

## $K_L \rightarrow \pi^0 \nu \bar{\nu}$ Beyond the Grossman–Nir Bound

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**George W.S. Hou**\* †

*National Taiwan University*

*E-mail:* wshou@phys.ntu.edu.tw

We point out that the commonly perceived Grossman–Nir bound of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.4 \times 10^{-9}$  may not actually apply. This is because the E787/E949  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  experiments have a “blind spot” for unobservable objects with mass around  $m_{\pi^0}$ , which is kinematically excluded from the signal region; the situation remains true for the NA62 experiment. Without such kinematic ability, the “blindman” approach of the KOTO experiment could in fact discover  $K_L \rightarrow \pi^0 X^0$ , where  $X^0$  is not observed but with  $m_{X^0} \sim m_{\pi^0}$ . We give an explicit model and discuss possible implications as an illustration.

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\*Speaker.

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## 1. Kaon: the Oldest Frontier

We have known the kaon since almost 70 years, its discovery being one of the defining moments for the emergence of particle physics. But quaint as it is, the search of the very rare  $K \rightarrow \pi \nu \bar{\nu}$  decays, which have yet to be established, remains a forefront quest. One fixation for almost 20 years has been the Grossman–Nir (GN) bound [1], which comes in two forms:

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \lesssim 4.3 \times \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \quad (1.1)$$

$$< 1.4 \times 10^{-9}. \quad (\text{GN bound}) \quad (1.2)$$

The factor of 4.3 in Eq. (1.1) arises mostly from  $\tau_{K_L}/\tau_{K^+}$  [1] and isospin, while Eq. (1.2), the commonly *perceived* “GN bound”, follows from inserting the E787/E949 value [2] for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ . Paraphrasing Taylor Swift, the theme of this talk is to “*Shake it off!*”, i.e. Eq. (1.2), as it is not fool-proof.

The KOTO experiment running at J-PARC, KEK aims at measuring the Standard Model (SM) value of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  predicted around  $3 \times 10^{-11}$ . But in KOTO’s own proposal and public presentations, it adheres strictly to the “GN bound”, such that their “business space” does not start until Eq. (1.2) is reached. Alas, KOTO has suffered inadvertent delays, first the 2011 earthquake and especially tsunami damage to J-PARC, and then the unfortunate radioactive leak at J-PARC. Because of the latter, the 2013 run was stopped at only 100 hours. Analysis of this data was finally announced at CKM2014, with sensitivity comparable to the precursor experiment, E391a [3]:

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8}, \quad (90\% \text{ C.L., E391a}) \quad (1.3)$$

which is still far above the GN bound of Eq. (1.2).

Renewed running of KOTO in 2015 hopes to finally breach the GN bound. We emphasize [4], however, that KOTO should “*Shake it off!*”, and be aware that above the GN bound of Eq. (1.2) lies a unique zone for discovery.

## 2. Blind Senses

“Stupid is as stupid does.” This is one famous line Forrest Gump’s mother always advises. But Forrest broke through *Running*, and eventually became rich! With an incoming  $K_L$ , which cannot be detected, and detecting two photons as the only handle for the outgoing  $\pi^0$ , but *without proper identification* because kinematics is unknown, KOTO is quite handicapped compared with its charged cousin, the  $K^+ \rightarrow \pi^+ + \text{nothing}$  experiments of E787/E949 and NA62.

Fig. 16 of the E787/E949 paper [2] has stared us in the face since a long time, with two signal boxes pinching the large spot exploding with bright red dots and blue triangles ... Because of the large  $K^+ \rightarrow \pi^+ \pi^0$  branching ratio, the  $K^+$  experiments have elected to *kinematically exclude* this blinding spot from the signal boxes, whether using stopped  $K^+$ , or decay in-flight. After all, the holy grail is the three-body  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay. E949 did make a study exploring the  $\pi^0$  window (the “chimney” of Fig. 18, Ref. [2]), but achieved a rather poor limit of  $\mathcal{B}(K^+ \rightarrow \pi^+ X^0) < 5.6 \times 10^{-8}$  for  $m_{X^0} \sim m_{\pi^0}$  where  $X^0$  is not observed (with  $X^0 \rightarrow \nu \bar{\nu}$  a possibility). When combined with Eq. (1.1), it leads to a much weaker bound than E391a on  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ , Eq. (1.3). The root

cause for this is because in the dedicated study [5], with 3 feeddown (non- $K_{\pi 2}$ ) events expected, 99 events appeared in the  $\pi^0 \rightarrow \nu\bar{\nu}$  (or  $X^0$ ) signal box, which was attributed to poor understanding of photonuclear interactions. The situation may not improve at NA62.

