NP models with extended gauge groups and extra dimensions: 
Impact on flavour observables in RS$_c$

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Deviations with respect to Standard Model predictions have recently shown up in angular distributions of the FCNC induced mode $B^0 \rightarrow K^{*0}\mu^+\mu^-$. Within New Physics models, such tensions might be explained by new contributions to the Wilson coefficients of the effective Hamiltonian governing this decay. I discuss the issue in the framework of the Randall-Sundrum model with custodial protection (RS$_c$), giving also predictions for other rare $B$ decays.
1. Introduction

Among rare $B$ decays, the mode $B \to K^* \ell^+ \ell^-$ plays a prominent role. Being a loop-induced process within the Standard Model (SM), possible new particles in the loops can modify the predictions for the numerous observables that can be measured, namely, the branching ratio, the forward-backward lepton asymmetry, the $K^*$ longitudinal polarization fraction in a few bins of $q^2$, the $\ell^+ \ell^-$ invariant mass, which were measured at the B factories for $\ell = e, \mu$. Recently, LHCb has found discrepancies with respect to SM predictions that could be hints of New Physics. Here I discuss this issue, describing the study performed in [1] within the Randall-Sundrum model [2] with custodial protection ($RS_c$) [3]. I also review the results obtained within $RS_c$ for the related modes $B \to K^{(*)}\nu\bar{\nu}$ [4], for which only upper bounds on the branching ratios are available [5, 6, 7].

2. $B \to K^* \ell^+ \ell^-$ and $B \to K^{(*)}\nu\bar{\nu}$ decays: effective Hamiltonians and general features

The $b \to s\ell^+\ell^-$ transition is described by the effective Hamiltonian

$$H^{eff} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_1 O_1 + C_2 O_2 + \sum_{i=3,6} C_i O_i + \sum_{i=7,10,3,5} [C_i O_i + C_i' O_i'] \right\} .$$

(2.1)

$G_F$ is the Fermi constant and $V_{ij}$ the elements of the Cabibbo-Kobayashi-Maskawa matrix. Among the operators in (2.1), I focus here only on $O_i^{(7)}$, $(i = 7, \ldots, 10)$:

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{\ell}_L \gamma^\mu \tau_\ell c_\beta b_\alpha) F_{\mu\nu}, \quad O_8 = \frac{g_s}{16\pi^2} m_b (\bar{\ell}_L \sigma^{\mu\nu}(\frac{\lambda^a}{2})_{\alpha\beta} b_\beta) G_{\mu\nu}^{a},$$

$$O_9 = \frac{e^2}{16\pi^2} (\bar{\ell}_L \gamma^\mu b_{\lambda\alpha}) \bar{\ell}_\mu \ell, \quad O_{10} = \frac{e^2}{16\pi^2} (\bar{\ell}_L \gamma^\mu b_{\lambda\alpha}) \bar{\ell}_\mu \gamma \nu \ell .$$

The corresponding primed operators are obtained reversing the quark field chirality. $\alpha, \beta$ are colour indices, $\lambda^a$ the Gell-Mann matrices, $F_{\mu\nu}$ and $G_{\mu\nu}^{a}$ denote the electromagnetic and the gluonic field strength tensors, $e$ and $g_s$ the electromagnetic and the strong coupling constants, $m_b$ is the $b$ quark mass. The operators proportional to the strange quark mass have been neglected. Only the unprimed operators appear in SM.

Taking into account the $K^*$ subsequent decay into $K\pi$, the fully differential decay width reads:

$$\frac{d^4\Gamma(B \to K^* \to K\pi\ell^+\ell^-)}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} = \frac{9}{32\pi} I(q^2, \theta_\ell, \theta_K, \phi) ,$$

(2.2)

$$I(q^2, \theta_\ell, \theta_K, \phi) = I_1 \sin^2 \theta_K + I_2 \cos^2 \theta_K + (I_3^2 + I_4^2) \cos 2\theta_\ell + I_5 \sin \theta_K \cos 2\phi + I_6 \sin 2\theta_\ell \cos \phi + I_7 \sin 2\theta_\ell \sin \phi + I_8 \sin 2\theta_\ell \cos 2\phi + I_9 \cos^2 \theta_K \cos 2\phi$$

$$+ I_{10} \sin 2\theta_K \sin 2\phi$$

(2.3)

