PROCEEDINGS OF SCIENCE

Deconvolution of very high-energy-gamma-ray image with the Richardson-Lucy algorithm

Ira Jung-Richardt[*]
Friedrich-Alexander University Erlangen-Nürnberg, Department of Physics, Erwin-Rommel-Str. 1, 91058 Erlangen, Germany
E-mail: `ira.jung@physik.uni-erlangen.de`

Sebastian Heinz
Friedrich-Alexander University Erlangen-Nürnberg, Department of Physics, Erwin-Rommel-Str. 1, 91058 Erlangen, Germany
E-mail: `heinz-sebastian@web.de`

Christian Stegmann
Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, D-15738 Zeuthen, Germany
E-mail: `christian.stegmann@desy.de`

The number of very high-energy-gamma-ray (VHE; > 100 GeV) sources has increased steadily in the last decades. The majority of these sources are extended and exhibit detailed structures. These structures and especially their correlations with data from different wavelengths may unveil the processes responsible for the gamma-ray emission. Multi wavelength studies, however, are hampered by the angular resolution of the measurements in the VHE gamma-ray regime which is worse compared to most of the other wavelength ranges roughly by a factor of 10.

To unveil the true morphology of VHE gamma-ray sources we apply the Richardson-Lucy deconvolution algorithm (RLA) to VHE gamma-ray images, and thus increase the angular resolution. We present detailed systematic studies on the deconvolution of simulated VHE gamma-ray data which show that deconvolution makes it possible to study structural details well below the angular resolution of the very high-energy gamma-ray experiment.

The 34th International Cosmic Ray Conference,
30 July- 6 August, 2015
The Hague, The Netherlands

---

[*] Speaker.

PoS(ICRC2015)804

1. Introduction

The detected very high-energy γ-ray (VHE; $>$100 GeV) sources belong to a variety of different source classes [1, 2]. Most of them are extended and have complicated structures. Detailed studies of their morphologies within different wavebands and correlations between those morphologies are important to reveal the γ-ray emission processes taking place in VHE γ-ray sources. The differences of angular resolution between VHE γ-ray and e.g. X-ray instruments hamper these analyses. The conventional way to determine the correlation of images of different wavebands is to degrade the angular resolution of the image with the better resolution to the lower resolution of the other image. This prohibits to study fine structures in sources. In this paper we present an alternative approach which is already common practice in astronomy and other scientific fields: deconvolution. We present detailed studies applying the Richardson-Lucy deconvolution algorithm [3, 4] to improve the angular resolution of very high-energy γ-ray images.

2. The Richardson-Lucy deconvolution algorithm

The observed image $I(x,y)$ in VHE γ-ray image is composed of the true source image $O(x,y)$ convolved with the points spread function $P(x,y)$ and the acceptance over the field of view $A(x,y)$. In addition, noise has to be taken into account because the measurements are not background free.

$$I(x,y) = A(x,y)\,(O \otimes P)\,(x,y) + A(x,y)\,B(x,y) \tag{2.1}$$

x and y stand for right ascension (RA) and declination (Dec), respectively. The published images are so-called *excess*-maps which are given by

$$E(x,y) = (O \otimes P)\,(x,y) \tag{2.2}$$

To obtain these maps, the measured image I is corrected for background and acceptance. The deconvolution algorithm is applied to the *excess*-map.

The Richardson-Lucy algorithm is an iterative process which is a Bayesian-based method. After the $n+1$-th iteration the deconvolved images is given by:

$$O^{n+1}(x,y) = \left[\frac{E(x,y)}{(P \otimes O^n)(x,y)} \times P^T(x,y)\right] O^n(x,y) \tag{2.3}$$

P^T is the transposed point spread function. $O^0(x,y)$ is assumed to be the uniform distribution for the first iteration.