What if Nature chooses to send in an  $X^0$  at  $\pi^0$  mass, i.e. through the “chimney”? The currently running NA62 uses  $K^+$  decay in flight, but it continues to kinematically exclude the region around  $m_{\pi^0}$ ; a second region of exclusion (same with E787/E949) for  $m_{\text{mis.}}^2 > 0.068 \text{ GeV}^2$  is due to the large branching ratio of  $K_{\pi 3}$  decay. Thus, if an “ $X^0$ ” lurks in these regions, we would not learn from E787/E949, nor by NA62. The curious thing for KOTO, in contrast, is in fact the lack of kinematic control of the  $K_L \rightarrow \pi^0 + \text{nothing}$ , hence the strategy is to “veto everything”. However, one cannot veto weakly interacting light particles (WILPs) — the  $\nu\bar{\nu}$  being the target. Thus, “Blind man Blessed by Senses.”: For  $K \rightarrow \pi X^0$  where  $X^0$  is a WILP that falls into the missing mass window, the  $K^+$  experiments would be oblivious, *but the  $K_L$  experiment can have a blunt feel of it!*

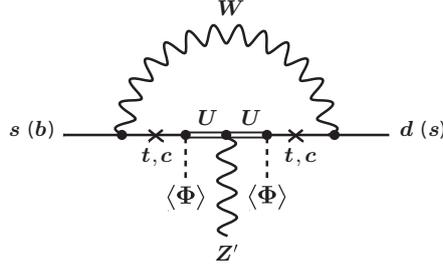
Though the GN relation of Eq. (1.1) is robust, the perceived GN bound of Eq. (1.2) does not apply. This is the surprising, almost “trivial” point, independent of model discussions. The KOTO experiment at J-PARC can discover  $K_L^0 \rightarrow \pi^0 X^0$  above the “Grossman–Nir bound” of  $1.4 \times 10^{-9}$ , starting with the 2015 run! If KOTO finds in the current run some “background” that cannot be removed, check whether it is consistent with a missing WILP with mass close to a  $\pi^0$ .

### 3. Explicit Model (existence proof)

To make the case more convincing, we offer an explicit model, which serves as an existence proof. Let us start by considering the gauged  $L_\mu - L_\tau$  leptonic force [6]. By taking the difference between the muon and tauon numbers, the U(1) gauge theory is anomaly free, while the associated  $Z'$  boson is the least probed of such type of leptonic gauge forces: any association with a gauged  $L_e$  force, i.e. the other two differences of gauged lepton numbers, would be much more tightly constrained.

Our original interest [4] was in rare  $t \rightarrow cZ'$  top decays. It is well known that a heavy  $Z'$  boson could account for the so-called  $P'_5$  anomaly uncovered by the LHCb experiment in  $B \rightarrow K^* \mu^+ \mu^-$  decays, where a  $3.7\sigma$  deviation from SM expectation is seen for  $1 \text{ fb}^{-1}$  data, and persists for  $3 \text{ fb}^{-1}$  (but unfortunately, the significance did not increase with more data). Invoking the gauged  $L_\mu - L_\tau$  force on the muon side, Altmannshofer *et al.* [7] (AGPY) constructed a model by adding vector-like quarks  $Q, D, U$  that couple to the  $Z'$  boson, where  $Q$  implies both left- and right-handed doublets, while  $D$  and  $U$  are left-right singlets. The exotic quarks mix with SM quarks via an exotic scalar field  $\phi$  with  $Z'$  charge, where  $\langle \phi \rangle$  generates  $m_{Z'}$  and induces  $bsZ'$  coupling at tree level. Note that, in this way, the  $U$  quark induces  $t_R \rightarrow c_R Z'$  transition (this can be effectively seen from inside the loop of Fig. 1) that is not linked with the  $P'_5$  anomaly, hence remain unconstrained. For a discussion of  $t \rightarrow cZ'$  decay, see the talk by Kohda [8]. We remark that, for the gauged  $L_\mu - L_\tau$  interactions,  $Z' \rightarrow \mu^+ \mu^-$  decay occurs with 1/3 branching fraction, much higher than the  $Z$  boson, and the ATLAS and CMS experiments are encouraged to pursue  $t \rightarrow cZ \rightarrow c\mu^+ \mu^-$ .

