

## Effect of decoherence on clean determination of $\sin 2\beta$ and $\Delta m_d$

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The important quantities of the  $B_d^0$  system, such as  $\sin 2\beta$  and  $\Delta m_d$ , are determined under the assumption of perfect quantum coherence. However, any real system interacts with its environment and this interaction can lead to decoherence. It is therefore desirable to re-examine the procedures of determination of  $\sin 2\beta$  and  $\Delta m_d$  in meson systems with decoherence. We find that the present values of these two quantities are modulated by the decoherence parameter  $\lambda$ . Re-analysis of  $B_d^0$  data from B-factories and LHCb can lead to a clean determination of  $\lambda$ ,  $\sin 2\beta$  and  $\Delta m_d$ .

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## 1. Introduction

The time evolution of neutral mesons is used to measure a number of important parameters in flavor physics. Here a perfect quantum coherence is usually assumed. However, any real system interacts with its environment and this interaction can lead to a loss of quantum coherence. The environmental effects may arise at a fundamental level, such as the fluctuations in a quantum gravity space-time background [1, 2]. They may also arise due to the detector environment itself. Irrespective of the origin of the environment, its effect on the neutral meson systems can be taken into account by using the ideas of open quantum systems [3, 4, 5]. This formalism enables the inclusion of effects such as decoherence and dissipation in a systematic manner [6].

We study the effect of decoherence on important observables in the  $B_d^0$  meson system, such as the CP violating parameter  $\sin 2\beta$  and the  $B_d^0 - \bar{B}_d^0$  mixing parameter  $\Delta m_d$ . We show that these parameters are affected by decoherence [7]. We also suggest a number of methods which will enable clean determination of the decoherence parameter along with the other observables quite easily at the LHCb or B-factories [7]. We also attempt determination of the decoherence parameter and  $\Delta m_d$  using Belle data on the time dependent flavor asymmetry of semi-leptonic  $B_d^0$  decays as given in Ref. [8].

## 2. Open time evolution of $B^0$ meson

We are interested in the decays of  $B^0$  and  $\bar{B}^0$  mesons as well as  $B^0 \leftrightarrow \bar{B}^0$  oscillations. To describe the time evolution of all these transitions, we need a basis of three states:  $|B^0\rangle$ ,  $|\bar{B}^0\rangle$  and  $|0\rangle$ , where  $|0\rangle$  represents a state with no  $B$  meson and is required for describing the decays. We use the density matrix formalism to represent the time evolution of the  $B^0$  system.  $\rho_{B^0(\bar{B}^0)}(0)$  is the initial density matrix for the state which starts out as  $B^0(\bar{B}^0)$ . The time evolution of these matrices is governed by the Kraus operators  $K_i(t)$  as  $\rho(t) = \sum_i K_i(t)\rho(0)K_i^\dagger(t)$  [9]. The Kraus operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment [10].

The time dependent density matrices are [7]

$$\begin{aligned} \frac{\rho_{B^0}(t)}{\frac{1}{2}e^{-\Gamma t}} &= \begin{pmatrix} a_{ch} + e^{-\lambda t}a_c & -a_{sh} - ie^{-\lambda t}a_s & 0 \\ -a_{sh} + ie^{-\lambda t}a_s & a_{ch} - e^{-\lambda t}a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix}, \\ \frac{\rho_{\bar{B}^0}(t)}{\frac{1}{2}e^{-\Gamma t}} &= \begin{pmatrix} a_{ch} - e^{-\lambda t}a_c & -a_{sh} + ie^{-\lambda t}a_s & 0 \\ -a_{sh} - ie^{-\lambda t}a_s & a_{ch} + e^{-\lambda t}a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix}, \end{aligned} \quad (2.1)$$

for  $B^0$  and  $\bar{B}^0$ , respectively. In the above equation,  $a_{ch} = \cosh\left(\frac{\Delta\Gamma t}{2}\right)$ ,  $a_{sh} = \sinh\left(\frac{\Delta\Gamma t}{2}\right)$ ,  $a_c = \cos(\Delta m t)$ ,  $a_s = \sin(\Delta m t)$ ,  $\Gamma = (\Gamma_L + \Gamma_H)/2$ ,  $\Delta\Gamma = \Gamma_L - \Gamma_H$ , where  $\Gamma_L$  and  $\Gamma_H$  are the respective decay widths of the decay eigenstates  $B_L^0$  and  $B_H^0$ . Also  $\lambda$  is the decoherence parameter, due to the interaction between one-particle system and its environment.

### 3. CP asymmetry in $B_d^0 \rightarrow J/\psi K_S$

We define the decay amplitudes  $A_f \equiv A(B^0 \rightarrow f)$  and  $\bar{A}_f \equiv A(\bar{B}^0 \rightarrow f)$ . The hermitian operator describing the decays of the  $B^0$  and  $\bar{B}^0$  mesons into  $f$  is

$$\mathcal{O}_f = \begin{pmatrix} |A_f|^2 & A_f^* \bar{A}_f & 0 \\ A_f \bar{A}_f^* & |\bar{A}_f|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3.1)$$

The probability,  $P_f(B^0/\bar{B}^0; t)$ , of an initial  $B^0/\bar{B}^0$  decaying into the state  $f$  at time  $t$  is given by  $\text{Tr} \left[ \mathcal{O}_f \rho_{B^0(\bar{B}^0)}(t) \right]$ .

