PROCEEDINGS OF SCIENCE

Correct mass and width of $f_0(500)$ meson from pion scalar form factor

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We propose a theoretical approach based on a model independent pion scalar form factor analysis and GKPY2 Roy-like equations [1] which allows to determine the parameters of the $f_0(500)$ meson, i.e. its mass and decay width to be $m_{f_0(500)} = (472 \pm 10)$ MeV and $\Gamma_{f_0(500)} = (524 \pm 22)$ MeV.

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1. Introduction

The pion scalar form factor (FF) is defined via a parametrization of the pion matrix element containing a scalar current

$$< \pi^i(p_2)|\hat{m}(\bar{u}u+\bar{d}d)|\pi^j(p_1)>= \delta^{ij}\Gamma_\pi(t), \tag{1.1}$$

$$t=(p_2-p_1)^2, \quad \hat{m}=(m_u+m_d)/2. \tag{1.2}$$

From theoretical considerations several properties of this form factor can be established:

- $\Gamma_\pi(t)$ is an analytic function for all $t\varepsilon\mathbb{R}$ besides the cut on the positive real axis from $t=4m_\pi^2$ to infinity.
- It obeys the reality condition
$$\Gamma_\pi(t)^* = \Gamma_\pi(t^*). \tag{1.3}$$
- The asymptotic behavior is
$$\Gamma_\pi(t)_{|t|\to\infty} \sim 1/t. \tag{1.4}$$
- In the elastic region $4m_\pi^2 \le t \le 16m_\pi^2$ the FF fulfills elastic unitarity condition
$$\mathrm{Im}\{\Gamma_\pi(t)\} = M_0^0\Gamma_\pi^*(t), \tag{1.5}$$
where M_0^0 denotes $I=J=0$ partial wave $\pi\pi$-scattering amplitude.

We adopt the normalization[1] $\Gamma_\pi(0)=1$.

2. Our method

We propose a fully solvable mathematical scheme developed for finding out the pion scalar FF in an explicit form. The dispersion relation with one subtraction

$$\Gamma_\pi(t) = 1+\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{Im}\{\Gamma_\pi(t')\}}{t'(t'-t)}dt' \tag{2.1}$$

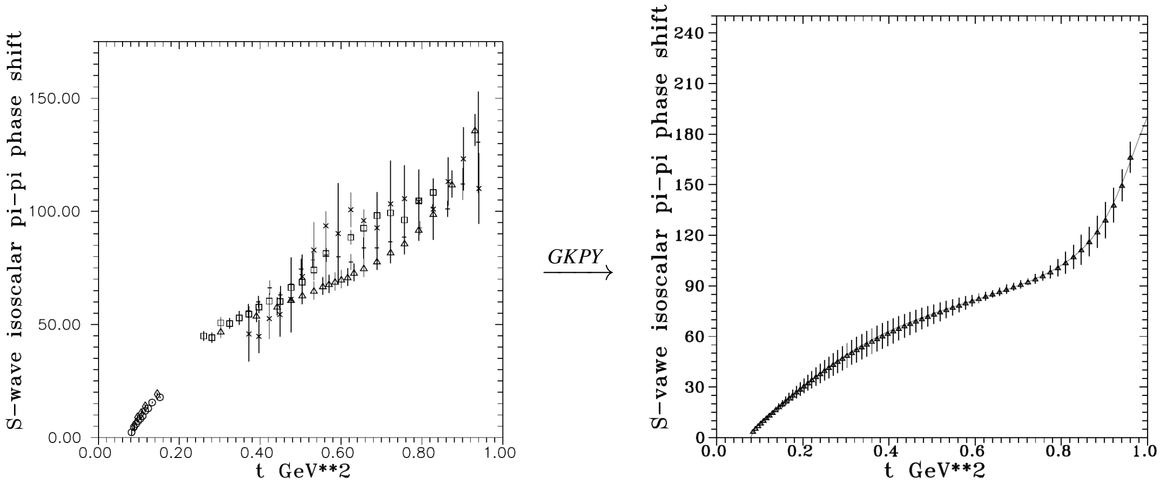
is combined with the elastic unitarity condition 1.5 to give the Muskhelishvili-Omnès integral equation [3, 4]. The latter has a known solution in the form (so-called FF phase representation)

