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Minimal anomaly-free chiral fermion sets and the unification of gauge couplings

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In this work we study which are the minimal anomaly-free chiral fermion sets, beyond the standard model, that lead to vector-like particles with respect to $\mathrm{SU}(3)$ and $\mathrm{U}(1)_{\mathrm{em}}$ after symmetry breaking. We focus on fermion multiplets with $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ dimensions less than or equal to 10 and 5, respectively. Furthermore, we study whether the addition of such chiral fermions allows for gauge coupling unification, at some high scale compatible with proton decay limits.

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1. Introduction

In the standard model (SM) there is no symmetry principle that relates the hypercharge, α_y, weak, α_w and strong, α_s, gauge couplings, $\alpha_{1,2,3} = \kappa_{1,2,3}\,\alpha_{y,w,s}$. However, at one-loop level and for $\kappa_i = (5/3, 1, 1)$ $\alpha_1 = \alpha_2$ around 10^{13} GeV and $\alpha_2 = \alpha_3$ around 10^{17} GeV. The gauge coupling unification (GCU) can be improved either by considering different normalization constants κ_i or by including extra particles in the theory. The last option introduces the issue of anomalies, i.e., the breaking of the symmetry of the Lagrangian at the quantum level. There are three types of anomalies: the triangular chiral anomaly [1, 2] that ensures the renormalizability of a theory; the mixed gauge-gravitational anomaly [3, 4] that ensures the general covariance of a theory and the Witten's anomaly [5] that imposes that any theory with $\mathsf{SU}(2)$ gauge group must have an even number of Weyl doublet to be mathematically consistent. These three anomalies must be absent in a consistent theory.

In the next two sections we will study [6] which are the minimal sets of chiral fermions, with arbitrary quantum numbers, that are free of anomalies and whether they lead to GCU.

2. Anomaly Cancellation

In this section we study which are the minimal sets of chiral fermions, beyond the SM particle content, that are free of anomalies. In order to preserve the parity symmetry, we will consider only those that lead to vector-like particles with respect to $\mathsf{U}(1)_{\text{em}}$ and $\mathsf{SU}(3)$. Since the SM is anomaly-free, the study is reduced to the contributions coming from the extra fermions. Without loss of generality, we consider the new fields as left-handed and with $(d_3(R), d_2(R))_{y_R}$ quantum numbers under $(\mathsf{SU}(3), \mathsf{SU}(2))_{\mathsf{U}(1)}$. The anomaly-free conditions, with respect to the SM gauge group, to be verified are the following:

$$[\mathsf{SU}(3)\text{-}\mathsf{SU}(3)\text{-}\mathsf{SU}(3)]: \qquad \sum_R A_3(R)\, d_2(R) = 0\,, \tag{2.1a}$$

$$[\mathsf{SU}(3)\text{-}\mathsf{SU}(3)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R\, t_3(R)\, d_2(R) = 0\,, \tag{2.1b}$$

$$[\mathsf{SU}(2)\text{-}\mathsf{SU}(2)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R\, t_2(R)\, d_3(R) = 0\,, \tag{2.1c}$$

$$[\mathsf{U}(1)\text{-}\mathsf{U}(1)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R^3\, d_2(R)\, d_3(R) = 0\,, \tag{2.1d}$$

$$[\text{gravity-gravity-}\mathsf{U}(1)]: \qquad \sum_R y_R\, d_2(R)\, d_3(R) = 0\,. \tag{2.1e}$$

The quantities $d_i(R)$, y_R, $A_i(R)$ and $t_i(R)$ are, respectively, the dimension, hypercharge, cubic anomaly and Dynkin index of the representation R with respect to the subgroup G_i of the SM; $A_i(R)$ and $t_i(R)$ are listed in Ref. [7].

The anomaly-free conditions given above are invariant under an overall rescaling of the hypercharge; this overall normalization will be important in determining the chiral sets in such a way that they lead to vector-like particles after electroweak symmetry breaking. Furthermore, the Witten's anomaly imposes that the sum of t_2 over all the representations in each set must be an integer number. To simplify our search we will consider representations with $\mathsf{SU}(2)$ dimensions up to 5 and $\mathsf{SU}(3)$ dimensions up to 10 and rational hypercharges.