Let us now consider  $B_d^0 \rightarrow J/\psi K_S$  decay. One can define a CP violating observable

$$\mathcal{A}_{J/\psi K_S}(t) = \frac{P_{J/\psi K_S}(\bar{B}_d^0; t) - P_{J/\psi K_S}(B_d^0; t)}{P_{J/\psi K_S}(\bar{B}_d^0; t) + P_{J/\psi K_S}(B_d^0; t)}. \quad (3.2)$$

Calculating the probabilities using Eqs. (2.1) and (3.1), we get [7]

$$\mathcal{A}_{J/\psi K_S}(t) = \frac{(|\lambda_f|^2 - 1) \cos(\Delta m_d t) + 2\text{Im}(\lambda_f) \sin(\Delta m_d t)}{(1 + |\lambda_f|^2) \cosh\left(\frac{\Delta \Gamma_d t}{2}\right) - 2\text{Re}(\lambda_f) \sinh\left(\frac{\Delta \Gamma_d t}{2}\right)} e^{-\lambda t}, \quad (3.3)$$

where  $\lambda_f = A(\bar{B}_d^0 \rightarrow J/\psi K_S)/A(B_d^0 \rightarrow J/\psi K_S)$ . The usual expression for  $\mathcal{A}_{J/\psi K_S}(t)$  is obtained by putting  $\lambda = 0$  in the above equation. With the approximations  $\Delta \Gamma_d \approx 0$ ,  $|\lambda_f| = 1$  and  $\text{Im}(\lambda_f) \approx \sin 2\beta$ , we get

$$\mathcal{A}_{J/\psi K_S}(t) = e^{-\lambda t} \sin 2\beta \sin(\Delta m_d t). \quad (3.4)$$

Therefore we see that the coefficient of  $\sin(\Delta m_d t)$  in the CP asymmetry is  $e^{-\lambda t} \sin 2\beta$  and not  $\sin 2\beta$ ! Thus the measurement of  $\sin 2\beta$  is masked by the presence of decoherence.

### 4. Determination of $\Delta m_d$

In order to determine  $\sin 2\beta$ , we need to know  $\Delta m_d$  and  $\lambda$ . A legitimate question at this stage is that whether the measurement of  $\Delta m_d$  also affected by the presence of decoherence? LHCb, CDF and D0 experiments determine  $\Delta m_d$  by measuring rates that a state that is pure  $B_d^0$  at time  $t = 0$ , decays as either as  $B_d^0$  or  $\bar{B}_d^0$  as function of proper decay time. In the presence of decoherence, the survival (oscillation) probability of initial  $B_d^0$  meson to decay as  $B_d^0(\bar{B}_d^0)$  at a proper decay time  $t$  is given by [7]

$$P_{\pm}(t, \lambda) = \frac{e^{-\Gamma t}}{2} \left[ \cosh(\Delta \Gamma_d t/2) \pm e^{-\lambda t} \cos(\Delta m_d t) \right]. \quad (4.1)$$

The positive sign applies when the  $B_d^0$  meson decays with the same flavor as its production and the negative sign when the particle decays with opposite flavor to its production.  $\Delta m_d$  is determined from the following time dependent asymmetry:

$$A_{\text{mix}}(t, \lambda) = \frac{P_+(t, \lambda) - P_-(t, \lambda)}{P_+(t, \lambda) + P_-(t, \lambda)} = e^{-\lambda t} \frac{\cos(\Delta m_d t)}{\cosh(\Delta \Gamma_d t/2)}. \quad (4.2)$$

Thus we see that in the limit of neglecting  $\Delta\Gamma_d$ , the otherwise pure cosine dependence of mixing asymmetry is modulated by  $e^{-\lambda t}$ .

Belle and BaBar experiments determine  $\Delta m_d$  by measuring time dependent probability  $P_+(t)$  of observing unoscillated  $B_d^0\bar{B}_d^0$  events and  $P_-(t)$  of observing oscillated  $B_d^0B_d^0/\bar{B}_d^0\bar{B}_d^0$  events for two neutral  $B_d$  mesons produced in an entangled state in the decay of the  $\Upsilon(4S)$  resonance. The expressions for  $P_{\pm}(t)$ , in the presence of decoherence, are the same as those given in Eq. (4.1), except that the proper time  $t$  is replaced by the proper decay-time difference  $\Delta t$  between the decays of the two neutral  $B_d$  mesons. Therefore, we see that the determination of  $\Delta m_d$  at LHCb, CDF, D0, Belle and BaBar experiments is also masked by the presence of  $\lambda$ .

It can be shown that the time independent observables  $r_d$  (measured by ARGUS and CLEO) and  $\chi_d$  (measured by the LEP experiments), used to determine  $\Delta\Gamma_d$ , are also affected by the presence of decoherence [7].

