

Effect of the Regular Galactic Magnetic Field on the Propagation of Galactic Cosmic Rays in the Galaxy

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The effects of the large-scale regular Galactic magnetic field (GMF) on the propagation of galactic cosmic rays (GCRs) in the interstellar space are numerically investigated. The calculations are made for the divergenceless GMF which consists of the spiral disk field, the toroidal halo field, and the out-of-plane field. We assume anisotropic diffusion coefficient of which the magnitude is inversely proportional to the strength of the GMF. We investigate the propagation of GCRs by assuming that the bulk of the GCRs are originated in supernova remnants (SNRs) and also by fully taking into account that the supernovae (SNe) occur discretely in both space and time. We obtained by the stochastic method the age distribution and the path length distribution of GCRs which arrived at the solar system with specified energies. The mean value of the age distribution at around 1 GeV is consistent with the age obtained by the observations of radioactive nuclei. We found the energy dependence of ages depends on the degree of anisotropy of the diffusion coefficients even with the same value of relevant parameters which reproduce the observed B/C ratio at 1 GeV/n. The source SNRs of GCRs have a tendency to be distributed in the direction along the general field lines of the GMF when the GCRs diffuse anisotropically. The energy dependence of the B/C ratio estimated with the path length distribution reproduces reliably the energy dependence of B/C obtained by recent observations in space.

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1. Introduction

Supernova remnants (SNRs) are believed to be the sources of galactic cosmic rays (GCRs) (Blandford & Eichler [3]) and this view is corroborated by recent TeV gamma-ray observations of SNRs (e.g. Muraishi et al. [13]; Aharonian et al. [2]). Supernovae (SNe) occur discretely in both space and time. Hence we have to take into account this discreteness in the study of the propagation process of GCRs. Recently, we have proposed a fully three-dimensional stochastic numerical method to calculate the age distribution and the path length distribution (PLD) of GCRs by directly incorporating this discreteness (Miyake, Muraishi, and Yanagita [12], hereafter we refer this paper as Paper I.). In that work we did not consider the existence of GMF explicitly and assumed the diffusion process of GCRs is an isotropic one and neglected possible energy changes of GCRs during the propagation process for simplicity.

In this work, we investigated the propagation of GCRs incorporating the regular GMF and assuming an anisotropic diffusion coefficient with respect to the regular GMF which depends on the strength of the GMF with the same stochastic method adopted in the previous work. The differences from the previous work in the resultant age distributions, PLDs, and the energy dependence of B/C ratios are discussed.

2. Models and Numerical Simulation

In this study, we numerically investigate the propagation of GCRs which are originated in SNRs in the Galaxy where the large-scale regular GMF exists. We adopted the large-scale regular GMF proposed by Jansson and Farrar [10], which is estimated from the observation by WMAP and the rotation measures of the extragalactic sources. The regular GMF consists of the disk component, the toroidal halo component, and the out-of-plane component as shown in Figure 1. We assume that the acceleration of GCRs in SNRs continues uniformly in time up to 10^5 years after the explosion of the parent SN, although it is suggested from recent studies that the escape time of GCRs from SNRs depends on their energy (e.g. Ohira et al. [14]; Caprioli et al. [5]; Drury [6]). The radius of SNRs at the age of 10^5 years is assumed to be 30 pc by following the Sedov model where the values of 10^{51} ergs, 10^9 cm sec $^{-1}$, and 1 proton cm $^{-3}$ for the total explosion energy, the velocity of the shock wave, and the ambient matter density respectively are adopted. We also assume that the SN occurs at random both in time and in space within the radius of 20 kpc from the center of the Galaxy. The frequency of occurrences of SN is assumed to be 3 times in 100 years in the Galaxy. The surface density of SN rate has a galactocentric radial dependence scaled to the molecular gas given by Williams & McKee [19], and has a Gaussian height distribution $P_{SN} = (2\pi\alpha_{SN}^2)^{-1/2} \exp\{-z^2/(2\alpha_{SN}^2)\}$ kpc $^{-1}$, where $\alpha_{SN} = 0.070$ kpc (Boulares & Cox [4]).