(the definition of the angles $\theta_K$, $\theta_\ell$ and $\phi$ can be found in [8, 9]). Analogous functions $\tilde{I}$ enter in the $B$ meson differential decay width $d^4\tilde{\Gamma}$, obtained replacing in (2.2) $I_{1,2,3,4,7} \to \tilde{I}_{1,2,3,4,7}$ and $I_{5,6,8,9} \to -\tilde{I}_{5,6,8,9}$ [9]. $I_1$ and $I_2$ depend on form factors parameterizing the $B \to K^*$ hadronic matrix elements. Introducing $S_0 = \frac{d\tilde{I}_1}{dq^2} + \frac{d\tilde{I}_2}{dq^2}$ and $A_0 = \frac{d\tilde{I}_1}{dq^2} - \frac{d\tilde{I}_2}{dq^2}$, one can define the lepton forward-backward asymmetry $A_{FB} = -\frac{3}{8} (2S_0^c + S_0^s)$, the longitudinal $K^*$ polarization fraction $F_L = S_0^c$. 


and the binned observables \( <S_i>_{[q_1^2, q_2^2]} \), with the numerators and denominators in \( S_i \) separately integrated over \( q^2 \in [q_1^2, q_2^2] \). LHCb has measured the observables \( P_{l=4,5,6,8} = \frac{S_{l=4,5,6,8}}{\sqrt{F_{l}(1 - F_{l})}} \) [10], finding that the measurement of \( P_{l} \), performed in bins of \( q^2 \), deviates from SM predictions for low \( q^2 \) values [11, 12].

For \( B \to K^{(*)}v \bar{v} \) decays, the most general \( b \to s v \bar{v} \) effective Hamiltonian is: \( H_{eff} = C_L O_L + C_R O_R \), where \( O_{L,R} = (\bar{b}s)_{V+A}(v \bar{v})_{V-A} \) [13]. In SM the contribution of \( O_R \) is negligible and \( C_L^{SM} = \frac{G_F}{\sqrt{2}} \sin \theta_W \alpha V_{tb}^* V_{ts} X(x_t) \). \( \alpha \) is the fine structure constant at the \( Z^0 \) scale and \( \theta_W \) the Weinberg angle. The function \( X \) depends on the ratio of the top and W masses \( x_t = m_t^2 / M_W^2 \) [14]. In NP scenarios, also \( O_R \) can be present, and \( C_{L,R} \) assume model specific values. It is useful to introduce two parameters, \( \epsilon^2 = \frac{|C_L|^2}{|C_L|^2 + |C_R|^2} \) and \( \eta = -\frac{\text{Re} (C_L C_R^*)}{|C_L|^2 + |C_R|^2} \), sensitive to deviations from SM where \( (\epsilon, \eta)_{SM} = (1, 0) \) [6]. \( \eta \) probes the presence of \( O_R \), while \( \epsilon \) measures the deviation of \( C_L \) from its SM value. Predictions in NP extensions can be expressed in terms of \( \eta \) and \( \epsilon \). In [4] the branching fractions and the spectra in the normalized neutrino pair invariant mass \( s_B = q^2 / m_B^2 \) have been computed and, for the decay \( B \to K^{(*)}v \bar{v} \), also the polarization fractions for longitudinally and transversely polarized \( K^* \): \( F_{L,T} = \frac{1}{\Gamma} \int_{d0}^{d0+1-2|\epsilon|^2} \frac{dL}{ds_B} \frac{dT_{L,T}}{ds_B} \). Denoting the branching ratio for a transversely polarized \( K^* \) as \( \mathcal{B}_{T} = \mathcal{B}(B \to K^{*+}_{h=1} v \bar{v}) + \mathcal{B}(B \to K^{*0}_{h=2} v \bar{v}) \), other observables are \( R_{K^*/K} = \frac{\mathcal{B}(B \to K^{*+}_{h=1} v \bar{v})}{\mathcal{B}(B \to K^{*0}_{h=2} v \bar{v})} \), sensitive to \( \eta \), and \( A_T = \frac{\mathcal{B}(B \to K^{*+}_{h=1} v \bar{v}) - \mathcal{B}(B \to K^{*0}_{h=2} v \bar{v})}{\mathcal{B}(B \to K^{*+}_{h=1} v \bar{v})} \), expected to be affected by a small hadronic uncertainty [6].

