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Precise Ratios of Decay Constants of Vector over Pseudoscalar $B_{(s)}$ Mesons

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The relative magnitude of the decay constants of the pseudoscalar and vector beauty mesons poses (in contrast to the case of charmed mesons) a long-standing puzzle. We revisit this problem within the framework of our recent improvements of the QCD sum-rule formalism for relating observable properties of mesons to quantum chromodynamics and are led to conclude that the decay constants of the beauty vector mesons are undoubtedly smaller than those of their pseudoscalar counterparts.

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1. Technical keystone of motivation: *The advantage of being* ration*al*

By chance, uncertainties of physical observables (*e.g.*, decay constants) may partially cancel in ratios of these quantities. This may render predictions for such ratios more precise than those for the individual quantities. We intend to exploit this serendipity to relate the $B_{(s)}^{(*)}$-meson decay constants.

2. QCD sum rules for beauty vector mesons

We analyze the decay constants f_V of the beauty *vector* mesons B^* and B_s^* — defined for vector mesons V with mass M_V and polarization vector $\varepsilon_\mu(p)$ in terms of heavy–light quark vector currents $j_\mu(x) = \overline{q}(x)\,\gamma_\mu\,Q(x)$ according to $\langle 0|\,j_\mu(0)|V(p)\rangle = f_V\,M_V\,\varepsilon_\mu(p)$ — by means of QCD sum rules; in particular, we'll be interested in the *relative* magnitude of the decay constants of such vector mesons and their pseudoscalar counterparts, as (in contrast to initial belief and the charmed-meson case) our preliminary results [1] provided a first hint that in the beauty sector the decay constants of the vector mesons are *smaller* than those (f_P) of the pseudoscalar ones. Starting from the two-point correlators

$$\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}\,p\,x}\,\langle 0|T\left(j_\mu(x)\,j_\nu^\dagger(0)\right)|0\rangle = \left(-g_{\mu\nu}+\frac{p_\mu\,p_\nu}{p^2}\right)\Pi(p^2)+\frac{p_\mu\,p_\nu}{p^2}\,\Pi_L(p^2)$$

subjected to *operator product expansion* (OPE), Borel transformation to a Borel variable, τ, and the postulate that above *effective thresholds* $s_{\rm eff}(\tau)$ all unknown contributions of excited and continuum hadron states equal those of perturbative QCD, we eventually deduce from $\Pi(p^2)$ the QCD sum rule

$$f_V^2\,M_V^2\,\mathrm{e}^{-M_V^2\,\tau} = \int\limits_{(m_Q+m)^2}^{s_{\rm eff}(\tau)} \mathrm{d}s\,\mathrm{e}^{-s\,\tau}\,\rho_{\rm pert}(s,\mu)+\Pi_{\rm power}(\tau,\mu)\equiv\Pi_{\rm dual}(\tau,s_{\rm eff}(\tau))\ .$$

The right-hand side of this relation forms the "dual correlator" $\Pi_{\rm dual}(\tau,s_{\rm eff}(\tau))$, which receives both perturbative contributions usually encoded in a dispersion integral of an appropriate spectral density

$$\rho_{\rm pert}(s)=\rho^{(0)}(s,m_b)+\frac{\alpha_{\rm s}(\nu)}{\pi}\,\rho^{(1)}(s,m_b)+\frac{\alpha_{\rm s}^2(\nu)}{\pi^2}\,\rho^{(2)}(s,m_b,\mu)+\cdots$$

and "power" contributions involving the vacuum condensates that parameterize all non-perturbative effects, and which fixes our predictions for *dual* mass and decay constant of the hadron under study:

$$M_{\rm dual}^2(\tau)\equiv-\frac{\mathrm{d}}{\mathrm{d}\tau}\log\Pi_{\rm dual}(\tau,s_{\rm eff}(\tau))\ ,\qquad f_{\rm dual}^2(\tau)\equiv\frac{\mathrm{e}^{M_V^2\,\tau}}{M_V^2}\,\Pi_{\rm dual}(\tau,s_{\rm eff}(\tau))\ .$$