Our stochastic method is based on the equivalence of a coupled set of stochastic differential equations (SDEs) to the Parker's convection-diffusion equation describing the propagation of GCRs. We solve numerically the coupled set of SDEs. We neglect any energy changes and the existence of the galactic wind for simplicity. Then, the coupled set of SDEs is written as

$$d\mathbf{r} = (\nabla \cdot \boldsymbol{\kappa}) dt + \sum_s \boldsymbol{\sigma}_s dW_s(t), \quad (2.1)$$

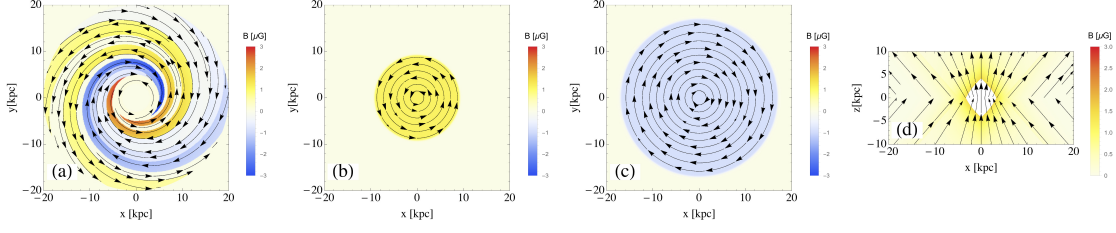


Figure 1: The large-scale regular GMF proposed by Jansson and Farrar [10]. (a) disk component; (b) northern toroidal halo component at $z = 1$ kpc; (c) southern toroidal halo component at $z = -1$ kpc; (d) out-of-plane component.

where \mathbf{r} indicates the position of pseudo-particle (Hereafter we refer to "pseudo-particle" as just "particle" if needed.), κ is the spatial diffusion coefficient tensor, $\sum_s \sigma_s^\mu \sigma_s^\nu = 2\kappa^{\mu\nu}$, and dW_s is the Wiener process given by a Gaussian distribution with a width of dt . We solved this SDE backwards in time (see Yamada et al. [20]; Zhang [22]). In other word, we follow the trajectory of a cosmic ray particle arrived at the solar system with a fixed energy backwards in time until the particle arrives at some active SNR. Here we assume the spatial diffusion process is anisotropic with respect to the direction of the regular GMF. We also assume the mean free path of the particle is inversely proportional to the strengths of the GMF. We adopted

$$\kappa_{\parallel} = \frac{1}{3} l_{m.f.p.} v, \quad (2.2)$$

$$\frac{\kappa_{\perp}}{\kappa_{\parallel}} = \begin{cases} 1 \\ 0.1 \end{cases}, \quad (2.3)$$

$$l_{m.f.p.} = l_0 \left(\frac{p}{1\text{GeV}/c} \right)^{\delta} \left(\frac{B}{1\mu\text{G}} \right)^{-1}, \quad (2.4)$$

where κ_{\parallel} and κ_{\perp} are the diffusion coefficients parallel and perpendicular to the magnetic field respectively, $l_{m.f.p.}$ is the mean free path of the particle, v is the velocity of the particle, p is the momentum of the particle, B is the regular GMF strength, and δ is the power law index related with the Alfvén wave spectrum. We assume the value of δ changing from $\frac{1}{3}$ for Kolmogorov-type model to $\frac{1}{2}$ for Kraichnan-type model ($\delta = \frac{1}{3}, 0.35, 0.40, 0.45, \frac{1}{2}$). The value of l_0 for each model is determined as shown in Table 1 so that our result of the B/C ratio at 1 GeV coincides with the observed value by AMS-02. This normalization may be allowed because the effect of the solar modulation on the ratio of B/C is estimated less than 6% with reasonable values for the modulation parameter according to the force field theory.

Table 1: The absolute values of the mean free path of the particle for each model of δ .

$\kappa_{\perp}/\kappa_{\parallel}$	δ				
	1/3	0.35	0.40	0.45	1/2
1.0	0.771	0.771	0.744	0.734	0.710
0.1	2.39	2.38	2.29	2.22	2.24

Note. All values in units of 10^{18} cm.