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Set	Particle content				
P1	$(\mathbf{d},\mathbf{1})_{5z/6}$	$\oplus$	$(\mathbf{d},\mathbf{2})_{-2z/3}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{3})_{z/6}$
P2	$(\mathbf{d},\mathbf{1})_{7z/6}$	$\oplus$	$(\mathbf{d},\mathbf{3})_{-5z/6}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{4})_{z/3}$
P3	$(\mathbf{d},\mathbf{1})_{3z/2}$	$\oplus$	$(\mathbf{d},\mathbf{4})_{-z}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{5})_{z/2}$
P4	$(\mathbf{d},\mathbf{2})_{4z/3}$	$\oplus$	$(\mathbf{d},\mathbf{3})_{-7z/6}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{5})_{z/6}$

Table 1: Minimal anomaly-free chiral fermion sets for $\mathbf{d} \leq 10$ and $d_2(R) \leq 5$. Due to Witten's anomaly, the $\mathsf{SU}(3)$ dimension in P1 and P4 needs to be an even number.

Sets with only one or two chiral fermions we end up with multiplets in the adjoint representation of $\mathsf{SU}(3)$ (octect) and $\mathsf{SU}(2)$ (triplet) with zero hypercharge or with vector-like solutions like $(\mathbf{d},\mathbf{d}')_y \oplus (\overline{\mathbf{d}},\mathbf{d}')_{-y}$, that we are not interested in (more details see [6]). For three multiplets and limiting the search to $\mathbf{d} \equiv d_3(R) \leq 10$ and $d_2(R) \leq 5$ we obtain four different sets of solutions, see Table 1. We see that $d_3(R)$ is equal for all multiplets within each set and it can take all possible values (1, 3, 6, 8 or 10) for P2 and P3 however in P1 and P4 the $\mathsf{SU}(3)$ dimension can only take even values due to the Witten's anomaly [5]. The $\mathsf{SU}(3)$ dimension equal for the three multiplets of each set because we chose $d_3(R) \leq 10$.

The overall rescaling z of the hypercharge is determined by the conditions for the electric charge cancellation

$$\sum_{p=1}^{3} \sum_{j_p} \left[j_p + y_p(z) \right]^m = 0\,, \tag{2.2}$$

where m is an odd positive integer number and $j_p = -s_p, -s_p+1, \ldots, s_p-1, s_p$ with s_p given in terms of the $\mathsf{SU}(2)$ dimension as $s_p = (d_2(R_p)-1)/2$. The value of z is then determined for $m=5$: $|z| = 0$, 1 or 3 independent of the set. For $\mathbf{d} = 1$ and 8 the three values of z are allowed however for $\mathbf{d} = 3$, 6 and 10 only $|z| = 1$ is viable.

3. Gauge Coupling Unification

In this section we study the possibility of having unification of the gauge couplings in a non-supersymmetric extension of the SM by the fermions in Table 1. We will consider the presence of just the Higgs and gauge fields of the SM.

The gauge couplings $\alpha_{1,2,3}$ evolve with the energy scale according to the renormalization group equations that, at one-loop level, have exact solution given by

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\,\kappa_i}\left(b_i^{\mathrm{SM}} + b_i^1 r_1 + b_i^2 r_2 + b_i^3 r_3 \right) \ln\left(\frac{\Lambda}{M_Z}\right). \tag{3.1}$$

At some high energy scale the three gauge couplings unify in a common value α_U, that we consider to be $\lesssim 1$ to ensure the perturbative regime.

The contribution of the extra three fermions is encoded in the parameters $r_{1,2,3}$ as

$$r_{1,2,3} = \frac{\ln\left(\Lambda/M_{1,2,3}\right)}{\ln\left(\Lambda/M_Z\right)}\,, \quad 0 \leq r_{1,2,3} \leq 1\,, \tag{3.2}$$

and becomes relevant above the thresholds $M_{1,2,3}$.

In Ref. [6] it is explained how to compute the one-loop beta coefficients, b_i, that are $b_1^{\rm SM} = 41/6$, $b_2^{\rm SM} = -19/6$ and $b_3^{\rm SM} = -7$ for the SM.

To study whether the sets in Table 1 lead to unification of the gauge couplings, we performed the so-called B-test. In this test we compare two quantities,

$$B \equiv \frac{\sin^2\theta_w - \dfrac{\kappa_2\,\alpha}{\kappa_3\,\alpha_s}}{\dfrac{\kappa_2}{\kappa_1} - \left(1+\dfrac{\kappa_2}{\kappa_1}\right)\sin^2\theta_w} \qquad \text{and} \qquad \widetilde{B} \equiv \frac{2\pi}{\alpha}\left[\frac{1}{\kappa_1} - \left(\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right)\sin^2\theta_w\right], \tag{3.3}$$

determined completely in terms of α^{-1}, α_s and $\sin^2\theta_w$ at M_Z scale, with those obtained from the remaining data. For $\kappa_1 = \kappa_2 = \kappa_3 = 1$ (no GUT embedding) and $\alpha^{-1} = 127.944 \pm 0.014$, $\alpha_s = 0.1185 \pm 0.0006$, $\sin^2\theta_w = 0.23126 \pm 0.00005$ [8] we get $B = 0.308 \pm 0.001$ and $\widetilde{B} = 431.4 \pm 0.1$.

