Rare $B \to K^{(*)}\mu^+\mu^-$ decay in covariant quark model

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The covariant quark model with infrared confinement is presented. The model is used to describe the hadronic effects in the rare decay $B \to K^{(*)}\mu^+\mu^-$ and predict value of several commonly used related observables.

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1. Introduction

Rare heavy hadron decays are nowadays a very active area of investigation with discovery potential. New high-luminosity machines today in operation allow for measurement of rare flavor-changing processes. Feynman diagrams of some of these processes may, in the loops, contain new hypothetical heavy particles. One reaction which might be sensitive to new physics is the $B \to K^{(*)}\mu^+\mu^-$ decay. The process has been measured by [1, 2, 3, 4], where some results also include information about differential and angular distributions. As of now, the standard model (SM) is confirmed with some tensions ($\sim 3\sigma$).

However any SM evaluation of the decay contains uncertainty related to the hadronic effects. The covariant quark model (CQM) is a Lagrangian-based effective field approach to hadronic interactions with limited number of free parameters. It is well suited to described the $B \to K^{(*)}\mu^+\mu^-$ decay and check the agreement between the data and the SM.

2. Covariant quark model (mesons)

The CQM Lagrangian introduces meson-quark interaction as follows

$$\mathscr{L}_{\text{int}} = g_M \cdot M(x) \cdot J_M(x), \quad J_M(x) = \int dx_1 \int dx_2 \, F_M(x, x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \Gamma_M \, q^a_{f_2}(x_2),$$

$$F_M(x, x_1, \ldots, x_n) = \delta\left(x - \sum_{i=1}^{n} w_i x_i\right) \Phi_H\left(\sum_{i<j}((x_i - x_j)^2)\right),$$

$$w_i = m_i / \sum_{j=1}^{n} m_j, \quad \bar{\Phi}_H(-k^2) = \exp\left(k^2/\Lambda_M^2\right), \tag{2.1}$$

where the meson field M interacts with a non-local quark current J_M. The symbol Γ_M corresponds to an appropriate string of Dirac matrices (depending on the particle spin) and the vertex function F_M is constructed in the way to be Lorentz invariant. It contains one free parameter Λ related to the size of the meson, the Gaussian shape is assumed for calculational convenience. The weights w_i are taken such as to match the quark-system barycenter with the meson position.

In addition to Λs, the CQM contains as free parameters four constituent quark masses and one infrared cutoff, so in total $N + 5$ parameters are needed to describe N hadrons. Their values (Table 1) are fixed by fits of the model to available experimental data. So-called compositeness condition [5, 6] is used to get a correct descriptions of hadrons as bound states of quarks and fix the values of the couplings g_M. It requires $Z_M^{1/2} = 0$, where the renormalization constant Z_M is interpreted as the matrix element between the physical state and the corresponding bare state. Making it zero implies that the physical state does not contain bare state and is therefore properly described as a bound state.

Parameter	$m_{u,d}$	m_s	m_c	m_b	λ_{cutoff}	Λ_B	Λ_K	Λ_K^*
Numerical value [in GeV]	0.241	0.428	1.67	5.05	0.181	1.96	1.02	0.75

Table 1: Parameters of the CQM model.

3. Computational techniques and infrared confinement.

A general Feynman graph with j external momenta, l loop integrations, m vertices and n quark propagators can be written as

$$\Pi(p_1,...,p_j) = \int [d^4 k]^{\ell} \prod_{i_1=1}^{m} \Phi_{i_1+n} \left(-K_{i_1+n}^2 \right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3}), \quad K_{i_1+n}^2 = \sum_{i_2} (\tilde{k}_{i_1+n}^{(i_2)} + \tilde{p}_{i_1+n}^{(i_2)})^2, \quad (3.1)$$

where $\tilde{k}_i$ refers to the linear combination of loop momenta k_i and $\tilde{p}_i$ to the linear combination of the external momenta p_i. We use the Schwinger representation of quark propagators