The effective threshold $s_{\rm eff}(\tau)$ is determined by minimizing, for polynomial *Ansätze* of low orders n

$$s_{\rm eff}^{(n)}(\tau)=\sum_{j=0}^{n} s_j^{(n)}\,\tau^j$$

with expansion coefficients $s_j^{(n)}$, the deviation of this predicted meson mass from its measured value

$$\chi^2\equiv\frac{1}{N}\sum_{i=1}^{N}\left[M_{\rm dual}^2(\tau_i)-M_V^2\right]^2$$

over a set of N equidistant discrete points τ_i in the admissible region of τ [2]. Our results' spread for polynomial orders $n=1,2,3$ enables us to estimate the *systematic* uncertainties inherent to the QCD sum-rule formalism [3]. Our novel ideas met great success when being applied to heavy mesons [4]. The actual application of this approach requires obvious numerical ingredients, collected in Table 1.

Table 1: Numerical parameter values required as input to the operator product expansions for beauty mesons.

Quantity		Numerical input value
Light-quark $\overline{\mathrm{MS}}$ mass	$\overline{m}(2\ \mathrm{GeV})$	(3.42 ± 0.09) MeV
Strange-quark $\overline{\mathrm{MS}}$ mass	$\overline{m}_s(2\ \mathrm{GeV})$	(93.8 ± 2.4) MeV
Bottom-quark $\overline{\mathrm{MS}}$ mass	$m_b \equiv \overline{m}_b(\overline{m}_b)$	(4247 ± 34) MeV [5]
Strong coupling	$\alpha_s(M_Z)$	0.1184 ± 0.0020
Light-quark condensate	$\langle \bar{q}q \rangle \equiv \langle \bar{q}q \rangle (2\ \mathrm{GeV})$	$-[(267 \pm 17)\ \mathrm{MeV}]^3$
Strange-quark condensate	$\langle \bar{s}s \rangle \equiv \langle \bar{s}s \rangle (2\ \mathrm{GeV})$	$(0.8 \pm 0.3) \times \langle \bar{q}q \rangle (2\ \mathrm{GeV})$
Two-gluon condensate	$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$	$(0.024 \pm 0.012)\ \mathrm{GeV}^4$

3. Decay-constant ratio of B^* and B mesons

Within the advanced formalism constructed and corroborated in a sequence of papers [2–4], the extraction of the decay constants from QCD sum rules proceeds along meanwhile well-paved paths:

- So far, our perturbative spectral density $\rho_{\mathrm{pert}}(s)$ has been derived, in terms of the heavy-quark pole mass, up to order $O(\alpha_s^2)$ or, equivalently, up to three-loop accuracy [6]. A reorganization of this perturbative expansion in terms of the $\overline{\mathrm{MS}}$ mass of the bottom quark bears the potential to improve the obviously confidence-inspiring *hierarchy* of the perturbative contributions [7].

- The unavoidable *truncation* of the (perturbative) spectral densities and the (non-perturbative) power contributions spoils the independence of QCD sum-rule extractions of any observables from the renormalization scale μ and provokes their (unphysical) μ dependence. Perturbative convergence and reproducibility of the B^*-meson's mass confine the acceptable values of μ to
$$3\ \mathrm{GeV} \le \mu \le 5\ \mathrm{GeV}\ .$$

- The allowed range of the *Borel variable* τ is defined by requiring the B- and B^*-meson masses and the B–B^* mass splitting to be predictable with an error less than 5 MeV over this τ region:
$$0.01\ \mathrm{GeV}^{-2} \le \tau \le 0.31\ \mathrm{GeV}^{-2} - 0.05\,\mu\ \mathrm{GeV}^{-3}\ .$$

In addition to the systematic errors, roughly measured by our algorithm for extracting an observable from a QCD sum rule [2,3], the limited precision of the input parameter values induces OPE-related *statistical* uncertainties. Our findings for f_{B^*} exhibit a linear dependence on the relevant OPE input,

$$f_{B^*}^{\mathrm{dual}}(m_b, \langle \bar{q}q \rangle, \langle \tfrac{\alpha_s}{\pi} GG \rangle) = (181.8 \pm 4_{\mathrm{syst}}) \times \left(1 - \frac{11}{181.8}\,\frac{m_b - 4.247\ \mathrm{GeV}}{0.034\ \mathrm{GeV}}\right)$$
$$\times \left(1 + \frac{7}{181.8}\,\frac{|\langle \bar{q}q \rangle|^{1/3} - 0.267\ \mathrm{GeV}}{0.017\ \mathrm{GeV}}\right) \times \left(1 - \frac{1}{181.8}\,\frac{\langle \frac{\alpha_s}{\pi} GG \rangle - 0.024\ \mathrm{GeV}^4}{0.012\ \mathrm{GeV}^4}\right) \mathrm{MeV}\ ,$$

