

Spin correlations of muons and tau leptons in $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ and of leptons in the final state of $\gamma\gamma \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

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Using the technique of helicity amplitudes, the electromagnetic process $e^+e^- \rightarrow \mu^+\mu^-$ is theoretically studied in the one-photon approximation. It is shown that, for the unpolarized electron and positron, the final muons are also unpolarized but their spins are strongly correlated. Explicit expressions for the components of the spin density matrix and correlation tensor of the final ($\mu^+\mu^-$) system are derived. It is demonstrated that here the spin correlations of muons have the purely quantum character, since one of the Bell-type incoherence inequalities for the correlation tensor components is always violated. In doing so, the qualitative character of the muon spin correlations does not change when involving the additional contribution of the weak interaction of lepton neutral currents through the virtual Z^0 boson.

Besides, the theoretical investigation of spin structure for the process $\gamma\gamma \rightarrow e^+e^-$ is performed as well. Here, quite similarly to the process $e^+e^- \rightarrow \mu^+\mu^-$, in the case of unpolarized photons the final electron and positron remain unpolarized, but their spins are strongly correlated. Explicit expressions for the components of the correlation tensor and for the relative fractions of singlet and triplet states of the final (e^+e^-) system are derived. Again, one of the Bell-type incoherence inequalities for the correlation tensor components is always violated and, thus, spin correlations of the electron and positron have the strongly pronounced quantum character.

Analogous considerations can be wholly applied as well, respectively, to the final tau leptons in the process $e^+e^- \rightarrow \tau^+\tau^-$ and to the final lepton pairs in the two-photon processes $\gamma\gamma \rightarrow \mu^+\mu^-$, $\gamma\gamma \rightarrow \tau^+\tau^-$ – all these reactions become possible at considerably higher energies.

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1. Helicity amplitudes for the annihilation process $e^+e^- \rightarrow \mu^+\mu^-$ and structure of the triplet states of the final $(\mu^+\mu^-)$ system

In the first non-vanishing approximation over the electromagnetic constant $e^2/\hbar c$, the process of conversion of the (e^+e^-) pair into the muon pair is described by the well-known one-photon Feynman diagram [1]. Due to the electromagnetic current conservation, the virtual photon with a time-like momentum transfers the total angular momentum $J = 1$ and negative parity. Since the internal parities of muons μ^+ and μ^- are opposite, the $(\mu^+\mu^-)$ pair is generated in the triplet states (total spin $S = 1$) with $J = 1$ and the orbital angular momenta $L = 0$ and $L = 2$.

The respective helicity amplitudes have the following structure:

$$f_{\Lambda'\Lambda}(\theta, \phi) = R_{\Lambda'\Lambda}(E) d_{\Lambda'\Lambda}^{(1)}(\theta) \exp(i\Lambda\phi), \quad (1.1)$$

where θ, ϕ are the polar and azimuthal angles of the flight direction of the positive muon (μ^+) in the c.m. frame of the reaction with respect to the initial positron momentum; $d_{\Lambda'\Lambda}^{(1)}(\theta)$ are the Wigner functions for $J = 1$; Λ is the difference of helicities of the positron and electron, coinciding with the projections of total spin and total angular momentum of the (e^+e^-) pair onto the direction of positron momentum in the c.m. frame; Λ' is the difference of helicities of the muons μ^+ and μ^- , coinciding with the projection of total angular momentum of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame (see, e.g., [1,2]).