$$\tilde{S}_q(k) = \left(m + \hat{k}\right) \int_0^\infty d\alpha \, e^{\left[-\alpha \left(m^2 - k^2\right)\right]} \tag{3.2}$$

and make use of couple of smart operator identities to simplify the computations. The first one

$$\int d^4 k \, P(k) e^{2kr} = \int d^4 k \, P\left(\frac{1}{2} \frac{\partial}{\partial r}\right) e^{2kr} = P\left(\frac{1}{2} \frac{\partial}{\partial r}\right) \int d^4 k \, e^{2kr}, \quad r = r(\alpha_i), \tag{3.3}$$

allows for an elegant loop momenta integration, the second one

$$\int_0^\infty d^n \alpha \, P\left(\frac{1}{2} \frac{\partial}{\partial r}\right) e^{-\frac{r^2}{a}} = \int_0^\infty d^n \alpha \, e^{-\frac{r^2}{a}} P\left(\frac{1}{2} \frac{\partial}{\partial r} - \frac{r}{a}\right), \quad a = a(\Lambda_H, \alpha_i), \tag{3.4}$$

simplifies the calculations following the trace evaluation: the derivative operator acts on unity instead of a more complex exponential function.

The final ingredient, the infrared cutoff, is introduced in relation with the integration over the Schwinger parameters. By inserting a unity in the form of a delta function, the multidimensional improper integral can by transformed to an integral over a simplex convoluted with only one improper integral. The cutoff is then applied on the upper integration limit

$$\Pi = \int_0^\infty d^n \alpha \, F\left(\alpha_1, \cdots, \alpha_n\right) = \int_0^{\infty \to \frac{1}{\lambda^2}} dt \, t^{n-1} \int_0^1 d^n \alpha \, \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \ldots, t\alpha_n), \tag{3.5}$$

where F stands for the whole structure of a given diagram. Consequently, Π becomes a smooth function with thresholds in quark loop diagrams and corresponding branch points removed. Universal value for all processes $\lambda = 0.181$ GeV is established and the integration is done numerically.

4. $B \to K^{(*)} \mu^+ \mu^-$ decay

The hadronic effects for the process are parametrized by a set of form factors, depending on the spin of the final state particle.

- Three form factors in the case of a scalar particle (K):

$$\left\langle P'_{[\bar{q}_3, q_2]}(p_2) \, | \bar{q}_2 O^\mu q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \right\rangle = F_+ \left(q^2\right) P^\mu + F_- \left(q^2\right) q^\mu, \tag{4.1}$$

$$\left\langle P'_{[\bar{q}_3, q_2]}(p_2) \, | \bar{q}_2 \left(\sigma^{\mu\nu} q_\nu\right) q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \right\rangle = \frac{i}{m_1 + m_2} \left(q^2 P^\mu - q \cdot P q^\mu\right) F_T \left(q^2\right). \tag{4.2}$$

(a)

(b)

(c)

