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Complex Interferometry Principles and its Potential in case of Reference Interferograms Availability

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Classical interferometry is one of the key methods in laser-produced plasma diagnostics. Its more advanced version, which allows *recording* and subsequent *reconstruction* of up to *three* sets of data using just *one* data object - a *complex interferogram* - was developed in the past and became known as *complex interferometry*. Employing this diagnostics, not only the usual *phase shift*, but also the *amplitude* of the probing beam as well as the fringe *contrast* (leading directly to the phase shift *time derivative*) can be reconstructed *simultaneously* from such a complex interferogram. In this paper it will be demonstrated that even in the case of a not particularly good diagnostic beam *quality* these three quantities can be reconstructed with a high degree of *accuracy* provided both the *diagnostic beam* as well as the corresponding *optical line* feature a reasonable *stability*. Such stability requirement is important as all together *four* shots need to be recorded: the *signal* complex interferogram, the *reference* interferogram as well as the *intensity* structures of the *signal* and *reference* part of the *diagnostic* beam.

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1. Introduction

Classical interferometry belongs to the key diagnostics of optically transparent objects (e.g., laser-produced plasmas). By analyzing classical interferograms one can obtain information about the *phase shift* between the *probing* and the *reference* part of the diagnostic beam. There are various ways how to perform such analysis. In this paper we shall concentrate solely on the usage of the fast Fourier transform (FFT), originally proposed for this purpose in its one-dimensional version by Takeda [1] and subsequently extended for two dimensions by Nugent [2], as it paved the way to a more advanced interferogram usability, which allows recording and subsequent reconstruction of up to *three* sets of independent data from just *one* data object - a *complex interferogram*. This technique became known as the *complex interferometry* (*CI*) [3],[4]. Using *CI* approach, not only the usual *phase shift* but also the *amplitude* and the *fringe contrast* (leading directly to the *phase shift time derivative)* can be recorded simultaneously and subsequently analyzed.

The original idea of *CI* technique dates back to measurements of *magnetic fields* spontaneously generated in laser-produced plasmas [5]. This approach provided results with spatial resolutions far superior to those of any similar experiments performed to that time [6]. A more detailed historic overview of circumstances leading to the birth of *CI* is available [7]. One example of the very first dedicated complex interferogram ever recorded during experiments with generation of spontaneous magnetic fields in laser produced plasma (including its analysis) is provided for illustration (Fig. 1).

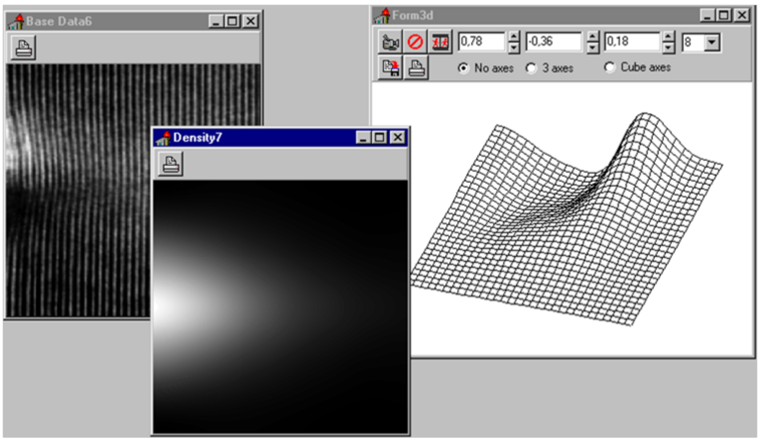

Figure 1: Example of the very first experimentally-obtained complex interferogram (on the left) including reconstructions of the plasma density (in the middle) and the magnetic field (on the right) profiles (M. Kalal and B. Luther-Davies, 1986, ANU, Canberra, Australia). It should be noted that in those early times no reference interferograms were employed.

Over the years this diagnostics has been continuously developed (in particular on the *analyzing software* side) and successfully employed in recording and analyzing various kind of phenomena. The *first* major upgrade was the usage of one additional - *reference* (i.e., *signal-free*) - interferogram in order to substantially improve the *phase shift* reconstruction [8]. The *second* major upgrade dealt with the quality of the *amplitude* reconstruction utilizing the *reference* interferogram [9]. However, it became apparent that such an approach would not be sufficient in order to *thoroughly* overcome the *non-homogeneous* structure of the diagnostic beam intensity across its cross-section (rather a very *practical* case). The way how to deal with this inadequacy will be presented in this paper.