We choose, without lost of generality, to scan r_2 and r_3 inside the allowed range and determine the scale of the remaining particle and the unification scale through the expressions

$$r_1 = \frac{B\,B'_{12} - B'_{23}}{\frac{b_2^1}{\kappa_2} - \frac{b_3^1}{\kappa_3} - B\left(\frac{b_1^1}{\kappa_1} - \frac{b_2^1}{\kappa_2}\right)} \qquad \text{and} \qquad \ln\left(\frac{\Lambda}{M_Z}\right) = \frac{\widetilde{B}}{B_1 - B_2}, \tag{3.4}$$

where

$$B'_{ij} = \frac{1}{\kappa_i}\left(b_i^{\rm SM} + b_i^2 r_2 + b_i^3 r_3\right) - \frac{1}{\kappa_j}\left(b_j^{\rm SM} + b_j^2 r_2 + b_j^3 r_3\right).$$

The results are given in Figures 1 and 2. For each set, the allowed intermediate mass scales (colored bars) for the multiplets, ordered as they appear in Table 1, and the unification scale (black bar) in terms of the SU(3) dimension are given. In our search we accepted only the solutions for which the unification scale is higher than 5×10^{15} GeV in order to be consistent with proton decay bounds.

Looking over the results one verifies that there is no gauge coupling unification for $z = 0$ in none of the sets. For P1, it is possible to have unification only when $\mathbf{d} = 8$ and the overall normalization is $z = 3$ (Figure 1). For P2 and $z = 1$ (first row left panel of Figure 2) it is possible to have unification for $\mathbf{d} = 3$, 6 and 8. In this case, the masses of $(\mathbf{d},\mathbf{4})_{-1}$ and $(\overline{\mathbf{d}},\mathbf{5})_{1/2}$ are above 10^9 GeV, while the mass of the weak singlet (blue bar) can take values around the TeV scale or even lower. For $z = 3$ (first row right panel of Figure 2) there are solutions only for $\mathbf{d} = 1$ and 8. The set P3 is very similar to P2 but with an additional solution for $\mathbf{d} = 10$ and $z = 1$ (second row left panel Figure 2). The content of P4 leads to gauge unification for $z = 1$ with any even SU(3) dimension (last row left panel of Figure 2) with the intermediate scales above 10^{10} GeV while for $z = 3$ there is unification only for $\mathbf{d} = 8$ (last row right panel of Figure 2) with intermediate scales higher than 10^{15} GeV.

From the results one sees that some of the new particles decouple from the theory at very high scales much larger than the electroweak scale. Since the charged scalars are very constrained by electroweak precision data, the generation of large masses for the extra fermions via vacuum expectation value of some extra scalar fields seems to be unfeasible. One possible way out is to generate them dynamically, what will be analyzed somewhere else.

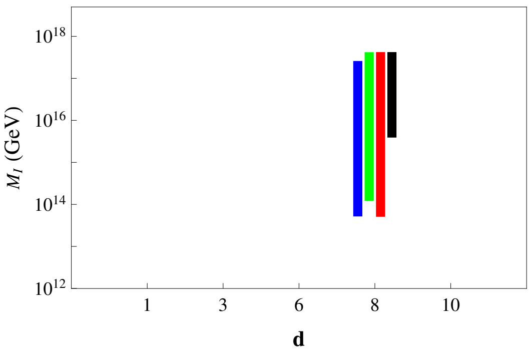

Figure 1: Intermediate, M_I, and unification, Λ, scales as function of SU(3) dimension, **d**, for P1 with $z = 3$. The colored bars are to the energy scales of $(\mathbf{d},\mathbf{1})_{\frac{5}{2}}$ (blue), $(\mathbf{d},\mathbf{2})_{-2}$ (green), $(\bar{\mathbf{d}},\mathbf{3})_{\frac{1}{2}}$ (red) and Λ (black).

4. Conclusions

In this work we studied which are the minimal sets of chiral fermions, beyond the SM, that are free of anomalies and lead to vector-like particles under SU(3) and $U(1)_{em}$ after symmetry breaking. We considered fermions with arbitrary quantum numbers under the SM gauge group and we restricted our search to $d_3(R) \leq 10$ and $d_2(R) \leq 5$. For such requirements we found four sets with three multiplets each.