(d)
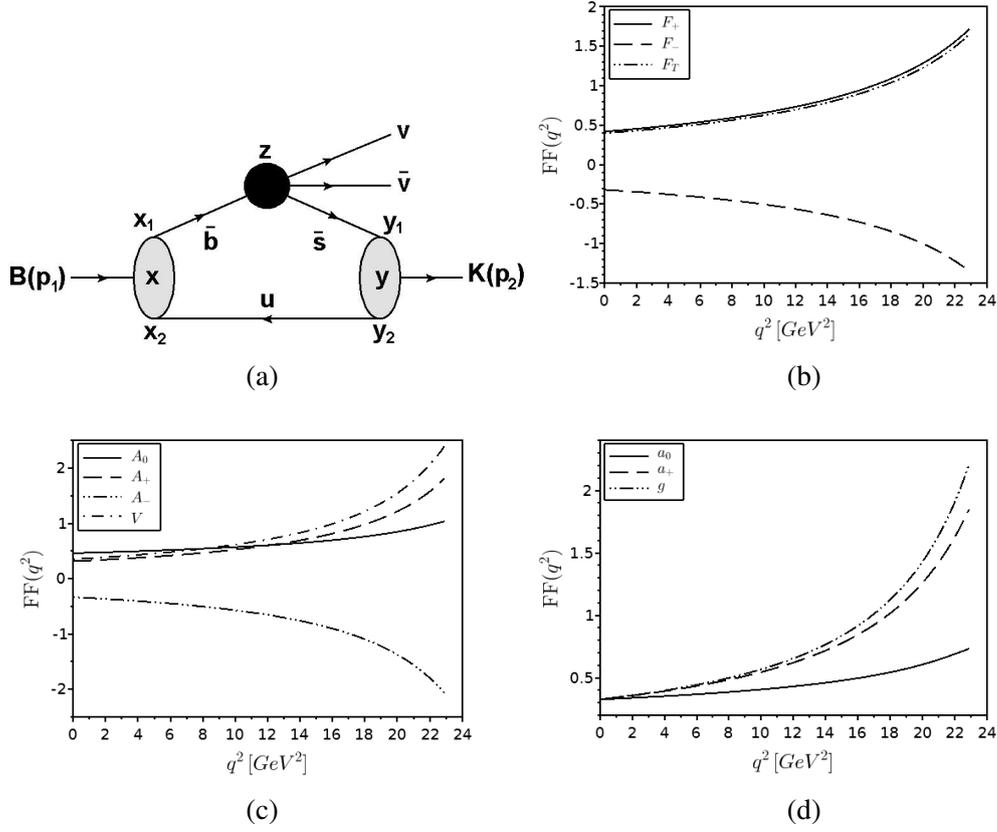
Figure 1: $B \to K^{(*)}\mu^+\mu^-$: Feynman diagram (a), $B \to K$ form factors (b) and $B \to K^*$ form factors (c-d).

- Seven form factors in the case of a vector particle (K^*):

$$\langle V_{[\bar{q}_3,q_2]}(p_2,\varepsilon_2)|\bar{q}_2 O^\mu q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \frac{\varepsilon_\nu^\dagger}{m_1+m_2}\left[-g^{\mu\nu}P\cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2)\right.$$
$$\left. + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V(q^2)\right], \qquad (4.3)$$

$$\langle V_{[\bar{q}_3,q_2]}(p_2,\varepsilon_2)|\bar{q}_2\left[\sigma^{\mu\nu}q_\nu(1+\gamma^5)\right]q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \varepsilon_\nu^\dagger\left[-\left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)P\cdot q\, a_0(q^2)\right.$$
$$\left. + \left(P^\mu P^\nu - q^\mu P^\nu \frac{p\cdot q}{q^2}\right)a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta\, g(q^2)\right]. \qquad (4.4)$$

The quark flavor transition and muon production is described by a four-fermion vertex using effective theory with local operators and Wilson coefficients. Their values are taken from the literature. The diagram of the process and the CQM form factors are shown on Fig 1.

5. Observables and results

We consider the cascade decay $B \to K^{(*)}(\to K\pi)\mu^+\mu^-$ in order to fully explore the available experimental information (for kinematics see Fig 2(a)). We use the helicity approach to derive

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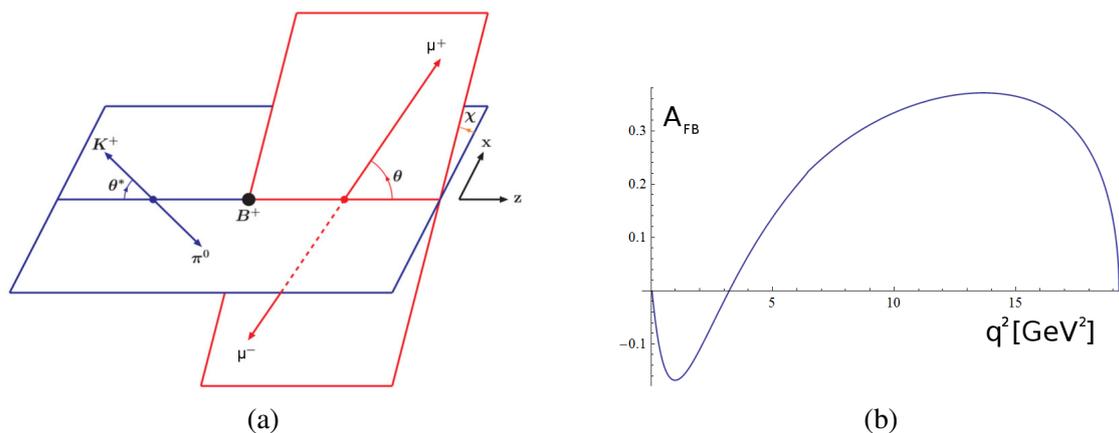
Figure 2: Kinematics of the $B \to K^{(*)}(\to K\pi)\mu^+\mu^-$ decay (a) and the CQM prediction for A_{FB} (b).