The path length X is calculated simultaneously with the SDE as $dX = \rho v dt$ where ρ is the density of the interstellar matter. The density ρ is calculated with the model of interstellar medium adopted by Higdon & Lingenfelter [9]. In this model, the interstellar medium consists of five major components, the molecular gas, the cold neutral gas, the warm neutral gas, the warm ionized gas, and the hot tenuous medium. The density of the molecular gas and the other components are given by Williams & McKee [19] and Ferrière [7] respectively, as a function of the galactocentric radius and the height from the galactic plane.

In our algorithm, first we generate a list of SNe which might have occurred during the last 5×10^8 yrs in our Galaxy so that their spatial position \mathbf{r}_{SN} and occurrence time t_{SN} are randomly distributed to be consistent with the conditions as stated before. Next we start to follow the sample path $\mathbf{r}(t)$ with a certain energy by the SDE with an appropriate time step Δt . The path length for each time step can be calculated by $v\rho\Delta t$. Sample paths start at the solar system which is located at $(-8.5 \text{ kpc}, 0 \text{ kpc}, 0 \text{ kpc})$ for a galactocentric Cartesian coordinate system (x, y, z) , and then run backwards in time until they hit some active SNR centered at the parent SN on the prepared list where the acceleration of CRs is still going on. If the sample path $\mathbf{r}(t)$ at some time t get into some SNR, in other words $\mathbf{r}(t)$ satisfies $|\mathbf{r}(t) - \mathbf{r}_{\text{SN}}| \leq 30 \text{ pc}$, $t_{\text{SN}} - 10^5 \text{ yrs} \leq t \leq t_{\text{SN}}$, then we stop to calculate the sample path and we record the position, the arrival time, and the total path length, and then we restart the same procedure again for another particle. We follow the sample path up to 5×10^8 yrs while the particle fails to hit some active SNR. By this numerical experiment, we can simultaneously obtain the age distribution and the PLD for particles observed with various energies in the solar system. We want to emphasize that the age of GCRs obtained by this method is not the escape time from the Galaxy but the time spent by the particles during their journey to the solar system from the SNR where they were born. Accordingly the value of this age should depend on the location of observers in our galaxy.

3. Results and Conclusions

Figure 2 shows the age distribution and the PLD of protons observed in the solar system with some fixed energies. We can see that the calculated age and path length are distributed over a significantly wide range depending on the particle energy. Furthermore, some sharp decreases or a cut-offs can be seen below a certain age or path length depending on the energy of particles. These remarkable features for the age distribution and PLD are similar to the results by our previous study (Paper I) and are caused by the discrete nature of SNe occurrences. The diffusion time of GCRs from their birth place to the solar system becomes shorter when the energy of particle becomes higher. The position of the sharp decrease shifts to lower range when the energy of particle becomes higher as expected.

We must notice that ages obtained by observations of radioactive nuclei with certain energy values may be the values averaged over distributions similar to those shown in left panel of Figure 2. Figure 3 shows the energy dependence of the mean values of the age and the path length thus obtained for several cases with different forms of the diffusion coefficient. The ages shown in the left panel of Figure 3 can be compared with observations. The value of $\sim 5 \times 10^7$ yrs at around 1 GeV agrees well with the age obtained by the observation of radioactive secondary nuclei (e.g. Yanasak et al. [21]). Figure 3 shows that both the mean age and the mean path length decrease