In Eq. (1.1), $R_{\Lambda'\Lambda}(E) = r_{\Lambda'}^{(\mu)}(E) r_{\Lambda}^{(e)}(E)$ ($\Lambda', \Lambda = +1, 0, -1$) – due to the factorizability of the Born amplitude, and here $r_{+1}^{(\mu)} = r_{-1}^{(\mu)} = r_1^{(\mu)}$, $r_{+1}^{(e)} = r_{-1}^{(e)} = r_1^{(e)}$ – owing to the space parity conservation in the electromagnetic interactions. Further, in accordance with the structure of electromagnetic current for the pairs (e^+e^-) and $(\mu^+\mu^-)$ in the c.m. frame [1], the following relations are valid:

$$r_0^{(\mu)} = \frac{m_\mu}{E} r_1^{(\mu)} = \sqrt{1 - \beta_\mu^2} r_1^{(\mu)}, \quad r_0^{(e)} = \frac{m_e}{E} r_1^{(e)}, \quad (1.2)$$

where m_μ, m_e are the muon and electron masses and β_μ is the muon velocity in the c.m. frame. Thus, since we always have $E \geq m_\mu \gg m_e$ for the given process, the contribution of (e^+e^-) -states with equal helicities can be neglected, i.e. $R_{\Lambda 0}(E) \approx 0$.

The one-photon diagram calculation gives (here e is the electron charge):

$$r_1^{(\mu)}(E) = r_1^{(e)}(E) = \frac{|e|}{\sqrt{2E}}. \quad (1.3)$$

Using the expressions (1.1)–(1.3) and the formulas for d -functions at $J = 1$ [1,2], we may find the effective cross section of the reaction $e^+e^- \rightarrow \mu^+\mu^-$ and the angular distribution of muon emission, normalized by unity, in the c.m. frame (see [3]).

Taking into account Eqs. (1.1)–(1.3), it is clear that, in the cases of total polarization of both the positron and electron along the positron momentum in the c.m. frame and in the direction being antiparallel to the positron momentum, the $(\mu^+\mu^-)$ system is produced, respectively, in the triplet states of the following form [3]:

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left(\frac{1 + \cos \theta}{2} | +1 \rangle - \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 - \cos \theta}{2} | -1 \rangle \right). \quad (1.4)$$

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left(\frac{1 - \cos \theta}{2} | +1 \rangle + \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 + \cos \theta}{2} | -1 \rangle \right). \quad (1.5)$$

Here β_μ is the velocity of each of the muons – just as in Eq. (1.2), and $| +1 \rangle, | -1 \rangle, | 0 \rangle$ are the states with the projection of total spin of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame, equaling $+1, -1$ and 0 , respectively.

2. Correlation tensor of the $(\mu^+\mu^-)$ pair and violation of "classical" incoherence inequalities

If the positron and electron are not polarized, then, since $r_0^{(e)} \approx 0$, the final state of the $(\mu^+\mu^-)$ pair represents an incoherent mixture of spin states $|\Psi\rangle^{(+1)}$ (1.4) and $|\Psi\rangle^{(-1)}$ (1.5), each of them being realized with the relative probability of $1/2$.

The components of the correlation tensor for two particles with spin $1/2$ are defined as: $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$ ($i, k \rightarrow \{1, 2, 3\} \rightarrow \{x, y, z\}$; the symbol $\langle \dots \rangle$ denotes the averaging over the quantum ensemble). For the final $(\mu^+\mu^-)$ pair under consideration, the axis z is directed along the momentum of μ^+ in the c.m. frame of the reaction $e^+e^- \rightarrow \mu^+\mu^-$, and the axis y – along the normal to the reaction plane.

It is easy to see that, in the case of unpolarized primary positron and electron, the produced muons μ^+ and μ^- are also unpolarized but their spins are correlated, and the correlation tensor components have the following form [3]:

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2 - \beta_\mu^2) \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{zz}^{(\mu^+\mu^-)} = \frac{2 \cos^2 \theta + \beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta},$$

$$T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{xz}^{(\mu^+\mu^-)} = -\frac{(1 - \beta_\mu^2)^{1/2} \sin 2\theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{xy} = T_{yz} = 0. \quad (2.1)$$