It would be the best of two worlds if the AGPY model for  $P'_5$  anomaly could also account for the muon  $g - 2$  anomaly. A second AGPY paper, Ref. [9], followed shortly after the first, where, probably to the authors' own surprise, they found that a not so well known process, called “neutrino



**Figure 1:** Effective  $dsZ'$  ( $sbZ'$ ) coupling, with  $Z'$  coupled to a vector-like  $U$  quark that mixes with  $c, t$  (“ $\times$ ” flips chirality) and connect with external  $d$ -type quarks via a  $W$  boson loop.

trident” production, or  $\nu_\mu N \rightarrow \nu_\mu \mu^+ \mu^- N$ , constrains the  $Z'$  to be rather light,

$$m_{Z'} \lesssim 400 \text{ MeV}. \quad (3.1)$$

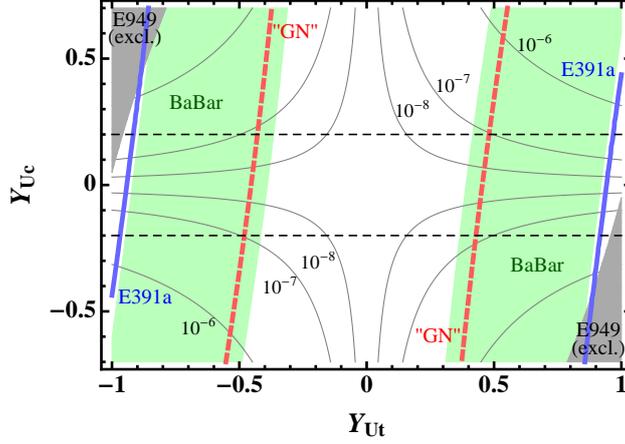
This precludes the  $Z'$  from being responsible for the  $P_5'$  anomaly, as a heavy  $Z'$  is needed. But it is rather intriguing to think that *New Physics* behind the muon  $g - 2$  anomaly could arise from a *light particle*! This is possible only because Eq. (3.1) implies the gauge coupling  $g' \lesssim 10^{-3}$ , far weaker than the weak coupling (but the  $Z'$  is not Dark Matter, as it decays quite fast). However, being lighter than the kaon inspired us to study the impact of this  $Z'$  on kaon and  $B$  physics.

The  $Z'$  model of Ref. [7] induces tree level FCNC for  $b, s$  and  $t$  quarks. For the  $Z'$  related to muon  $g - 2$ , its lightness and the very precise measurements in  $B$  and  $K$  sectors make it rather “precarious”. To avoid fine-tuning, it is prudent to decouple the  $Q$  and  $D$  quarks, which can be achieved by discrete  $Z_2$  charge assignments. But even with only the  $U$  quark, as illustrated in Fig. 1, a SM  $W$ -boson loop around the effective  $ttZ'$  ( $t$  can be interchanged with  $c$ ) coupling turns it into [4]  $bsZ'$  and  $sdZ'$  couplings, with help of chirality flip from  $m_{t(c)}$ . The  $sdZ'$  coupling can precisely lead to  $K \rightarrow \pi Z'$  decay, where the light  $Z'$  is a candidate for the  $X^0$  of previous section, that could lead to the surprise evasion of the GN bound of Eq. (1.2), if  $m_{Z'} \sim m_{\pi^0}$ . In fact, it was through this model that we stumbled upon the aforementioned observation.

#### 4. Where Else? — an Illustration

The explicit model also provides an illustration [4] of potential links of  $K \rightarrow \pi X^0$  decay with related phenomenology, in particular with hints or possibilities in rare  $B$  and  $K$  decays. The case can be separated into  $m_{Z'} < 2m_\mu$ , with  $Z' \rightarrow \nu\bar{\nu}$  only, and  $400 \text{ MeV} \gtrsim m_{Z'} > 2m_\mu$ , where  $Z' \rightarrow \nu\bar{\nu}$  and  $\mu^+\mu^-$ .

It is interesting to note that the BaBar experiment has a mild hint for the analogous  $B^+ \rightarrow K^+ \nu\bar{\nu}$  decay. Note that  $\mathcal{B}(B^+ \rightarrow K^+ \pi^0) \ll \mathcal{B}(K^+ \rightarrow \pi^+ \pi^0)$ , therefore there is no analogy of a  $B^+ \rightarrow K^+ \pi^0$  induced “blinding spot”. The BaBar experiment has lead the way by conducting a binned  $s_B \equiv m_{\nu\bar{\nu}}^2/m_B^2$  search [10], separating missing mass  $q^2 \equiv m_{\nu\bar{\nu}}^2$  into 10 bins. With 471M  $B\bar{B}$  pairs, they reported a *two-sided* 90% confidence interval. The lower 90% C.L. bound is driven by a mild excess in the lowest  $s_B$  bin. Translating the BaBar allowed region into the  $Y_{Ut}-Y_{Uc}$  plain, where  $Y_{Ui}$  (treated as real) are Yukawa couplings of  $\phi$  that mix exotic  $U$  quark with quarks  $i$  in