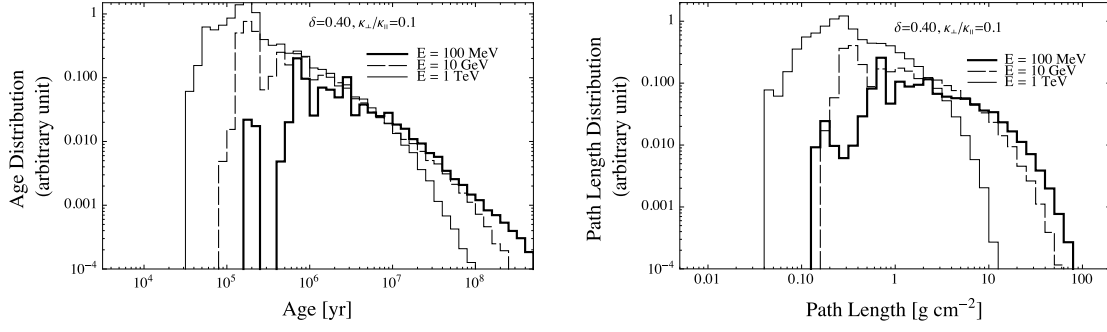


Figure 2: Calculated age distribution (left panel) and PLD (right panel) of GCR protons observed in the solar system with some fixed energies for the case of $\delta = 0.40$ and $\kappa_{\perp}/\kappa_{\parallel} = 0.1$. The thick line, the long-dashed line, and the thin line indicate the age distribution of protons which arrive at the solar system with a fixed energy of 100 MeV, 10 GeV, and 1 TeV, respectively.

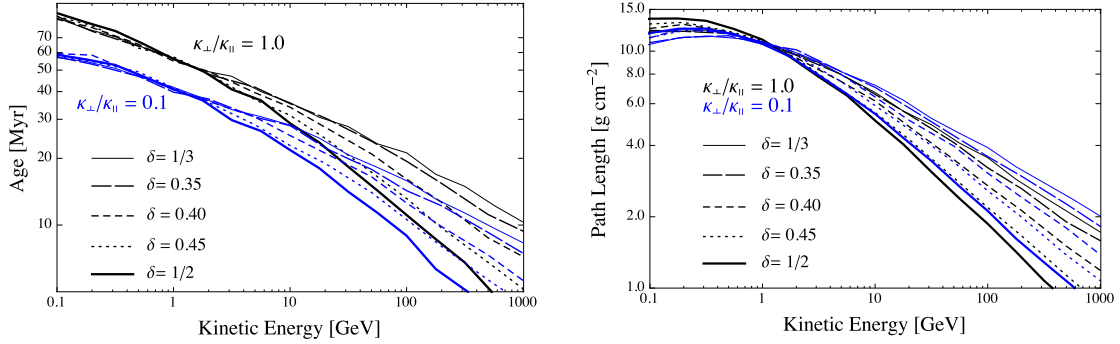


Figure 3: Energy dependence of the mean age (left panel) and the mean path length (right panel) of GCR protons for $\kappa_{\perp}/\kappa_{\parallel} = 1.0$ (black line) and $\kappa_{\perp}/\kappa_{\parallel} = 0.1$ (blue line). The thin line, the long dashed line, the short dashed line, the dotted line, and the thick line indicate the mean values for the case of $\delta = \frac{1}{3}$, 0.35, 0.40, 0.45, and $\frac{1}{2}$, respectively.

with energy in energy region above 1 GeV depending on the form of the diffusion coefficient as anticipated. This tendency of the energy dependence of those mean values coincides qualitatively with the results by our previous study based on the simplified propagation model (Paper I).

As we stated in the section 2, we have normalized the numerical factor multiplied to the mean free path so that the calculated B/C ratio at 1 GeV/n coincides with the observed value by AMS-02. Accordingly the mean value of the path length at 1 GeV/n is the same value for all models of the diffusion coefficient as seen in the right panel of Figure 3. However the mean age at 1 GeV/n differs depending on the ratio $\kappa_{\perp}/\kappa_{\parallel}$ as recognized by the left panel of Figure 3. This difference may come from the difference in the mean density of the interstellar matter seen by GCRs which depends on the degree of an anisotropy. For the case of an anisotropic diffusion of $\kappa_{\perp}/\kappa_{\parallel} = 0.1$, the time interval that GCRs stay in the halo region where the matter densities are lower than that of the disk region may be shorter compared with the case of an isotropic diffusion of $\kappa_{\perp}/\kappa_{\parallel} = 1.0$. This is because the particles have a tendency that they propagate along the magnetic field lines of the GMF that is dominated by the disk component which lies along the $x - y$ plane. Accordingly for the ratio $\kappa_{\perp}/\kappa_{\parallel} = 0.1$, the expected age of the particles with the same value of the path length becomes shorter than that for the ratio $\kappa_{\perp}/\kappa_{\parallel} = 1.0$. Sample paths of GCRs shown in the upper