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2. Complex interferometry generalization

A full mathematical description of *CI basics* (without any *reference* interferograms considered) can be found in [3] and [4]. However, as these particular publications might be somewhat difficult to acquire, one can also look into the easier ones to obtain ([5] and [6]). In particular, the latter of these two contains all the necessary details (but not using the *CI terminology*, which was introduced only later on).

Our present goal is to perform the *third* upgrade of the *CI* theory (the first two were already mentioned in the *Introduction*) in order to further improve the *amplitude* reconstruction. The main reason why the *second* upgrade [9] was nor sufficient lies in the fact that the *amplitudes* of the *signal* and *reference* part of the *diagnostic* beam were (by default) assumed to be *exactly* the *same* (pixel by pixel) on the *recording* device (without any object influencing the diagnostic beam, of course). This could be, in principle, achieved (after an extremely careful setup) for interferometers with an *amplitude* division (e.g., *Michelson*, *Mach-Zehnder*). In the case of the *phase front* division (e.g., *Nomarski*) it is not possible at all. As a result of this fact the formulae (13) and (14) in [9], while *correct*, are not very *practical*.

Therefore, a new approach needs to be invented for the purpose of the most precise *amplitude* reconstructions even in the case of a not very high quality of the diagnostic beam. It will be shown that it can be done. Provided the *stability* of the *diagnostic* beam as well as the *interferometer setup* between the *signal* and *reference* shots would be sufficient. On top of that *two* more *reference* shots will be required to record *intensity structures* of the *signal* and *reference* part of the *diagnostic* beam.

Due to this *fundamental upgrade* it seems useful to perform the derivation of *final results* from scratch, making it easier to the reader to follow. With only occasional references to already published work. Let us start by the form of an *instantaneous* intensity $i(y,z,t)$ of an *interference field* falling in the time t on the point (y,z) of the *detector*:

$$\begin{aligned} i(y,z,t) &= a_s^2(y,z,t)f(t) + a_r^2(y,z,t)f(t) + \\ &+ 2a_s(y,z,t)a_r(y,z,t)\cos[2\pi(\omega_0 y + \nu_0 z) + \varphi(y,z,t)]f(t)\,. \end{aligned} \tag{2.1}$$

Here $a_s(y,z,t)$ is the *amplitude* of the *signal* part of the diagnostic beam; $a_r(y,z,t)$ is the *amplitude* of the *reference* part of the diagnostic beam (in this *CI* upgrade it will be assumed that even the *reference* part can have its own *spatial* as well as *temporal* structure); $\varphi(y,z,t)$ represents the *phase shift* between the *signal* and *reference* part of the diagnostic beam; ω_0 and ν_0 are the *spatial frequencies* in y and z direction, respectively. $f(t)$ represents a temporal profile of the diagnostic beam *intensity* and it is *normalized* to satisfy the following criteria:

$$f(t) = f_s(t) + f_a(t); \quad f(t) \geq 0; \quad \int_{-\infty}^{+\infty} f(t)dt = \int_{-\infty}^{+\infty} f_s(t)dt = 1\,, \tag{2.2}$$

where the functions $f_s(t)$ and $f_a(t)$ represent the *symmetric* and *anti-symmetric* part of the function $f(t)$, respectively. Time $t = 0$ should be looked for in order to *minimize* the anti-symmetric part.

With time $t = 0$ properly selected and taking into account that the *center of symmetry* of $f_s(t)$ as well as *anti-symmetry* of $f_a(t)$ will be *identical*, the following formulae would hold:

$$\int_{-\infty}^{+\infty} t^{2n+1} f_s(t)\, dt = 0; \quad \int_{-\infty}^{+\infty} t^{2n} f_a(t)\, dt = 0; \quad n = 0, 1, 2... \tag{2.3}$$

The total *superimposed* form of the interference field in the (y,z) plane of the detector will be a *time integral* over the diagnostic beam *duration* which can be extended to *infinity* on both sides of the time interval (as outside of the pulse duration $f(t) = 0$):

$$i(y,z) = \int_{-\infty}^{+\infty} i(y,z,t) dt\,. \tag{2.4}$$

The values $i(y,z)$ represent the *structural* information of the *complex interferogram* to be processed. It should be noted, however, that these structures might *differ* from the *recorded* ones depending on the *response* of the detector in use. This is very true, e.g., when a *film* would be employed as a recording medium (with a subsequent *digitization*). In case of CCD detectors (and working below their *saturation* limits) this effect can be usually regarded as *insignificant* (a simple multiplicative constant having no real influence on the recorded data structure).