The true value of  $\Delta m_d$ , along with  $\Delta\Gamma_d$ , can be determined by a three parameter  $(\Delta m_d, \Delta\Gamma_d, \lambda)$  fit to the time dependent mixing asymmetry  $A_{\text{mix}}(t, \lambda)$  defined in Eq. (4.2). This in turn will enable a determination of true value of  $\sin 2\beta$  using Eq. (3.3).

## 5. Estimation of $\lambda$ : An Example

We make an attempt of a clean determination of  $\lambda$ ,  $\Delta m_d$  and  $\Delta\Gamma_d$  using the experimental data of the time dependent flavor asymmetry of semi-leptonic  $B_d^0$  decays as given in Ref. [8]. We perform a  $\chi^2$  fit to  $A_{\text{mix}}(\Delta t, \lambda)$ , using the efficiency corrected distributions given in Table I of Ref. [8]. First, the fit is done by assuming no decoherence, i.e.,  $\lambda = 0$ . In this case, we find  $\Delta m_d = (0.489 \pm 0.010)$  ps<sup>-1</sup> and  $\Delta\Gamma_d = (0.087 \pm 0.054)$  ps<sup>-1</sup> with  $\chi^2/d.o.f = 8.42/9$ . We then redo the fit including decoherence. This gives  $\lambda = (-0.012 \pm 0.019)$  ps<sup>-1</sup> along with  $\Delta m_d = (0.490 \pm 0.010)$  ps<sup>-1</sup> and  $\Delta\Gamma_d = (0.144 \pm 0.088)$  ps<sup>-1</sup> with  $\chi^2/d.o.f = 8.02/8$ . Thus we see that the decoherence parameter  $\lambda$  is very loosely bound. The upper limit on  $\lambda$  is 0.03 ps<sup>-1</sup> at 95% C.L. We also find in this example that  $\Delta m_d$  is numerically unaffected where as  $\Delta\Gamma_d$  can be affected by inclusion of decoherence. Given the wealth of data coming from LHCb and expected from the KEK Super B factory, a clear picture is expected to emerge.

## 6. Comments on approximations made

The decoherence is expected to emerge from a scale much finer than that of the flavor physics. Hence for an accurate determination, one should include all the known effects, such as CP violation in mixing and decay width  $\Delta\Gamma_d$ , which are usually neglected in the extraction of  $\sin 2\beta$  and  $\Delta\Gamma_d$ .

In the determination of  $\sin 2\beta$  one should also take into account the penguin contributions. The theoretical precision for the extraction of CP violating phase  $\sin 2\beta$  from the CP asymmetry of  $B_d^0 \rightarrow J/\psi K_S$  decay is limited by contributions from doubly Cabibbo-suppressed penguin topologies which cannot be calculated in a reliable way within QCD [11, 12]. However,  $B_s^0 \rightarrow J/\psi K_S$  is related to  $B_d^0 \rightarrow J/\psi K_S$  through  $U$ -spin symmetry of strong interactions and it offers a tool to control the penguin effects [13].

## 7. Decoherence in $B_s$ systems

The present analysis can easily be extended to the  $B_s^0$  system as well. The expression for the time dependent CP asymmetry in the mode  $B_s^0 \rightarrow J/\psi\phi$  will be a function of four parameters:  $\lambda$ ,  $\sin 2\beta_s$ ,  $\Delta m_s$  and  $\Delta\Gamma_s$ . The time dependent mixing asymmetry defined in Eq. (4.2) will determine  $\lambda$ ,  $\Delta m_s$  and  $\Delta\Gamma_s$ . These two time-dependent asymmetries should be re-analysed using a four parameter fit for a clean determination of  $\sin 2\beta_s$ ,  $\Delta m_s$ ,  $\Delta\Gamma_s$  and  $\lambda$ . Also, like  $\sin 2\beta_d$ , the extraction of  $\sin 2\beta_s$  from time dependent CP asymmetry in the mode  $B_s^0 \rightarrow J/\psi\phi$  is restricted due to penguin pollution. The penguin contribution to  $B_s^0 \rightarrow J/\psi\phi$  can be estimated using decays  $B_d^0 \rightarrow J/\psi\rho$  and  $B_s^0 \rightarrow J/\psi K^*$  [11, 14].

## 8. Conclusions

We study the effect of decoherence on two important observables  $\sin 2\beta$  and  $\Delta m_d$  in a neutral meson system. We find that the asymmetries which determine these quantities are also functions of the decoherence parameter  $\lambda$ . Hence it is imperative to measure  $\lambda$  for a clean determination of these quantities. We suggest a re-analysis of the data on the above asymmetries for an accurate measurement of all the three quantities  $\lambda$ ,  $\sin 2\beta$  and  $\Delta m_d$ . The present analysis can easily be extended to the  $B_s^0$  system as well. Thus a detailed study of  $B^0$  observables can lead to tests of physics at scales much higher than those typical of flavor physics.

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