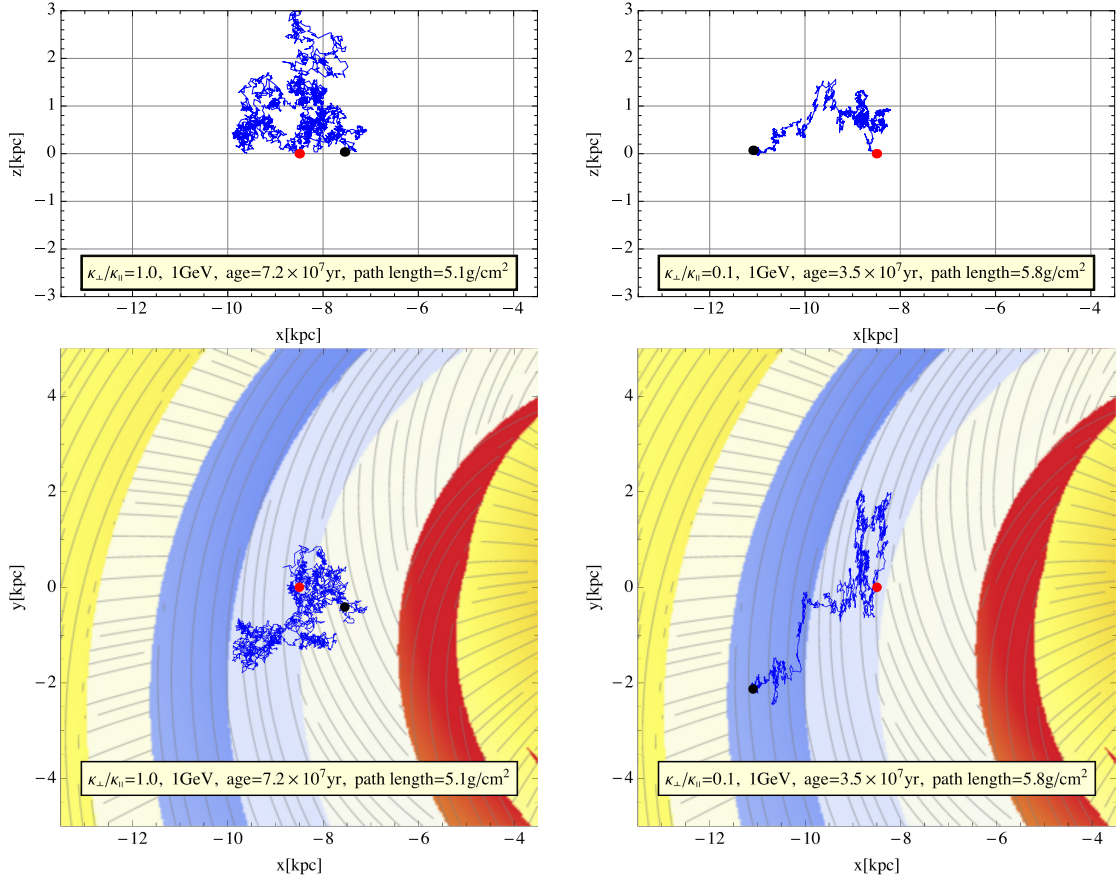


Figure 4: Sample trajectory of GCR proton with 1 GeV in the solar system for $\kappa_{\perp}/\kappa_{\parallel} = 1.0$ (left panels) and $\kappa_{\perp}/\kappa_{\parallel} = 0.1$ (right panels). The red point and the black point indicate the position of the solar system and the source of the sample particle arriving at the solar system, respectively.

two panels of Figure 4 demonstrate the difference in degree of confinement in the disk region between two different degree of anisotropy. Two lower panels in Figure 4 show the sample paths projected on the $x - y$ plane for two particles shown in the upper panels together with the magnetic field lines. It is clearly seen that the particle has a tendency to propagate along the magnetic field lines for the ratio $\kappa_{\perp}/\kappa_{\parallel} = 0.1$ as stated above. This finding may be checked when we compare the calculation of the abundance ratio of the radioactive secondary nuclei to the stable secondary nuclei, e.g. $^{10}\text{Be}/^9\text{Be}$, to observations of the abundance ratio.