We studied also whether the addition, to the SM, of such anomaly-free sets would lead to gauge coupling unification. We performed an approximate study where the non-supersymmetric running of gauge couplings is taken at one-loop level. Among all the possibilities only 16 solutions lead to successful gauge unification. In Ref. [6] we extended the study presented here to the case where the chiral fermions were part of SU(5) representations, with dimensions less than 70. It was also studied the gauge and gravitational unification at string scale for both scenarios, fermion with arbitrary quantum numbers and fermions in multiplets of SU(5) representations.

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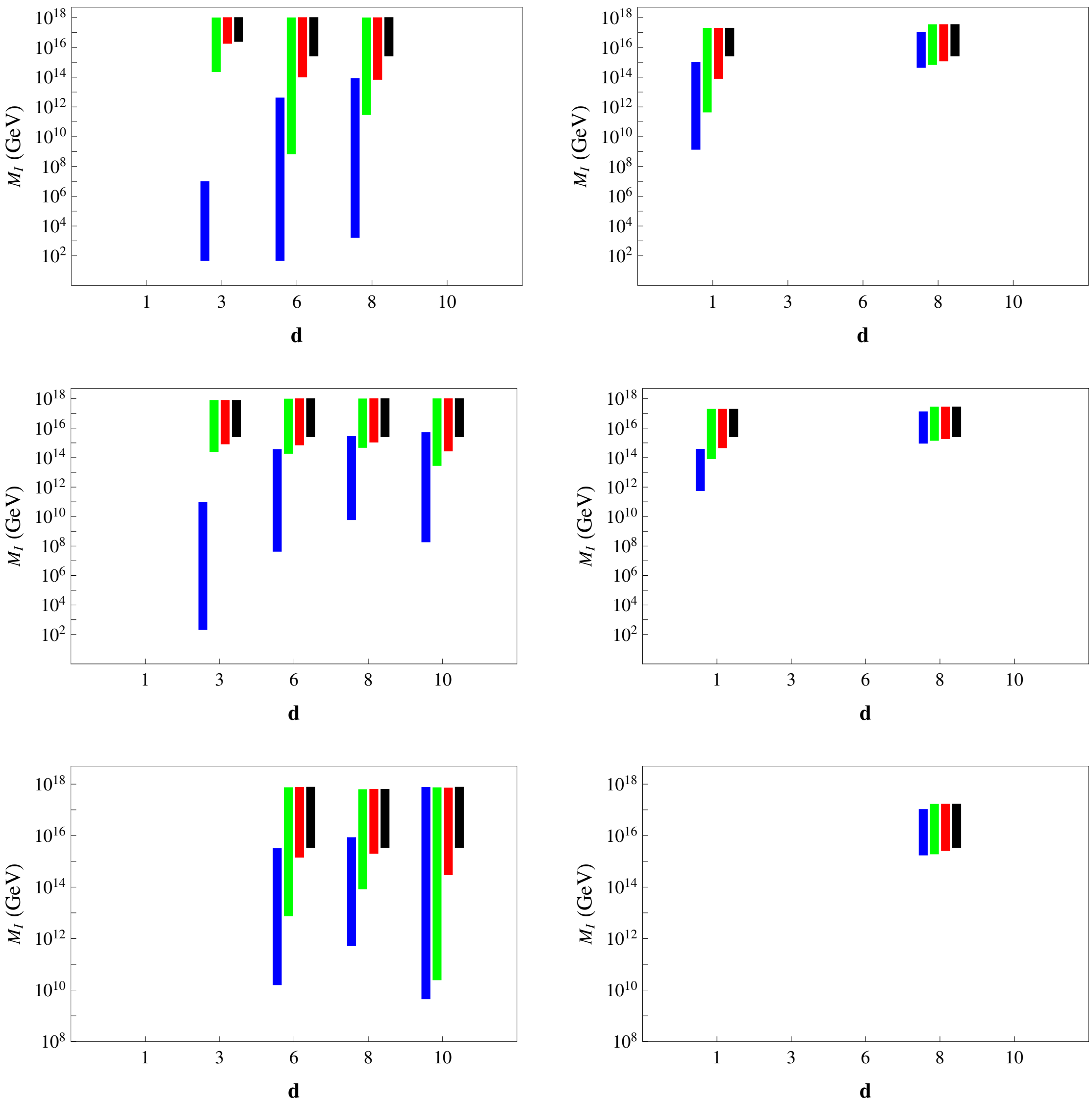

Figure 2: Intermediate, M_I, and unification, Λ, scales as function of the SU(3) dimension, **d**, for P2 (first row), P3 (second row) and P4 (last row) with $z = 1$ (left panels) and $z = 3$ (right panels). For each value of **d** the colored bars correspond to the energy scales of the first (blue), second (green) and third (red) multiplets in Table 1, Λ is in black.