If the applied number of iterations is too high the Richardson-Lucy algorithm tends to produce artifacts, i.e. low significant structure fluctuations are interpreted as real structures which leads to large fluctuations in the deconvolvsed structure. Therefore, it is essential to study in detail the influence of the algorithm on VHE γ-ray data. In the following sections, the results of the paper of S. Heinz et al. [5] are summarized as well as new results on simulated RX-J1713.7-3946 γ-ray data. In the latter case, X-ray data were taken as a template for the simulation of the γ-ray data. This allows to study the maximal achievable correlation and the minimal size of structures that can be evaluated.

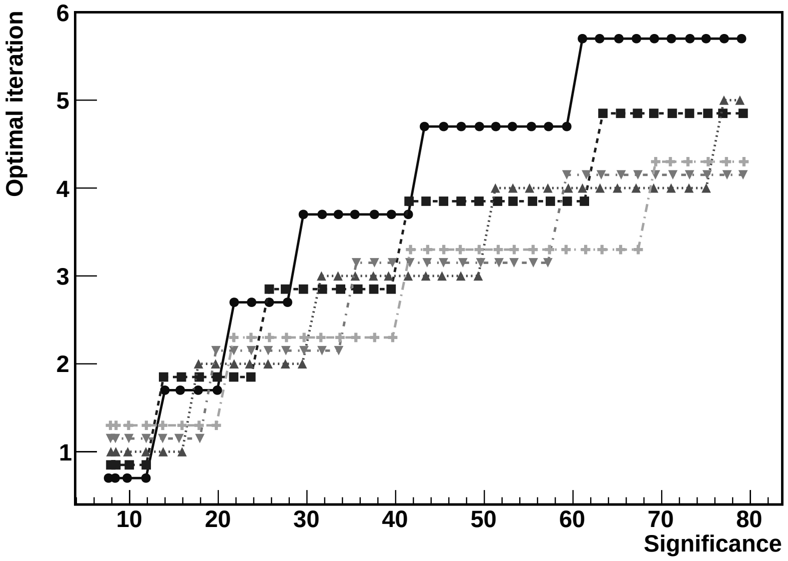

Figure 1: The optimal number of iterations *Nopt* as a function of significance for a ring-like source morphology. The graphs are shifted on the ordinate to prevent overlap. The ring width was kept constant at 0.2° while the radii varied. The outer radii are 0.3° (dots, shift -0.2) , 0.4° (squares, shift -0.1) , 0.5° (triangles pointing upwards, shift 0.0), 0.6° (triangles pointing downwards, shift 0.1) and 0.7° (crosses shift 0.2).[1]

3. Detailed studies

The following paragraphs give a summary of the results presented in S. Heinz et al. [5]). In this paper the effects of Richardson-Lucy algorithm when applied to simulated γ-ray *excess* maps were systematically studied . The simulations took the acceptance of C1713 herenkov-telescopes, the typical background rate and the signal-to-background ratio into account (for further detail see [5]).

The simulated maps were then deconvolved and the deconvolution quality determined, which was defined as the relative error between the deconvolved $N_{\text{excess}}(x_i, y_j)$ and the true morphology $N_{\text{true}}(x_i, y_j)$.

$$RE = \sqrt{\frac{\sum_{i,j}^{n_i,n_j} \left(N_{\text{excess}}(x_i, y_j) - N_{\text{true}}(x_i, y_j)\right)^2}{\sum_{i,j}^{n_i,n_j} N_{\text{true}}(x_i, y_j)^2}} \qquad (3.1)$$

x_i and y_i is the center of the right ascension and the declination bin, respectively and $N_{\text{excess}}(x_i, y_j)$ are the number of gammas in the simulated map and $N_{\text{true}}(x_i, y_j)$ the number of gammas of the true source distribution.