3. Randall-Sundrum model with custodial protection

The RS model is defined in a five-dimensional spacetime with metric \( ds^2 = e^{-2ky} \eta_{\mu \nu} dx^\mu dx^\nu - dy^2 \), where \( \eta_{\mu \nu} = \text{diag}(+1, -1, -1, -1, -1) \), \( x \) denote the ordinary 4D coordinates and \( y \) varies in the range \( 0 \leq y \leq L \) (\( y = 0 \) is called UV brane, \( y = L \) IR brane). The parameter \( k \) is fixed to \( k = 10^{19} \) GeV to adress the hierarchy problem through a geometrical mechanism. The custodially protected variant of the model is based on the group \( SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{L,R} \) [3]. The discrete \( P_{L,R} \) symmetry implies a mirror action of the two \( SU(2)_{L,R} \) groups, preventing large \( Z \) couplings to left-handed fermions. The group is broken to the SM group by boundary conditions (BC) on the UV brane; moreover, Higgs-driven spontaneous symmetry breaking occurs, as in SM. All fields can propagate in the bulk, except for the Higgs localized close to the IR brane.

Due to the compactification of \( y \), towers of Kaluza-Klein (KK) excitations exist for all particles. The zero modes are identified with SM particles. To distinguish particles having a SM correspondent from those without it, Neumann BC on both branes (++) are imposed, while Dirichlet BC on the UV brane and Neumann BC on the IR one (−) are chosen for fields without SM partners.

The enlarged gauge group leads to new gauge bosons. In the case of \( SU(2)_L \) and \( SU(2)_R \) they are \( W^\mu_{\alpha \beta} \) and \( W^\mu_{\alpha \beta} \) (\( \alpha = 1, 2, 3 \)), respectively, while the \( U(1)_X \) gauge field is \( X_\mu \). Charged gauge bosons are defined as \( W^\mu_{\alpha \beta} = W^\mu_{\alpha \beta} + W^\mu_{\alpha \beta} \). As for neutral fields, \( W^3_\mu \) and \( X \) mix to give \( Z_\mu \) and \( B_\mu \) mixes with \( W^3_\mu \) giving \( Z \) and \( A \) fields. Zero modes and higher KK modes of gauge fields also
mix. Neglecting modes with KK number larger than 1, mixings occur, \((W_{L}^{\pm(0)}) W_{L}^{\pm(1)} W_{R}^{\pm(1)} \to (W^{\pm} W_{H}^{\pm} W_{H}^{\pm})\) and \((Z^{(0)} Z_{X}^{(1)} Z_{X}^{(1)} \to (Z Z_{H} Z_{H}^{'}))\) [15].

In the Higgs sector, the Higgs field \(H(x,y)\) transforms as a bidoublet under \(SU(2)_{L} \times SU(2)_{R}\) and as a singlet under \(U(1)_{X}\). It contains two charged and two neutral components. Only one of the two neutral fields, \(h_{0}\), has a non-vanishing vacuum expectation value \(v = 246.22\) GeV, as in SM.

Moving to fermions, SM left-handed doublets fit in bidoublets of \(SU(2)_{L} \times SU(2)_{R}\), together with two new fermions. Right-handed up-type quarks are singlets; neutrinos are only left-handed. Right-handed down-type quarks and charged leptons transform as \((3,1) \oplus (1,3)\) \(SU(2)_{L} \times SU(2)_{R}\) multiplets in which additional new fermions are also present. The relation \(Q = T_{L}^{1} + T_{R}^{1} + Q_{X}\) holds among the electric charge \(Q\), the third component of the \(SU(2)_{L}\) and \(SU(2)_{R}\) isospins \(T_{L}^{3}\) and the charge \(Q_{X}\). The profiles of zero-mode fermions involve the fermion bulk mass, which is the same for fermions in the same \(SU(2)_{L} \times SU(2)_{R}\) multiplet.

As in SM, quark flavour eigenstates undergo a rotation to give mass eigenstates. Denoting by \(\mathcal{U}_{L(R)}\) the rotation matrices of up-type left (right) and down-type left (right) quarks, respectively, the CKM matrix is \(V_{CKM} = \mathcal{U}_{L}^{\dagger} \mathcal{D}_{L}\). Their matrix elements are involved in the Feynman rules of tree-level flavour-changing neutral currents that exist in the model, mediated by \(Z, Z', Z_{H}\), and by the first KK mode of the photon and of the gluon. Such elements depend on the 5D Yukawa couplings \(\lambda_{ij}^{u,d}\) of up and down-type quarks, constrained to reproduce quark masses and CKM elements. Adopting the assumption of real and symmetric \(\lambda^{u,d}\) matrices, one is left with six independent entries among their elements, namely \(\lambda_{12}^{u}, \lambda_{13}^{u}, \lambda_{23}^{u}, \lambda_{12}^{d}, \lambda_{13}^{d}, \lambda_{23}^{d}\), which, together with the bulk mass parameters, represent the set of numerical inputs of our study.