Just as it should hold for triplet states, the "trace" of the correlation tensor is equal to unity: $T^{(\mu^+\mu^-)} = T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1$. Let us note that the "trace" of the correlation tensor T determines the angular correlation between flight directions for the products of decay of two unstable particles with spin $1/2$ under the space parity nonconservation (see [4-8]). In particular, for the decays of the muons μ^- and μ^+ ($\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$, $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$), produced in the process $e^+e^- \rightarrow \mu^+\mu^-$, we obtain [3] (in accordance with the general formula for angular correlation [4,5,7,8]):

$$dW^{(\mu^+\mu^-)} = \frac{1}{2} \left(1 + \frac{\alpha_1 \alpha_2 T}{3} \cos \delta \right) d(-\cos \delta) = \frac{1}{2} \left(1 - \frac{1}{27} \cos \delta \right) d(-\cos \delta). \quad (2.2)$$

Here α_1 and α_2 are the angular asymmetry coefficients for the decays of the first (μ^-) and second (μ^+) particle (here we have: $\alpha_1 = -1/3$, $\alpha_2 = +1/3$ [9]), $\cos \delta = \mathbf{n}_1 \mathbf{n}_2$, \mathbf{n}_1 and \mathbf{n}_2 are unit vectors along the momenta of particles formed in the first and second decay (in our case – e^- and e^+), which are defined, respectively, in the rest frames of the first and second unstable particle and specified with respect to a unified system of spatial coordinate axes [7,8] (for further details, see also [3]).

Regarding the components (2.1) of the correlation tensor, we observe here the violation of the "classical" incoherence inequalities (for incoherent mixtures of factorizable states of two particles with spin 1/2, the modulus of sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity [4,5]). Indeed, according to Eqs. (2.1), one of the incoherence inequalities is always violated at $\theta \neq 0$ [3, 10] :

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1 - T_{yy}^{(\mu^+\mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2 \theta} > 1. \quad (2.3)$$

Thus, we see that the spin correlations of muons in the process $e^+e^- \rightarrow \mu^+\mu^-$ have the strongly pronounced quantum character.

Certainly, the above consideration can be wholly applied also to the process $e^+e^- \rightarrow \tau^+\tau^-$ – with the replacements $m_\mu \rightarrow m_\tau$, $\beta_\mu \rightarrow \beta_\tau$.

At very high energies $E \gg m_\mu$ (m_τ), when $\beta_\mu, \beta_\tau \rightarrow 1$, the nonzero components of the correlation tensor for the final lepton pair take – in accordance with Eqs. (2.1) – the following values:

$$T_{xx} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}, \quad T_{yy} = -\frac{\sin^2 \theta}{1 + \cos^2 \theta}, \quad T_{zz} = 1, \quad (2.4)$$

and we see that one of the incoherence inequalities is still violated : $T_{xx} + T_{zz} \geq 1$.

Finally, it should be noted that at very high energies the annihilation processes $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$ are conditioned not only by the electromagnetic interaction through the virtual photon, but also by the weak interaction of neutral currents through the virtual Z^0 boson [9]. The interference of amplitudes of the purely electromagnetic and weak interaction leads to the charge asymmetry in lepton emission and to the effects of space parity violation. Due to the contribution of weak interaction, in the case of unpolarized electron and positron the final leptons acquire the longitudinal polarization; the polarization vectors of the positively and negatively charged leptons are the same, and their average helicities have different signs on account of the opposite directions of momenta in the c.m. frame (see [3]).

The structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at very high energies – see Eqs. (2.4). In doing so, the nonzero components of the correlation tensor are: $T_{zz} = 1$, $T_{xx} = -T_{yy}$, as before (but, of course, the expression for T_{xx} changes [3]). Again, one of the incoherence inequalities for the correlation tensor components is violated: $T_{xx} + T_{zz} > 1$ [3] .

3. Spin structure of the two-photon process $\gamma\gamma \rightarrow e^+e^-$ and correlation tensor of the (e^+e^-) pair

Now let us consider another electromagnetic process – the electron-positron pair production by two photons: $\gamma\gamma \rightarrow e^+e^-$.