the angular distributions and formulas for observables. They are expressed in terms of so-called helicity amplitudes (details in Ref. [7]).

The observables are chosen such as to be sensitive to possible new physic, but not sensitive to hadronic effects. In addition, they need to be well experimentally accessible. In this text we present our results on the branching fractions (Tab. 2), the lepton forward-backward asymmetry A_{FB} and the K^* longitudinal polarization F_L. The A_{FB} and F_L are coefficient q^2-dependent functions if front of the definite angular terms

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{d\cos\theta_l \, dq^2} = \frac{3}{4}F_L(1-\cos^2\theta_l) + \frac{3}{8}(1-F_L)(1+\cos^2\theta_l) + A_{FB}\cos\theta_l. \tag{5.1}$$

The predicted behavior of A_{FB} in the whole allowed kinematic range is shown in Fig. 2(b). In experiments, integrated forms of these observables are used (numerator and denominator integrated separately) and are measured in various q^2 ranges. We chose the biggest one $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and present the comparison in Table 3 (K^* particle only).

6. Discussion and outlook

The CQM describes the measured data fairly well, maybe with the exception of the branching fraction for the $B \to K\mu^+\mu^-$ reaction. In all cases the predictions have the correct order of magni-

Process	Branching fraction		Ref.
	CQM	Data	
$B \to K^*\mu^+\mu^-$	1.27×10^{-6}	$(1.05 \pm 0.10) \times 10^{-6}$	[8]
$B \to K\mu^+\mu^-$	7.18×10^{-7}	$(3.4 \pm 0.5) \times 10^{-7}$	[8]
$B \to K^*\nu\bar{\nu}$	1.36×10^{-5}	$< 5.5 \times 10^{-5}$	[8]
$B \to K\nu\bar{\nu}$	0.60×10^{-5}	$< 4.9 \times 10^{-5}$	[8]
$B \to K^*\gamma$	3.74×10^{-5}	$(4.21 \pm 0.18) \times 10^{-5}$	[9]

Table 2: Branching fractions for $B \to K^{(*)}\mu^+\mu^-$ and similar processes.

	CQM	[1]	[2]	[3]
$<A_{FB}>$	0.022	$0.26^{+0.27}_{-0.30}\pm0.07$	$-0.06^{+0.13}_{-0.14}\pm0.04$	$0.29^{+0.20}_{-0.23}\pm0.07$
$<F_L>$	0.75	$0.67\pm0.23\pm0.05$	$0.55\pm0.10\pm0.03$	$0.69^{+0.19}_{-0.21}\pm0.08$

Table 3: Integrated observables $<A_{FB}>$ and $<F_L>$ for the $B \to K^*\mu^+\mu^-$ decay.

tude and for what concerns angular observables in the K^* case, the experimental uncertainties are still important and situation is not conclusive.

A more detailed analysis concerning $B \to K^{(*)}\mu^+\mu^-$ in the framework of the CQM is about to be published. It includes a larger set of observables (also so-called "clean" observables P_i) and provides results in various bins.

Since the CQM proves to be a suitable tool for describing heavy hadron decays, we intend to apply it to other recently measured processes, e.g. $B_s \to \Phi\mu^+\mu^-$ or $B_s \to K_S K^*(892)^0$.

Acknowledgment

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