4. \(B \to K^{\ast} \ell^{+} \ell^{-}\) and \(B \to K^{(\ast)} \nu \bar{\nu}\) decays in RS

In the RS model the Wilson coefficients, \(C^{RS} = C^{SM} + \Delta C\), have been derived in [16], except for \(C_{7}\) and \(C_{8}\) computed in [1] with the same assumptions adopted in [16]. Different computational schemes for \(C_{7}^{(i)}\) were used in [17].

The new contributions \(\Delta C\) are obtained scanning the parameter space. In [1, 4] the quark bulk mass parameters and the independent entries of the matrices \(\lambda^{u,d}\) have been fixed imposing quark masses and CKM constraints, as well as constraints derived in [18] using the measurements of the coupling \(Zbb\), of the \(b\)-quark left-right asymmetry parameter and of the forward-backward asymmetry for \(b\) quarks. The parameter space is further reduced imposing that \(\mathcal{B}(B \to K^{\ast} \mu^{+} \mu^{-})\) and \(\mathcal{B}(B \to X_{s} \gamma)\) lie within the \(2\sigma\) range of the measurements [19, 20]. For further details I refer to [1].

In Fig. 1 SM and RS predictions for \(A_{FB}\) and \(P'_{K}\) are compared, varying the model parameters and including the uncertainty on the form factors computed in [21] using light-cone QCD sum rules [22]. The form factor uncertainty has an impact on the SM results, except for the position of the zero in \(A_{FB}(q^{2})\), almost free of uncertainty. In RS, deviations from SM are small, and discrepancy with data is found as well. For \(B \to K^{\ast} \ell^{+} \ell^{-}\) no data are available at present [1].

Considering the modes \(B \to K^{(\ast)} \nu \bar{\nu}\), in Fig. 2 the \((\epsilon, \eta)\) correlation plot shows that the largest value of \(\eta\) in RS, is \(\eta = -0.075\), compared to the SM value \(\eta = 0\). This is the consequence of the non vanishing role of the operator \(O_{R}\) in the model. The branching ratios in RS, \(\mathcal{B}(B^{0} \to K^{0} \nu \bar{\nu})_{RS} \in [3.45 - 6.65] \times 10^{-6}\) and \(\mathcal{B}(B^{0} \to K^{0} \nu \bar{\nu})_{RS} \in [6.1 - 14.3] \times 10^{-6}\), span a range larger
**Figure 1:** $A_{FB}$ (left) and $P_5'$ (right) in $B^0 \to K^0 \mu^+ \mu^-$. The green band is the SM result, including the uncertainty of the form factors. The red and blue vertical bars are the RS$_c$ result, without or with the uncertainty in form factors. The black dots, with their error bars, are the LHCb measurements in [11].

**Figure 2:** Left: Correlation between the parameters $\eta$ and $\epsilon$ in the RS$_c$ model (blue curve). The red dot corresponds to SM. Right: Correlation between $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})$ and $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})$ (blue curve) normalized to the corresponding SM values (red dot) obtained for the central value of the form factors.

than in SM, $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})_{SM} = (4.6 \pm 1.1) \times 10^{-6}$, $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})_{SM} = (10.0 \pm 2.7) \times 10^{-6}$. The right panel of Fig. 2 displays the results in correspondence to the central value of the form factors, showing an anticorrelation between the branching ratios of the two modes in RS$_c$.

The pattern of correlations among the various observables is interesting [4]. $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})$ and $F_L$ are correlated, while $\mathcal{B}(B^0 \to K^0 \nu \bar{\nu})$ and $A_T$ are anticorrelated, as well as $R_{K^*/K}$ and $F_L$, a pattern that can be viewed as a specific feature of RS$_c$. Similar features appear in the decays $B_s \to (\phi, \eta, \eta' f_0(980)) \nu \bar{\nu}$ [4].

### 5. Conclusions

In the RS$_c$ model, deviations with respect to SM predictions are found in several observables relative to the modes $B \to K^* \ell^+ \ell^-$ and $B \to K^{(*)} \nu \bar{\nu}$, even though small. Correlations among observables exist, that can be used to discriminate this model from other NP scenarios.

**Acknowledgments.** I thank P. Biancofiore, P. Colangelo and E. Scrimieri for collaboration on the topics discussed here, and A. J. Buras for useful discussions.
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