$$\Gamma_\pi(t) = P_n(t)\exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_\Gamma(t')}{t'(t'-t)}dt'\right], \tag{2.2}$$

where $P_n(t)$ is an arbitrary polynomial (normalized at $t=0$ GeV^2) and δ_Γ is the FF phase.

Next the δ_Γ phase is related with the S-wave isoscalar $\pi\pi$-scattering phase shift in the $4m_\pi^2 \le t \le 16m_\pi^2$ region. From the parametrized form of the partial wave scattering amplitude $M_0^0 = e^{i\delta_0^0}\sin\delta_0^0$ and the elastic unitarity condition one obtains an expression for the imaginary part of $\Gamma_\pi(t)$, $\mathrm{Im}\{\Gamma_\pi\} = e^{i\delta_0^0}\Gamma_\pi^*\sin\delta_0^0(t)$, which needs to be identical to $\mathrm{Im}\{\Gamma_\pi(t)\} \equiv |\Gamma_\pi(t)|\sin\delta_\Gamma(t)$ for every t. A comparison implies

$$\delta_0^0 \equiv \delta_\Gamma. \tag{2.3}$$



(a) (b)

Figure 1: Raw experimental data on δ_0^0 (a) and the same data after the GKPY procedure (b).

The existing data on δ_0^0 are very scattered. However, with help of the rigorous theoretical approach by Kaminski et al. [1] the data can be reprocessed with errors dramatically reduced, Fig 1. To find an appropriate description of δ_0^0, analytic properties of $\Gamma_\pi(t)$ are exploited. It can be shown that $\Gamma_\pi(t)$ has a square-root type branch point at $t = 4m_\pi^2$, which can be removed by applying the conformal mapping into the variable q

$$q = \sqrt{\frac{t-4}{4}} \qquad (m_\pi = 1). \tag{2.4}$$

Neglecting higher branch points, $\Gamma_\pi(q)$ has only poles and zeros. Taking in addition into account the reality condition 1.3 a general parametrization of $\tan \delta_\Gamma$ can be worked out, leading to

$$\delta_\Gamma(q) \equiv \delta_0^0(q) = \arctan \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \dots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \dots}, \tag{2.5}$$

where A_i are real numbers (A_1 being the S-wave isoscalar $\pi\pi$ scattering length a_0^0). Next a fit to the (reprocessed) data is performed (Fig 1(b)) where the number of parameters is chosen such as to minimize the χ^2/ndf. The minimum is reached with five coefficient having the following numerical values

$$\begin{aligned}
A_1 &= 0.23456 \pm 0.00778, \\
A_3 &= 0.11595 \pm 0.00296, \\
A_5 &= -.01180 \pm 0.00031, \\
A_2 &= -.10376 \pm 0.00373, \\
A_4 &= -.00288 \pm 0.00046.
\end{aligned} \tag{2.6}$$

[1]The ChPT predicts the norm to be $0.99 \pm 0.02 m_\pi^2$ [2].

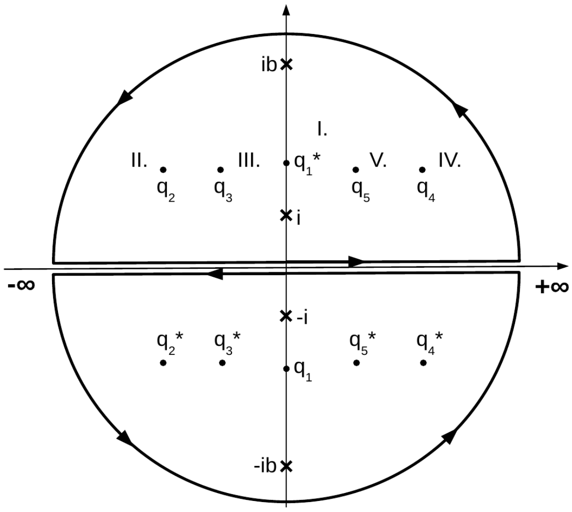


Figure 2: Poles (×) and branch points (•) of the integrands of Eqs. 2.8 with contours of integrations in the upper and the lower half q-planes, respectively.