From the *integral* formula (2.4) it becomes obvious that there is no way how to reconstruct the original *instantaneous* values $a_s(y,z,t)$, $a_r(y,z,t)$, and $\varphi(y,z,t)$ for every value of t. Instead, the *Taylor expansion* of these quantities in *time* can be used:

$$\begin{aligned} a_s(y,z,t) &= a_s(y,z) + a_s'(y,z)t \\ a_r(y,z,t) &= a_r(y,z) + a_r'(y,z)t \\ \varphi(y,z,t) &= \varphi(y,z) + \varphi'(y,z)t \end{aligned} \tag{2.5}$$

This approach is equivalent to the assumption about the *linear* change of these quantities during the diagnostic pulse duration τ. This requirement can be always satisfied by taking τ sufficiently *short*. Also, for the purpose of the correct phase shift *reconstruction*, it is important to make sure that the *maximal* change of the phase shift during the diagnostic pulse duration will not exceed 2π (thus less than one fringe):

$$\varphi'_{max}(y,z)\tau < 2\pi\,. \tag{2.6}$$

Taking into account the formulae (2.3) it can be shown that under the above conditions the *phase shift* changes will play the *most significant* role and changes of the *amplitudes* can be *neglected*.

Considering only these most relevant terms the *generalized* interferogram formula would be:

$$\begin{aligned} i(y,z) = a_s^2(y,z) + a_r^2(y,z) + \\ +2a_s(y,z)a_r(y,z)\,|q(y,z)|\cos\left[2\pi(\omega_0 y + \nu_0 z) + \varphi_{total}(y,z)\right], \end{aligned} \tag{2.7}$$

where

$$q(y,z) = \int_{-\infty}^{+\infty} f(t)\exp[i\varphi'(y,z)t]dt = \int_{-\infty}^{+\infty} f_s(t)\cos[\varphi'(y,z)t]dt + i\int_{-\infty}^{+\infty} f_a(t)\sin[\varphi'(y,z)t]dt \tag{2.8}$$

and

$$\varphi_{total}(y,z) = \varphi_p(y,z) + \varphi_{derr}(y,z) + \varphi_{serr}(y,z)\,. \tag{2.9}$$

In the expression for the *total* phase shift (2.9) the meaning of its *partial* contributions is the following: $\varphi_p(y,z)$ stands for the *pure* phase shift caused by the *object itself* (this is the quantity we are looking for !!!); $\varphi_{derr}(y,z)$ stands for the *error* caused by the *diagnostic system* itself (interferometer setup and the diagnostic beam wave front quality); $\varphi_{serr}(y,z)$ stands for the *systematic error* caused by the *degree of asymmetry* of the diagnostic pulse $f(t)$.

In the case of the *symmetric* diagnostic beam profile $f(t) = f_s(t)$ the $q(y,z)$ becomes the function with *real* and *positive* values only:

$$q(y,z) = |q(y,z)| = q_r(y,z) = \int_{-\infty}^{+\infty} f_s(t)\cos[\varphi'(y,z)t]dt\,; \quad 0 < q(y,z) \leq 1\,. \tag{2.10}$$

For the more general cases of working with *non-symmetric* diagnostic beam profiles $f(t)$, where the function $q(y,z)$ will become *complex*, we also denoted the integral (2.10) as the corresponding *real* part $q_r(y,z)$. For typical *symmetric* diagnostic beam profiles $f_s(t)$ the function $q(y,z)$ is a *monotonically decreasing* function of $\varphi^{'}(y,z)$. This makes finding the *inversion* process possible, provided the *exact* time profile of $f_s(t)$ is available - either *analytically* or *numerically* (by sampling the $f_s(t)$ time profile). The reconstructed $\varphi^{'}(y,z)$ can be further employed, e.g., for determining the plasma expansion *velocity* [9].