but insensitivity to the renormalization scale in its range (Fig. 1). Averaging over assumed Gaussian distributions of all the OPE parameters but a flat distribution of the scale μ eventually yields (Fig. 2)

$$f_{B^*} = \left(181.8 \pm 13.1_{\mathrm{OPE}} \pm 4_{\mathrm{syst}}\right) \mathrm{MeV}\ .$$

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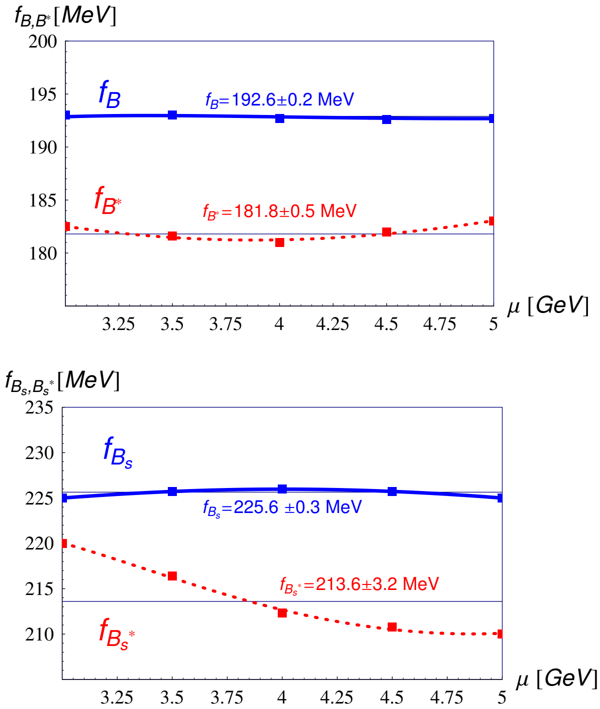

Figure 1: $B^{(*)}$ (top) and $B_s^{(*)}$ (bottom) meson decay constants $f_{(s)}^{(*)}$ as function of the renormalization scale μ.

The *ratios* of decay constants benefit from huge *cancellations* among their OPE uncertainties. Their remaining OPE errors arise primarily from the gluon condensate and their total errors are dominated by the *systematic* uncertainties. Confronting f_{B^*} with our earlier finding [5] for the B-meson's decay constant f_B, we find the B^*-meson's decay constant to lie 2.7σ *below* the B-meson's one [8] (Fig. 2):

$$\frac{f_{B^*}}{f_B} = 0.944 \pm 0.011_{\rm OPE} \pm 0.018_{\rm syst} = 0.944 \pm 0.021 \;\lneqq 1\ .$$

4. Decay-constant ratio of B_s^* and B_s mesons

Without going into details, let us now mimic the treatment of nonstrange beauty mesons for the case of strange beauty mesons, stressing the unequal aspects. The response of $f_{B_s^*}$ to 1σ variations is

$$f_{B_s^*}^{\rm dual}(\mu=\overline{\mu}, m_b, \langle\bar{s}s\rangle, \langle\tfrac{\alpha_s}{\pi}GG\rangle) = (213.6 \pm 6_{\rm syst}) \times \left(1 - \frac{13.2}{213.6}\,\frac{m_b - 4.247\ {\rm GeV}}{0.034\ {\rm GeV}}\right)$$
$$\times \left(1 + \frac{11.8}{213.6}\,\frac{|\langle\bar{s}s\rangle|^{1/3} - 0.248\ {\rm GeV}}{0.033\ {\rm GeV}}\right) \times \left(1 - \frac{1}{213.6}\,\frac{\langle\frac{\alpha_s}{\pi}GG\rangle - 0.024\ {\rm GeV}^4}{0.012\ {\rm GeV}^4}\right) {\rm MeV}\ .$$

