On the correlation of the angular and lateral deflections of electrons after multiple scattering allowing for energy losses

M. Giller, R. Legumina and A. Śmiałkowski
University of Łódź, Poland

Abstract
In the model of multiple small angle scattering the correlation coefficient of a fast particle angle and its lateral deflection from the initial direction is large: $\rho = 0.87$. For electrons in EAS it is much smaller: $\rho \approx 0.5$. This paper explains why the difference is so large.

**Model 1:** Multiple scattering (Coulomb) of electron by small angles, electron energy $E = \text{const.}$
Electron makes $N$ steps $\Delta z$ being scattered by a small angle $\phi$ in each step, with the dispersion $\sigma_\phi$.

**Definition:** Correlation coefficient $\rho = \langle \eta x \rangle / \sigma_e \Delta z$, between final angle $\eta$ and lateral deflection $x$.
From
\[
\langle \eta z \rangle = \alpha^z \Delta z, \quad \langle \eta^2 z \rangle = \alpha^z \Delta z \quad \text{and} \quad \langle x + \Delta z \rangle = x + \eta \Delta z,
\]
where $z = N \Delta z$, we obtain that $\rho = \frac{\alpha}{\sqrt{2}} \approx 0.866$

A very strong correlation!

**Model 2:** Heitler-like model of electromagnetic cascade – electrons and photons, $E = \text{const.}$
After each step $\Delta z$ electron emits a photon, photon creates two electrons $\text{(e-e)}$. Let $\eta_i(z)$ be the angle of an electron at level $z$ and $\eta_o(z)$ the angle of an electron at $z$, the parent of which at $z-\Delta z$ is an electron (photon). We have:
\[
\eta_i(z) = 2 \eta_o(z), \quad \langle \eta^2_i(z) \rangle = \langle \eta^2_o(z) \rangle + \langle \eta^2_o(z) \rangle, \quad \text{and} \quad \rho = \frac{\alpha}{\sqrt{2}} \approx 0.866
\]
From this we obtain:
\[
\langle \eta^2(z) \rangle = \frac{\alpha^z}{\Delta z}, \quad \langle x(z) \rangle = \frac{\alpha^z}{\Delta z}, \quad \langle \eta(z) x(z) \rangle = \frac{\alpha^z}{\Delta z}
\]
so that again $\rho = \frac{\alpha}{\sqrt{2}} \approx 0.866$

Parent photons do not influence $\rho$ if $E = \text{const.}$

**Model 3:** Multiple scattering of electron by small angles, electrons loses energy: $E = \text{E(z)}$.
After $N$ steps electron energy equals
\[
\eta_i = \sum_i \eta_i, \quad E_{i+1} = E_i - E_{\text{photon}} + \cdots + E_{\text{photon}}, \quad \text{where} \quad E_{\text{photon}} \quad \text{is a scattering angle for} \quad \text{some fixed} \quad E_{\text{photon}}.
\]
Assuming $\Delta z = 0$ so that $N \Delta z = \text{const.}$ and $E_{\text{photon}} = 21$ MeV we obtain:
\[
\langle \eta^2(z) \rangle = \frac{1}{X_e} \int \frac{E(z)}{E(z')} \left( \frac{E(z)}{E(z')} \right)^2 dz' \quad \text{and} \quad \langle \eta(z) x(z) \rangle = \frac{1}{X_e} \int \left( \frac{z-z'}{X_e} \right)^2 \left( \frac{E(z)}{E(z')} \right)^2 dz'
\]
Assuming $E(z)$ one can find the above variances and compute the correlation coefficient $\rho$.

**Fig. 1:**
Comparison of $\rho_{\text{EAS}}$ and for $\rho_{\text{single electron}}$ as a function of the ratio of the final to initial electron energy. $\rho_{\text{EAS}}$ - Bremsstrahlung only, $\rho_{\text{ion}}$ - ionisation only. A completely different behaviour of $\rho$ in the two cases can be seen.

**Fig. 2:**
Correlation coefficient $r$ as a function of electron energy in units of the critical energy $\beta$. Green curve – bremsstrahlung energy losses only, blue – ionisation, red – both processes. $E_i = 100$ GeV.

Looking for the reason of the difference:
To study the possible effect of photons in the framework of some more realistic models we consider two of them:

**Model 4:** Multiple scattering of a single electron in steps
At each step electron energy is diminished by a constant factor $k < 1$, so that $E_i = E_{i-1} / k$ and $\langle \eta^2_i \rangle = U_{i-1} k^{-2}$. We obtain:
\[
\langle \eta^2_i \rangle = U_{i-1} (k^{-2} - 1) (1 - k^{-3}), \quad \langle x_i \rangle = U_{i-1} \Delta z k^{-3} \sum_{j=1}^{i-1} j k^j
\]
and the correlation coefficient for $N = \infty$ equals
\[
\rho_k = \frac{k^2 (1 - k^{-2})}{\sqrt{2} \Delta z}.
\]
For $k = 0.5$ we obtain $\rho_k = 0.447$.

Thus, even if the energy - loss process considered here is similar to bremsstrahlung we obtain a much smaller value when continuous, mean bremsstrahlung losses were assumed leading to $\rho = 0.707$.

Also: MC simulations of electron propagation were done where emission of high-energy photons was chosen randomly. No mean effect on $\rho$ was found (Fig. 3).

**Model 5:** Heitler-like model of electromagnetic cascade – electrons and photons, as Model 2 but $E_i = E_i (1/2)^i$.

Note: electron energy decrease in each step is the same in both Model 4 and Model 5.

For $N = \infty$ we obtain
\[
\rho_k = \frac{1.184 U_{i-1} k^4}{4}, \quad \rho_{\text{Bremsstrahlung}} = 0.448 U_{i-1} (1-2k^3),
\]
so that $\rho = 0.60$ independently of $N$.

Thus, going from Model 4 (single electron) to Model 5 (cascade with photons) decreases correlation by factor $0.30 \approx 0.67$.

By analogy, we may expect that going from multiple scattering Model 3 (single electron) to EAS (cascade) would decrease $\rho$ by roughly the same factor:

at $E_i / i = 1$ this ratio is $\rho = 0.73$ (Fig. 3),

- what we consider as a reasonable agreement.

**Conclusion:**
Weak correlation of electron angles and their lateral deflections in EAS, when compared with multiple scattering models, is caused by electron energy losses – mainly those for ionisation - and by the parent photons.