In the case of the *symmetric* diagnostic beam profile $f_s(t)$ no *systematic error* $\varphi_{serr}(y,z)$ will be generated. However, in more *practical* cases (with some degree of *asymmetry* of the diagnostic beam profile $f(t)$), it would be very useful to have some method for determining the value of this systematic error $\varphi_{serr}(y,z)$. And, indeed, it can be done. Such a possibility is directly related to the fact that the $\varphi^{'}(y,z)$ function can be reconstructed reasonably well even in the case of $f(t)$ *asymmetry* paving the way to determination of the *imaginary* part of $q(y,z)$ (i.e., $q_i(y,z)$) from the integral:

$$q_i(y,z) = \int_{-\infty}^{+\infty} f_a(t)\sin[\varphi'(y,z)t]dt\,. \tag{2.11}$$

This approach can be fully justified by taking into account the fact that both *anti-symmetric* functions $f_a(t)$ and $\sin[\varphi^{'}(y,z)t]$ go through *zero values* at the time $t = 0$ (unlike $f_s(t)$ and $\cos[\varphi^{'}(y,z)t]$), making valid the following inequality:

$$|q_i(y,z)| \ll q_r(y,z)\,. \tag{2.12}$$

Provided the function $|q(y,z)|$ can be reconstructed from complex interferograms, the set of information needed for evaluation of $\varphi_{serr}(y,z)$ will become complete. Considering (2.12) $|q(y,z)|$ can be regarded as a *reasonable approximation* for the real part $q_r(y,z)$. This will provide the way of reconstructing $\varphi^{'}(y,z)$ from the *inversion* of the integral (2.10). Subsequently, the imaginary part $q_i(y,z)$ can be calculated using (2.11). Finally, the $\varphi_{serr}(y,z)$ itself can be evaluated:

$$\varphi_{serr}(y,z) = \arcsin\left(\frac{q_i(y,z)}{|q(y,z)|}\right)\,. \tag{2.13}$$

3. Analysis of complex interferograms employing reference interferograms

In this Section it will be shown how the *complex interferograms* can be *analyzed* using FFT approach and what advantage will be gained from the use of *additional* sets of *reference* data (*interferograms* as well as *intensities* of the *signal* and *reference* part of the *diagnostic* beam). As this approach is based on the *same* already published basic principles (e.g., [3]-[6] and [9]) it won't be repeated in all its details. Instead, only the most relevant results will be used in order to start the analysis. It should be emphasized, however, that this time so far the *most general* method providing the *most accurate* results will be developed.

In order to obtain information about all three *fundamental* quantities (i.e., *phase shift*, *amplitude*, and *phase shift time derivative*) through modification of the *signal* part of the diagnostic beam by the *object* under observation, two specific quantities available from the *Fourier space* corresponding to a complex interferogram (2.7) need to be *reconstructed* using the usual technique: $v(y,z)$ (*visibility* - from the *side* lobe) and $b(y,z)$ (*background* - from the *central* lobe):

$$v(y,z) = a_s(y,z)a_r(y,z)\,|q(y,z)|\,\exp(i\varphi_{total})\,, \tag{3.1}$$

$$b(y,z) = a_s^2(y,z) + a_r^2(y,z)\,. \tag{3.2}$$

In the cited literature so far only the *individual* complex interferograms (already containing some *signal*) had been considered for processing. Having no corresponding *reference* interferogram (*signal-free*) counterpart (with only two exceptions - [8] and [9]). Under such circumstances, however, a *reliable* reconstruction of the three fundamental quantities was possible only by assuming a very good *quality* of the diagnostic beam as well as an interferometer *setup* (optical alignment and quality of its components). Also, the corresponding complex interferograms needed to contain some *signal-free* parts (for *normalization*). However, such requirements are, in practice, rarely fulfilled to the full extent.

In the past, there was a good reason for this (limited) approach, though, as in majority of cases the *diagnostic* beam was derived from the *main* laser beam (usually with a subsequent conversion to a *higher harmonic*) to keep the *jitter* between these two beams at its *minimum*. Under such circumstances, however, firing a signal-free shot was not regarded as *economical*. Also, the time delay between two such shots (with the *high* energy level) would be rather *long* (tens of minutes), thus not very *stable* from shot to shot. And using CW alignment laser as a substitute was far from satisfactory for *CI* requirements. Only recently, after solving the problem of mutual *synchronization*, major laser installations (e.g., Prague Asterix Laser System - PALS) started to employ fully *independent* diagnostic beams (usually Ti:Sa) capable of firing the reference shot with a minimum delay. Which opened the door to manifest a full *CI potential*.

In order to be able to compensate for *typical errors* both in the *phase shift* as well as the *amplitude* reconstruction, the *reference* interferograms come very handy. In the generalizations published so far it was assumed (by default) that two interfering parts of the diagnostic beam in the *absence* of any *object* would have *exactly the same* (pixel by pixel) (y,z) structure in the interference/detector plane. This could be, in principle, achieved (after a very careful setup) for interferometers with an *amplitude* division (e.g., *Michelson*, *Mach-Zehnder*). In the case of the *phase front* division (e.g., *Nomarski*) it is practically impossible (with the degree of precision required). Total *energies* between individual shots could *vary*, but their *ratio* needs to be taken into account.