In the paper by S. Heinz et al. [5] different source morphologies, e.g. point, elliptical and shell-type ones were simulated and the influence of the deconvolution and the optimal number of iterations was evaluated. It was found, that ”the number of optimal iterations (N_{opt}) itself depends strongly on the signal-to-noise ratio and on the significance of the source. For typical VHE sources

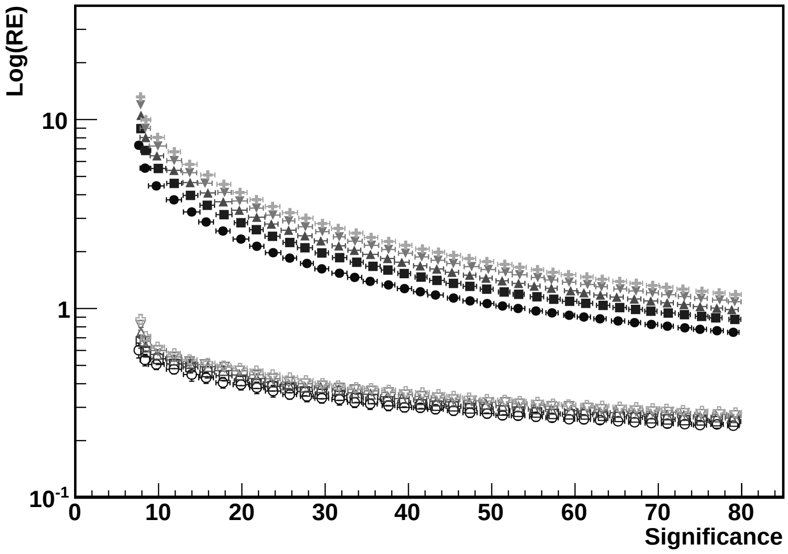

Figure 2: *RE* versus significance before and after deconvolution for a ring-like source morphology. Filled markers label *RE* before RLA application and open markers after. The ring width was kept constant at 0.2° while the radii varied. The outer radii are 0.3° (dots) , 0.4° (squares) , 0.5° (triangles pointing upwards), 0.6° (triangles pointing downwards) and 0.7° (crosses).[1]

N_{opt} is in the order of 2 to 3." [1]. The number of optimal iterations is defined as the number where the deconvolved source image describes best the true source distribution and has the smallest amount of artifacts. This is demonstrated in Fig. 1 and 2 for a shell-type source morphology. In this case, the number of optimal iterations as a function of the significance is shown. As expected, with increasing significance a higher number of iterations should be applied to obtain the optimum. Fig 2 shows the improvement obtained by deconvolution.

Since the number of iterations determines the improvement of the angular resolution, a minimal number of iterations is necessary to make deconvolution a reasonable option (for further details see [5]). Having the results of the simulation studies, described above, at hand, it is therefore possible to evaluate which of the already detected shell-type supernova remnants are viable deconvolution candidates: "The study of the five known shell-type supernova remnants has shown that for two of them deconvolution is feasible"[1] (RX J1713.7−3946 [6] and HESS J1731−347)[7]. In addition, this detailed study "indicates that deconvolution enables us to analyze the spatial structure of the processes within the interior of SNRs."[1]

4. Study of the maximal achievable correlation

Correlation studies between different wavebands have to be performed with care. Important issues to take into account are, among others, to thoughtfully chose the area to be compared and to ensure that the binning of the images is identical.

---

[1] Reprinted from Astroparticle Physics, 36, S. Heinz,I. Jung,C. Stegmann, "Systematic studies of the Richardson-Lucy deconvolution algorithm applied to VHE gamma data', 146, Copyright (2012), with permission from Elsevier.