The identity $\arctan(z) = i[\ln(1-iz) - \ln(1+iz)]/2$ allows to insert the phase in the logarithmic form into the FF expression 2.2, obtaining

$$\Gamma_\pi(t) = P_n(t) \exp\left[\frac{q^2+1}{2\pi i}\int_{-\infty}^{\infty} \frac{q' \ln \frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2+1)(q'^2-q^2)} dq'\right]. \tag{2.7}$$

Now theory of residues is used (Fig. 2). The integral (I) in 2.7 can be split and evaluated separately for the upper and lower half-plane $I = I_1 + I_2$ ($b \equiv -iq$)

$$I_1 = \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{q'-q_1^*}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq', \quad I_2 = \int_{-\infty}^{\infty} \frac{q' \ln \frac{q'-q_1}{(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq', \tag{2.8}$$

where the integration over the (infinite) half-circles is vanishing since the integrands decrease rapidly enough for $|q| \to \infty$. The computation of residua allows for evaluation of the integrals $I_{1,2} = 2\pi i \sum_{n=1}^{2} Res_n$, finally giving

$$I = \frac{2\pi i}{q^2+1} ln\left[\frac{q-q_1}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)} \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{i-q_1}\right]. \tag{2.9}$$

Substituting 2.9 into 2.7 leads to an explicit expression for the pion scalar form factor. We identify the pole $q = -q_3$ on the second Riemann sheet in the t-variable as the $f_0(500)$ resonance, thus determining its mass and width to be

$$m_{f_0(500)} = (472 \pm 10)\,\text{MeV}, \tag{2.10}$$

$$\Gamma_{f_0(500)} = (524 \pm 22)\,\text{MeV}. \tag{2.11}$$

These results are in agreement with those obtained by [5, 6]. The predicted behavior of the scalar pion form factor on the interval $-3\text{GeV}^2 < t < 3\text{GeV}^2$ is shown in Fig. 3.

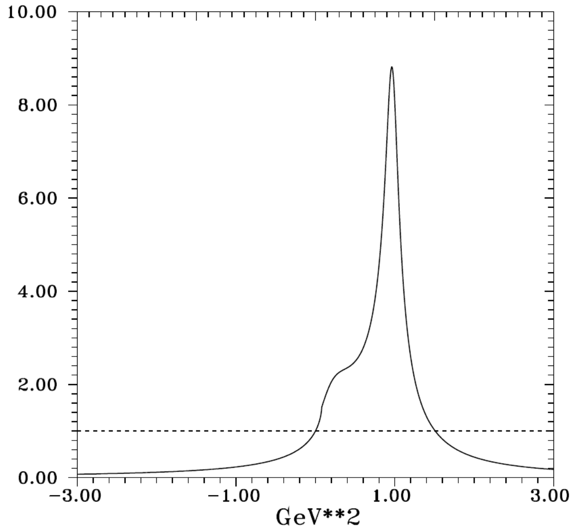


Figure 3: Behavior of the pion scalar form factor in the region $-3\text{GeV}^2 < t < 3\text{GeV}^2$.

3. Summary and conclusion

We have demonstrated that analytic properties of $\Gamma_\pi(t)$ together with the GKPY procedure allow for a prediction of the $\Gamma_\pi(t)$ behavior and an unambiguous determination of the $f_0(500)$ resonance parameters with small model dependence. Our numbers confirm some of the previously published determinations.

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