Unlike the B^* meson, the B_s^* meson exhibits a pronounced dependence on the renormalization scale,

$$f_{B_s^*}^{\rm dual}(\mu) = 213.6\ {\rm MeV}\left(1 - 0.12\log\frac{\mu}{\overline{\mu}} + 0.11\log^2\frac{\mu}{\overline{\mu}} + 0.43\log^3\frac{\mu}{\overline{\mu}}\right),$$

introducing a kind of *average* $\overline{\mu}$ of the renormalization scale: $\overline{\mu} = 3.86$ GeV. The resulting $f_{B_s^*}$ reads

$$f_{B_s^*} = (213.6 \pm 18.2_{\rm OPE} \pm 6_{\rm syst})\ {\rm MeV}\ .$$

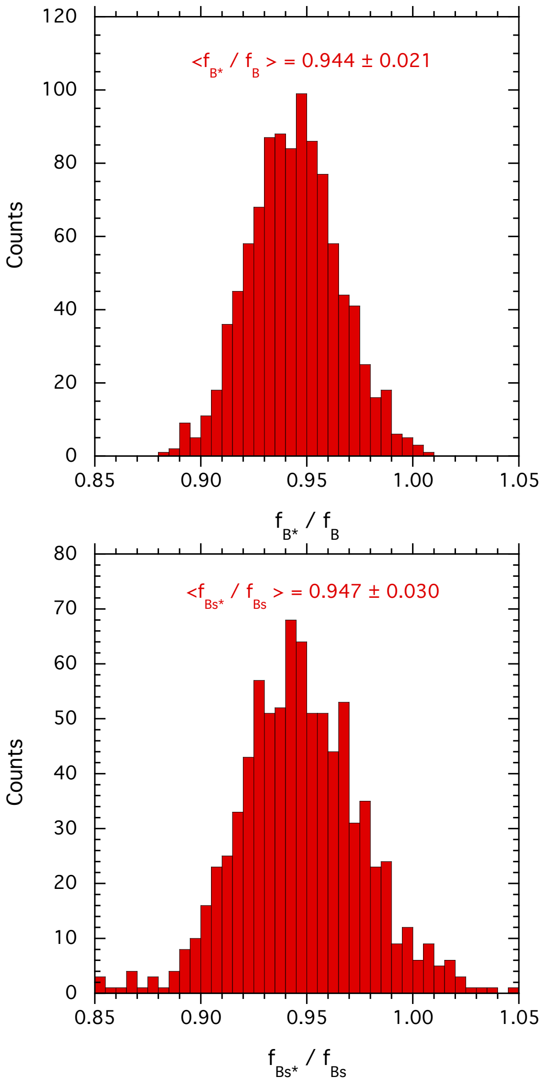

Figure 2: Gaussian-like distributions of vector vs. pseudoscalar decay-constant ratios for nonstrange, f_{B^*}/f_B (top), and strange, $f_{B_s^*}/f_{B_s}$ (bottom), beauty mesons from a bootstrap study relying on 1000 generated events.

For completeness, the corresponding relations of the pseudoscalar strange beauty meson B_s are

$$f_{B_s}^{\rm dual}(m_b,\langle\bar{s}s\rangle,\langle\tfrac{\alpha_s}{\pi}GG\rangle)=(225.6\pm 3_{\rm syst})\times\left(1-\frac{14.1}{225.6}\,\frac{m_b-4.247\text{ GeV}}{0.034\text{ GeV}}\right)$$
$$\times\left(1+\frac{11.5}{225.6}\,\frac{|\langle\bar{s}s\rangle|^{1/3}-0.248\text{ GeV}}{0.033\text{ GeV}}\right)\times\left(1+\frac{1}{225.6}\,\frac{\langle\frac{\alpha_s}{\pi}GG\rangle-0.024\text{ GeV}^4}{0.012\text{ GeV}^4}\right)\text{MeV}$$

for the behaviour of f_{B_s} under 1σ variations of all crucial OPE parameters and, as our f_{B_s} prediction,

$$f_{B_s}=(225.6\pm 18.3_{\rm OPE}\pm 3_{\rm syst})\text{ MeV}\,.$$

In the case of strange beauty mesons, their decay-constant ratio is thus 1.7σ *below* unity [8] (Fig. 2):

$$\frac{f_{B_s^*}}{f_{B_s}}=0.947\pm 0.023_{\rm OPE}\pm 0.020_{\rm syst}=0.947\pm 0.030\ \lnapprox 1\,.$$