**Figure 2:** For  $m_{Z'} = 135$  MeV ( $Z' \rightarrow \nu\bar{\nu}$  100%), bounds for  $\mathcal{B}(K^+ \rightarrow \pi^+ Z') < 5.6 \times 10^{-8}$  (E949, dark grey exclusion region) and  $\mathcal{B}(K_L \rightarrow \pi^0 Z') < 2.6 \times 10^{-8}$  (E391a, blue solid), the usual “GN bound” of  $\mathcal{B}(K_L \rightarrow \pi^0 Z') \mathcal{B}(Z' \rightarrow \nu\bar{\nu}) < 1.4 \times 10^{-9}$  (red dashed), and  $2\sigma$  range for  $\mathcal{B}(B^+ \rightarrow K^+ Z') \mathcal{B}(Z' \rightarrow \nu\bar{\nu}) = (0.35_{-0.15}^{+0.6}) \times 10^{-5}$  (BaBar, light green allowed region) on the  $Y_{Uc}$ - $Y_{Ut}$  plane. The horizontal lines mark “reasonable”  $Y_{Uc}$  range, and in the backdrop we plot  $\mathcal{B}(t \rightarrow cZ')$  contours.

SM, in Fig. 2 we compare with the E949 explicit bound for  $K^+ \rightarrow \pi^+ Z'$ , the direct search bound from E391a, and the commonly perceived “GN bound” of Eq. (1.2). We see that the BaBar two-sided 90% confidence interval could by itself be interpreted as a refutation of the “GN bound” of Eq. (1.2), and that pushing below the E391a bound could lead to possible discovery.

Although Belle pioneered the  $B^+ \rightarrow K^+ \nu\bar{\nu}$  search, its followup [11] study just added 40% data but followed the same analysis, including a cut on high  $p_{K^+}$  for sake of rejecting  $B \rightarrow K^* \gamma$ , which precisely cuts against the  $B \rightarrow K^{(*)} Z'$  possibility. With existing full dataset, Belle should follow the more sophisticated path as the BaBar paper, the binned  $m_{\text{mis.}}^2$  analysis, at least as a crosscheck of the BaBar result. If a mild excess is also uncovered, then one should attempt a B factory combination. This should be followed up at Belle II, whether KOTO makes a discovery or not.

There is a second kinematic exclusion window for  $K^+ \rightarrow \pi^+ \nu\bar{\nu}$  search,  $m_{\text{mis.}} > 260$  MeV, due to  $K^+ \rightarrow \pi^+ \pi\pi$  background. In Ref. [4], we had used the  $m_{Z'} = 285$  MeV case to make estimates of what is allowed by a not so constraining study by NA48/2 experiment [12]. The case has been superseded, however, by the recent dark boson search result by LHCb [13] in  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$  decay. But for the latter, the constraint becomes weakest for  $m_{Z'} \sim 2m_\mu$  (where efficiency vanishes), or  $m_{Z'} \sim 335$  MeV. We have pointed out [4] that there seems to be some mild excess above the mean for  $1.0 \text{ GeV}^2 < q^2 < 6.0 \text{ GeV}^2$  in the  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$  result [15] from LHCb, which should be treated more carefully, as already done in Ref. [13] for  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ .

A more detailed discussion of the implications of LHCb data would be forthcoming [14]. One outcome is that  $\mathcal{B}(t \rightarrow cZ')$  for light  $Z' \rightarrow \mu^+ \mu^-$  would be suppressed, allowing  $10^{-6}$  only for not quite natural, fine-tuned  $V_{Uc}$  and  $V_{Ut}$  values. The lower branching ratio values can probably only be probed at a 100 TeV pp collider. Looking back at Fig. 2, a suppressed  $\mathcal{B}(t \rightarrow cZ')$  seems also the case for 100%  $Z' \rightarrow \nu\bar{\nu}$ , where one does not even know how to probe a low branching ratio for a monochromatic rare  $t \rightarrow c + \text{nothing}$  decay.

## 5. Conclusion

In conclusion, we have pointed out that  $K_L \rightarrow \pi^0 + \text{nothing}$  can occur above the commonly perceived Grossman-Nir bound of  $1.4 \times 10^{-9}$ . If KOTO sees early events, they should try hard to eliminate it as background, but only to a certain degree and not overly zealous. If it becomes definitely established above the GN Bound, then likely there is a “ $\pi^0$ ” mass object that would slip through NA62, and the best confirmation would likely come from Belle II (and also a BaBar–Belle combined analysis). When KOTO reaches below GN Bound, the concept is still effective, with KOTO, NA62, LHCb, and maybe Belle(II) all in the game.

Above all: *Run, KOTO, Run!* (Jenny’s call)

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