As already mentioned in the section above, RX J1713.7−3946 [6] .and HESS J1731−347 [7] are the two candidates which are feasible to deconvolve in order to obtain new and meaningful physical insights into the processes that take place in the supernova remnant (see [5]. To determine the correlation coefficient between X-ray and VHE γ-ray data one has to be aware of the fact, that statistical fluctuations are present in the image. To get a handle on those and to obtain the correlation coefficient for the case that the VHE γ-ray image has the identical morphology as the source in the X-rays, simulations have been performed. The X-ray image of the *XMM-Newton* X-ray telescope was taken as a template for the VHE γ-ray morphology. Background rate, acceptance, signal-noise ratio, angular resolution and the significance were chosen according to the H.E.S.S. analysis. The VHE γ-ray *excess*-map was then simulated accordingly. In Fig. 3 the correlation coefficients before and after the deconvolution as a function of the binsize are shown. For each of the data points 100 simulations have been performed and the error on the points denote the RMS value. After deconvolution the correlation coefficient stays almost stable at about 0.9 whereas for the not deconvolved image the correlation coefficient decreases below 0.02 degrees^2 which is comparable to the points spread function of the H.E.S.S. analyses. In summary, this means that deconvolution allows for studying morphological structures below the angular resolution of the experiment which is identical to the conclusion gained by Heinz et al, [5], In addition, we determined the correlation coefficient of an identical match of X-ray and γ-ray morphologies to 0.9.

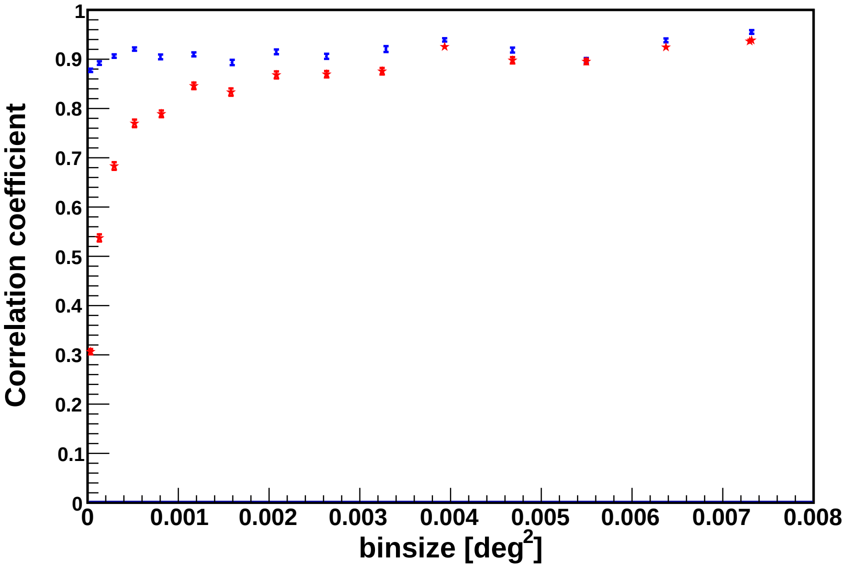

Figure 3: The correlation coefficient between X-ray data of *XMM-Newton* X-ray telescope and simulated VHE-γ-ray data before (red points) and after the deconvolution of the simulated VHE-γ–ray data (blue points) as a function of bin size, the area for which the coefficient was calculated. For each data point 100 simulations have been performed. The error bar denotes the RMS value.

5. Conclusion

In this paper we summarized the results of the paper by Heinz. et al. [5] that derived the conclusion ”that deconvolution makes it possible to study structural details well below the angular resolution of the very high-energy γ-ray experiment”[1]. Systematic studies on simulated γ-ray maps of the supernova remnant RX J1713.7$-$3946 have revealed that under the assumption of an identical morphology of the source in the γ-ray and the X-ray regimes a correlation coefficient of 0.9 is obtained and that deconvolution allows to study morphological structures down $\sim 3\times10^{-5}$ degrees^2.

References

[1] F.A. Aharonian et al., Rep. Prog. Phys. 71 (2008) 6901

[2] J. A. Hinton, W. Hofmann, Annu. Rev. Astron. Astr. 47 (2009) 523

[3] W. H. Richardson, J. Optical Soc. America 62 (1972) 55

[4] L. B. Lucy, Astron. J. 79 (1974) 745

[5] S. Heinz, I. Jung, C. Stegmann, Astroparticle Physics 36 (2012) 146

[6] F.A. Aharonian et al., Astron. Astrophys. 457 (2006) 899

[7] F.A. Aharonian et al., Astron. Astrophys. 477 (2008) 353