5. Observations, conclusions, and comparison

Our QCD sum-rule analysis of the decay constants of $B_{(s)}^{(*)}$ mesons [8] provides a lot of insights:

- As in the case of the pseudoscalar heavy mesons [4], the highly unsatisfactory convergence of the perturbative expansion formulated in terms of the pole mass of the heavy quark found also for vector heavy mesons enforces conversion of the OPE to the $\overline{\rm MS}$ quark-mass definition [8].
- Acceptable reproduction of the experimentally measured masses of the beauty vector mesons by our QCD sum-rule dual predictions necessitates a correlation between the upper boundary of the adoptable Borel-variable range and the renormalization scale chosen for evaluation [8].
- Very accurate reproduction of the meson masses and their splitting, enabled by our concept of extraction of an observable [2], is imperative for the smallness of the systematic uncertainties.
- A study of beauty-meson decay constants within the realm of lattice-regularized QCD carried out practically simultaneously to our analysis gets [9], in perfect agreement with our findings,

$$\frac{f_{B^*}}{f_B} = 0.941 \pm 0.026 \,, \qquad \frac{f_{B_s^*}}{f_{B_s}} = 0.953 \pm 0.023 \,.$$

So, the outcomes of the present study add a great deal of credibility to our initial observation [1]: the decay constants of vector beauty mesons are, beyond doubt, smaller than those of their pseudoscalar counterparts; hence, we are no longer stunned by our inability to reproduce the claims of Refs. [10].

References

[1] W. Lucha, D. Melikhov, and S. Simula, EPJ Web Conf. **80** (2014) 00046, arXiv:1410.6684 [hep-ph]; arXiv:1411.3890 [hep-ph]; arXiv:1411.7844 [hep-ph].

[2] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **79** (2009) 096011, arXiv:0902.4202 [hep-ph]; J. Phys. G **37** (2010) 035003, arXiv:0905.0963 [hep-ph]; Phys. Lett. B **687** (2010) 48, arXiv:0912.5017 [hep-ph]; Phys. Atom. Nucl. **73** (2010) 1770, arXiv:1003.1463 [hep-ph]; W. Lucha, D. Melikhov, H. Sazdjian, and S. Simula, Phys. Rev. D **80** (2009) 114028, arXiv:0910.3164 [hep-ph].

[3] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **76** (2007) 036002, arXiv:0705.0470 [hep-ph]; Phys. Lett. B **657** (2007) 148, arXiv:0709.1584 [hep-ph]; Phys. Atom. Nucl. **71** (2008) 1461; Phys. Lett. B **671** (2009) 445, arXiv:0810.1920 [hep-ph]; D. Melikhov, Phys. Lett. B **671** (2009) 450, arXiv:0810.4497 [hep-ph].

[4] W. Lucha, D. Melikhov, and S. Simula, J. Phys. G **38** (2011) 105002, arXiv:1008.2698 [hep-ph]; Phys. Lett. B **701** (2011) 82, arXiv:1101.5986 [hep-ph]; Phys. Lett. B **735** (2014) 12, arXiv:1404.0293 [hep-ph].

[5] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **88** (2013) 056011, arXiv:1305.7099 [hep-ph].

[6] K. G. Chetyrkin and M. Steinhauser, Phys. Lett. B **502** (2001) 104, arXiv:hep-ph/0012002; Eur. Phys. J. C **21** (2001) 319, arXiv:hep-ph/0108017.

[7] M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005, arXiv:hep-ph/0108135.

[8] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **91** (2015) 116009, arXiv:1504.03017 [hep-ph].

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[9] B. Colquhoun *et al.* (HPQCD Coll.), Phys. Rev. D **91** (2015) 114509, arXiv:1503.05762 [hep-lat].

[10] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys. Rev. D **88** (2013) 014015, arXiv:1305.5432 [hep-ph]; **89** (2014) 099901(E); **91** (2015) 099901(E); S. Narison, Int. J. Mod. Phys. A **30** (2015) 1550116, arXiv:1404.6642 [hep-ph]; D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B **635** (2006) 93, arXiv:hep-ph/0602110; D. Bečirević, A. Le Yaouanc, A. Oyanguren, P. Roudeau, and F. Sanfilippo, arXiv:1407.1019 [hep-ph].