Based on the above analysis it became clear that a *new* approach should be developed for the *most accurate* reconstructions even in the case of a not very high *quality* of the diagnostic beam. The good news is that it can be done. Provided the *stability* of the diagnostic beam between the *reference* and *signal* shots would be sufficient. The same holds for the interferometer setup stability.

Let us denote the *signal* and *reference* part of the diagnostic beam in the case of the *reference* interferogram (*signal-free*) by the *lower* index 0. In that case the *effect* of the *object* on the *amplitude* of the *signal* part of the diagnostic beam generating the *complex interferogram* can be expressed the following way:

$$a_s(y,z) = f(y,z)\, a_{s0}(y,z)\,, \tag{3.3}$$

where $f(y,z)$ is the *multiplicative* function containing the *amplitude* effect information ($f(y,z) = 1$ with no effect present). Denoting $s(y,z)$ as the *ratio* between *intensities* of the *reference* - $I_{r0}(y,z)$ - and the *signal* - $I_{s0}(y,z)$ - part of the diagnostic beam for the *signal-free* shots (*two* additional separate shots to record these intensities are required and shot-to-shot energy fluctuations need to be taken into account)

$$s(y,z) = \frac{I_{r0}(y,z)}{I_{s0}(y,z)} = \frac{a_{r0}^2(y,z)}{a_{s0}^2(y,z)}\,, \tag{3.4}$$

the following set of the *most accurate* general solutions can be found:

$$\varphi_p(y,z) = \arctan\left[\frac{\mathrm{Im}\frac{v(y,z)}{v_0(y,z)}}{\mathrm{Re}\frac{v(y,z)}{v_0(y,z)}}\right] - \varphi_{serr}(y,z) \tag{3.5}$$

$$f(y,z) = \sqrt{\frac{1}{p}\frac{b(y,z)}{b_0(y,z)}\left[s(y,z)+1\right] - s(y,z)} \tag{3.6}$$

$$|q(y,z)| = \frac{\frac{1}{p}\left|\frac{v(y,z)}{v_0(y,z)}\right|}{\sqrt{\frac{1}{p}\frac{b(y,z)}{b_0(y,z)}\left[s(y,z)+1\right] - s(y,z)}} \tag{3.7}$$

where $v_0(y,z)$ and $b_0(y,z)$ are the corresponding *visibility* and *background* quantities reconstructed from the *reference* interferogram and p is the *ratio* between *energies* of the diagnostic pulses used for recording of the *signal* and *reference* interferograms.

The $\varphi_p(y,z)$ is the *pure* phase shift, free of any *errors* due to the *alignment* of the interferometer as well as imperfections in shifting of the side lobe to the center of the Fourier space (as no such shifting is necessary). Also the *systematic error* $\varphi_{serr}(y,z)$ caused by the diagnostic beam *asymmetric* profile $f(t)$ was corrected using (2.13) as well as the imperfections of the *phase front* structure of the diagnostic beam (as they are the *same* for the *signal* and the *reference* shots).

The $f(y,z)$ function is a *pure* effect on the *amplitude* of the probing beam from the object under observation (regardless of the *quality* of the *diagnostic* beam). For even better practical results in the *amplitude* reconstruction the *hanning* function should be applied to both interferograms.

And eventually the $|q(y,z)|$ function is evaluated (its usage was discussed in detail in the previous Section).

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4. Concluding remarks

In this paper so far the *most advanced* version of the *complex interferometry* theoretical analysis was presented. It was demonstrated that even in the case of a not particularly good diagnostic beam *quality* the *three fundamental quantities* related to the *three degrees of freedom* (characteristic for *CI*) can be reconstructed with a high degree of *accuracy* provided both the *diagnostic beam* as well as the corresponding *optical line* feature a reasonable shot to shot *stability*. Such stability requirement is important as all together *four* shots need to be recorded: the *signal* complex interferogram, the *reference* interferogram as well as the *intensity* structures of the *signal* and *reference* part of the *diagnostic* beam. These *additional* intensity structures (to be recorded *separately*) are necessary due to the fact that their required *ratio* cannot be determined *unambiguously* from the reference interferogram only. Two solutions with *mutually inverted* values can be found (not included in this paper). But due to the *symmetry* there is no way how to decide which contribution comes from the *signal* and which from the *reference* part of the diagnostic beam.

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