differential cross-section of charge transfer $\left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0}$ and the data on the total cross-sections σ_{pp} and σ_{np} that the main contribution into the cross-section $\left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0}$ is provided namely by the spin-dependent part $\left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0}$.

If the differential cross-section $\frac{d\sigma}{dt}$ is given in the units of $mbn / (\frac{GeV}{c})^2$ and the total cross-sections are given in mbn , then the spin-independent part of the "forward" charge transfer cross-section may be expressed in the form :

$$\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} \approx 0.0512 (\sigma_{pp} - \sigma_{np})^2 (1 + \alpha^2). \quad (3.9)$$

Using Eq. (3.9) and the data from the works [9–11], we obtain the following estimates of the ratio $\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} / \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0}$ at different values of the neutron laboratory momentum k_n :

$$1) k_n = 0.7 \frac{GeV}{c}; \quad \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} = 268 \text{ mbn} / (\frac{GeV}{c})^2; \quad \sigma_{pp} - \sigma_{np} = -22.6 \text{ mbn};$$

$$\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} / \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} \approx 0.1 (1 + \alpha^2).$$

$$2) k_n = 1.7 \frac{GeV}{c}; \quad \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} = 37.6 \text{ mbn} / (\frac{GeV}{c})^2; \quad \sigma_{pp} - \sigma_{np} = 10 \text{ mbn};$$

$$\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} / \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} \approx 0.136 (1 + \alpha^2).$$

$$3) k_n = 2.5 \frac{GeV}{c}; \quad \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} = 17.85 \text{ mbn} / (\frac{GeV}{c})^2; \quad \sigma_{pp} - \sigma_{np} = 5.5 \text{ mbn};$$

$$\left. \frac{d\sigma_{np \rightarrow pn}^{(si)}}{dt} \right|_{t=0} / \left. \frac{d\sigma_{np \rightarrow pn}}{dt} \right|_{t=0} \approx 0.085 (1 + \alpha^2).$$

So, it is well seen that, assuming $\alpha^2 \lesssim 1$, the spin-dependent part $\left. \frac{d\sigma_{np \rightarrow pn}^{(sd)}}{dt} \right|_{t=0}$ provides at least $(75 \div 90)\%$ of the total magnitude of the "forward" nucleon charge-exchange cross-section (see also [1–4]).

The predominant role of the spin-dependent part in the differential cross-section of the "forward" nucleon charge-exchange reaction $n + p \rightarrow p + n$ is confirmed also by the experimental data on the differential cross-sections of "forward" deuteron charge-exchange breakup processes $d + p \rightarrow (pp) + n$ and $n + d \rightarrow (nn) + p$, obtained in Dubna (JINR, Veksler and Baldin Laboratory of High Energy Physics) – see [4] and references therein .

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