The spatial distribution of the parent SNRs of GCRs which arrived at the solar system is obtained by our simulation as described in Section 2. It is anticipated that the source SNRs have a tendency to be distributed in the direction along the general field lines of the GMF when the GCRs diffuse anisotropically, because the particles move faster along the magnetic fields lines than in the direction perpendicular to the field lines. Figure 5 shows the resultant spatial distribution of the source SNRs projected on the galactic plane for the same two cases of anisotropy shown in Figure 4 overlaid with the magnetic field lines of the GMF. The distribution for the ratio $\kappa_{\perp}/\kappa_{\parallel} = 1.0$ is almost isotropic centered at the solar system, while in the case of $\kappa_{\perp}/\kappa_{\parallel} = 0.1$ the sources are distributed along the field lines anisotropically as anticipated. This anisotropic distribution might

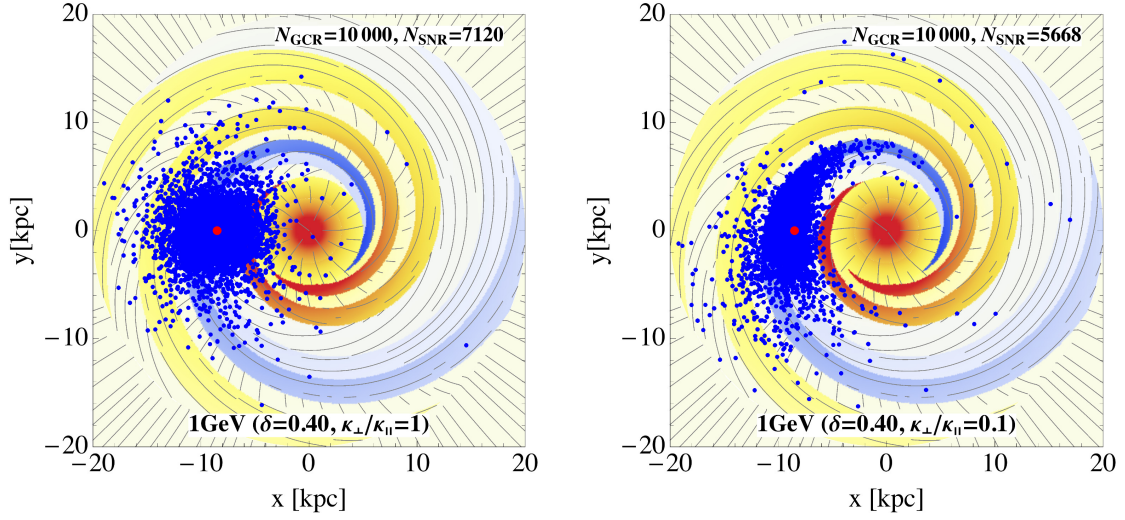


Figure 5: Spatial source distribution of GCR protons with 1 GeV in the solar system for $\kappa_{\perp}/\kappa_{\parallel} = 1.0$ (left panel) and $\kappa_{\perp}/\kappa_{\parallel} = 0.1$ (right panel). The red point and the blue point indicate the position of the solar system and the sources of the sample particle arriving at the solar system, respectively.