The spin state of the electron-positron system is described, in the general case, by the two-particle spin density matrix :

$$\hat{\rho}^{(e^-e^+)} = \frac{1}{4} \left[\hat{I}^{(e^-)} \otimes \hat{I}^{(e^+)} + \hat{I}^{(e^-)} \otimes (\hat{\sigma}^{(e^+)} \mathbf{P}^{(e^+)}) + (\hat{\sigma}^{(e^-)} \mathbf{P}^{(e^-)}) \otimes \hat{I}^{(e^+)} + \sum_{i=1}^3 \sum_{k=1}^3 T_{ik} \hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)} \right], \quad (3.1)$$

where \hat{I} is the two-row unit matrix, $\mathbf{P}^{(e^-)}$ and $\mathbf{P}^{(e^+)}$ are the polarization vectors of the electron and positron, respectively, T_{ik} – components of the correlation tensor ($T_{ik} = \langle \hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)} \rangle$), $i, k = \{1, 2, 3\} = \{x, y, z\}$).

In the absence of correlations, we have: $T_{ik} = P_i^{(e^-)} P_k^{(e^+)}$.

The process $\gamma\gamma \rightarrow e^+e^-$ is described by two well-known Feynman diagrams [1]. Within the first nonvanishing approximation over the electromagnetic constant (Born approximation), in the case of unpolarized primary photons the final electron and positron prove to be also unpolarized, but their spins are correlated. Thus, in the above formula for the spin density matrix $\hat{\rho}^{(e^+e^-)}$ (3.1) we have: $\mathbf{P}^{(e^-)} = \mathbf{P}^{(e^+)} = 0$; meantime, the components of the correlation tensor of the (e^+e^-) pair, generated by unpolarized γ quanta, may be calculated by applying the results of the paper [11]. Finally we obtain the following expressions :

$$T_{zz} = 1 - \frac{2(1-\beta^2)[\beta^2(1+\sin^2\theta)+1]}{1+2\beta^2\sin^2\theta-\beta^4-\beta^4\sin^4\theta}, \quad (3.2)$$

$$T_{yy} = \frac{(1-\beta^2)[\beta^2(1+\sin^2\theta)-1]-\beta^2\sin^4\theta}{1+2\beta^2\sin^2\theta-\beta^4-\beta^4\sin^4\theta}, \quad (3.3)$$

$$T_{xx} = \frac{(1-\beta^2)[\beta^2(1+\sin^2\theta)-1]+\beta^2\sin^4\theta}{1+2\beta^2\sin^2\theta-\beta^4-\beta^4\sin^4\theta}. \quad (3.4)$$

Here the axis z is aligned along the positron momentum in the c.m. frame, the axis x lies in the reaction plane and the axis y is directed along the normal to the reaction plane; $\beta = \frac{v}{c}$, $1-\beta^2 = \frac{m_e c^2}{E_+}$, where \mathbf{v} is the positron velocity and E_+ is the positron (or electron) energy in the c.m. frame; θ is the angle between the positron momentum and the momentum of one of the photons in the c.m. frame.

Meantime, the differential cross section of the process $\gamma\gamma \rightarrow e^+e^-$ in the c.m. frame has the following form [1, 2] (here $r_0 = \frac{e^2}{m_e c^2}$):

$$\frac{d\sigma}{d\Omega} = r_0^2 \frac{1-\beta^2}{4} \beta \left[\frac{1+2\beta^2\sin^2\theta-\beta^4-\beta^4\sin^4\theta}{(1-\beta^2\cos^2\theta)^2} \right]. \quad (3.5)$$

The "trace" of the correlation tensor of the final (e^+e^-) pair is determined by the formula :

$$T = T_{xx} + T_{yy} + T_{zz} = 1 - \frac{4(1-\beta^2)}{1+2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}. \quad (3.6)$$

In doing so, the relative fraction of the triplet states is as follows [5] :

$$W_t = \frac{T+3}{4} = 1 - \frac{1-\beta^2}{1+2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}, \quad (3.7)$$

and the relative fraction of the singlet state (total spin $S = 0$) equals

$$W_s = \frac{1-T}{4} = 1 - W_t = \frac{1-\beta^2}{1+2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}. \quad (3.8)$$

At $\beta \ll 1$ we have: $W_t \approx 0$, $W_s \approx 1$, i. e. the (e^+e^-) pair is generated in the singlet state only. Meantime, if $\beta \approx 1$ and $2\sin^2\theta - \sin^4\theta > 1 - \beta^2$, we obtain, on the contrary : $W_t \approx 1$, $W_s \approx 0$ – almost pure triplet state of the pair.

Let us remark also that the generation of singlet (e^+e^-) pairs is provided only by states of two photons with negative space parity, which are antisymmetric over polarizations and correspond to the total spin 1 ; if both the photons are unpolarized, the relative fraction of such photon pairs amounts to $1/4$. Meantime, the generation of triplet (e^+e^-) pairs is provided only by states of two photons with positive space parity, which are symmetric over polarizations and correspond to the total spins 0 and 2 ; if both the photons are unpolarized, the relative fraction of such photon pairs equals $3/4$ (see the detailed analysis, e.g., in [12]) .

4. Violation of "classical" incoherence inequalities for correlation tensor components in the processes $\gamma\gamma \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

Let us consider now the particular cases $\theta = 0$ and $\theta = \pi$. In accordance with the formula for the differential cross section $\frac{d\sigma}{d\Omega}$ (3.5), here we obtain :

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} r_0^2 \beta (1 + \beta^2). \quad (4.1)$$

In the ultrarelativistic limit formula (4.1) turns to $\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}$.

According to the general expressions (3.2)–(3.4) for the correlation tensor components, at $\theta = 0$ and $\theta = \pi$ we have :

$$T_{zz} = 1 - \frac{2(1+\beta^2)(1-\beta^2)}{1-\beta^4} = -1; \quad T_{xx} = T_{yy} = -\frac{1-\beta^2}{1+\beta^2}. \quad (4.2)$$

In doing so, the "trace" of the correlation tensor (3.6) takes the value

$$T = 1 - \frac{4}{1+\beta^2} = -\frac{3-\beta^2}{1+\beta^2}, \quad (4.3)$$

and the relative fractions of the triplet states W_t (3.7) and the singlet state W_s (3.8) amount to

$$W_t = \frac{T+3}{4} = \frac{\beta^2}{1+\beta^2}, \quad W_s = \frac{1-T}{4} = \frac{1}{1+\beta^2}. \quad (4.4)$$

At nonrelativistic velocities $W_t \approx 0$, $W_s \approx 1$, in accordance with the general case ; meantime, in the ultrarelativistic limit ($\beta \rightarrow 1$) we have : $W_t = W_s = \frac{1}{2}$.

It should be stressed that in the process $\gamma\gamma \rightarrow e^+e^-$ we observe again – just as in the above-considered reaction $e^+e^- \rightarrow \mu^+\mu^-$ – the violation of the "incoherence" inequalities for diagonal components of the correlation tensor, established previously at the classical level [5] (see Sec. 2). Indeed, for the cases $\theta = 0$ and $\theta = \pi$ we obtain, in particular (since $\beta < 1$) :

$$|T_{zz} + T_{xx}| = |T_{zz} + T_{yy}| = \frac{2}{1+\beta^2} > 1. \quad (4.5)$$

Thus, the spin correlations of the final electron and positron in the considered two-photon process also have the strongly pronounced quantum character – quite similarly to the spin correlations of the final leptons in the processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$.

Finally, one should remark that analogous results hold as well for the processes of generation of a muon pair and a tau-lepton pair by two photons : $\gamma\gamma \rightarrow \mu^+\mu^-$, $\gamma\gamma \rightarrow \tau^+\tau^-$, which become possible at considerably higher energies .

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