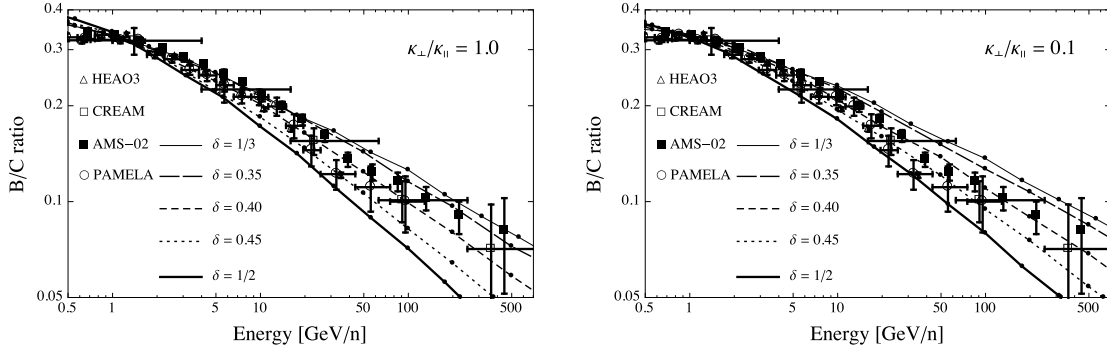


Figure 6: Calculated B/C ratios for some cases with different forms of the diffusion coefficient overlaid with the observational data by HEAO3, CREAM (compilation by Maurin et al. [11] and the references cited therein), AMS-02 (Oliva et al. [15]), and PAMELA (Adriani et al. [1]). The line symbols are as in Figure 3.

be observed as an anisotropy of the arrival direction of GCRs.

Finally we calculated the B/C ratio as a function of energy by the weighted slab method using the PLDs obtained in our simulation as shown in Figure 2 as example. We assume only C, N, and O are the parents nuclei of B. The source abundances of each parent nuclei were taken from Strong & Moskalenko [17]. Relevant nuclear data were referred to Garcia-Munoz et al. [8], Webber et al. [18], and Ramaty et al [16]. The results for two cases of anisotropic diffusion are shown in Figure 6 together with some recent observational results. In both cases, the value of δ which reproduces well the observed ratio is in between 1/3 (for Kolmogorov-type model) and 1/2 (for Kraichnan-type model).

Acknowledgements

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References

- [1] O. Adriani, G. C. Barbarino, G. A. Bazilevskaya, et al., *ApJ*, 791, 11, 2014.
- [2] F. Aharonian, A. G. Akhperjanian, A. R. Bazer-Bachi, et al., *ApJ*, 636, 777, 2006.
- [3] R. Blandford & D. Eichler, *Phys. Rept.*, 154, 1, 1987.
- [4] A. Boulares & D. P. Cox, *ApJ*, 365, 544, 1990.
- [5] D. Caprioli, E. Amato, & P. Blasi, *Astropart. Phys.*, 33, 160, 2010.
- [6] L. O'C. Drury, *MNRAS*, 415, 1807, 2011.
- [7] K. Ferrière, *ApJ*, 497, 759, 1998.
- [8] M. Garcia-Munoz, J. A. Simpson, T. G. Guzik, J. F. Wefel, & S. H. Margolis, *ApJS*, 64, 269, 1987.
- [9] J. C. Higdon & R. E. Lingenfelter, *ApJ*, 582, 330, 2003.
- [10] F. Jansson & G. R. Farrar, *ApJ*, 757, 14, 2012.
- [11] D. Maurin, F. Melot, & R. Taillet, *ArXiv e-prints*: 1302.5525, 2013.
- [12] S. Miyake, H. Muraishi, & S. Yanagita, *A&A*, 573, 2015. (Paper I)
- [13] H. Muraishi, T. Tanimori, S. Yanagita, et al., *A&A*, 354, L57, 2000.
- [14] Y. Ohira, K. Murase, & R. Yamazaki, *A&A*, 513, A17, 2010.
- [15] A. Oliva & AMS Collaboration, *Proc. of 33rd ICRC (Rio de Janeiro)*.
- [16] R. Ramaty, B. Kozlovsky, R. E. Lingenfelter, & H. Reeves, *ApJ*, 488, 730, 1997.
- [17] A. W. Strong & I. V. Moskalenko, *Adv. Space Res.*, 27, 717, 2001.
- [18] W. R. Webber, A. Soutoul, J. C. Kish, & J. M. Rockstroh, *ApJS*, 144, 153, 2003.
- [19] J. P. Williams & C. F. McKee, *ApJ*, 476, 166, 1997.
- [20] Y. Yamada, S. Yanagita, & T. Yoshida, *Geophys. Res. Lett.*, 25, 2353, 1998.
- [21] N. E. Yanasak, M. E. Wiedenbeck, R. A. Mewaldt, et al., *ApJ*, 563, 768, 2001.
- [22] M. Zhang, *ApJ*, 513, 409, 1999.