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The effect of a thermal medium on the transport properties of an interacting pion gas

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Both the first and second order transport coefficients have been studied using kinetic theory approach by solving the relativistic transport equation. The first order transport coefficients such as shear viscosity, bulk viscosity and thermal conductivity are evaluated using the Chapman-Enskog approximation and the second order transport coefficients namely the relaxation times of dissipative flows and the heat viscous coupling lengths have been estimated using the Grad's 14-moment method. The effects of the medium have been implemented through a temperature dependent $\pi\pi$ cross-section obtained by including one-loop self-energies in the propagators of the exchanged ρ and σ mesons. To account for early chemical freeze out in heavy ion collisions, a temperature dependent pion chemical potential has been introduced in the distribution function. Both of these are found to affect the temperature dependence of the transport coefficients in a significant way.

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1. Introduction

The study of transport coefficients of hot hadronic matters has been attracting much interest and attention in recent years. The experimentally measured elliptic flow v_2 of hadrons in Au+Au collision at Relativistic Heavy Ion Collider (RHIC), can be interpreted in terms of viscous hydrodynamics with a small value of η/s, which is close to the quantum bound $1/4\pi$ [1], η and s being the coefficient of shear viscosity and entropy density respectively. Such kind of results indicate the strongly interacting nature of the matter created in heavy ion collisions. This interpretation based on the measured elliptic flow v_2 of hadrons in terms of viscous hydrodynamics however depends sensitively on the value of η/s. The behaviour of η and the bulk viscous coefficient ζ as a function of temperature is particularly relevant in the context of non-ideal hydrodynamic simulations of heavy ion collisions. A lot of interest has been generated, leading to quite a few estimates of the transport coefficients of both partonic [2, 3] as well as hadronic [4, 5, 6, 7] constituents of strongly interacting matter. The effects of heat flow in heavy ion collisions has received much less attention. This is presumably on account of the fact that the net baryon number in the central rapidity region at the RHIC and LHC is very small. However, at FAIR energies or in the low energy runs at RHIC the baryon chemical potential is expected to be significant and heat conduction by baryons may play a more important role. Based on such a scenario a few studies of heat conduction by pions have been carried out. Using the experimental $\pi\pi$ cross-section the thermal conductivity of a pion gas was estimated in [8, 4, 9] whereas in [10] a unitarized scattering amplitude was employed.

As already understood the created matter in heavy ion collisions undergoes dissipative processes on its way to space time evolution and hence requires a non-ideal theory to describe its kinematics. The first order theories of dissipative fluid dynamics that include the coefficients of viscosity and thermal conductivity do not suffice this description since they face severe causality violation problem. Hence we need a causal second order theory where the corresponding relaxation times τ go as input in the viscous hydrodynamic equations [11, 12]. They indicate the time taken by the fluxes to relax to their steady state values and consequently play an important role in determining the space-time evolution of relativistic heavy ion collisions. The first order transport coefficients go as inputs in these relaxation times. The temperature dependence of the relaxation times have been estimated in [4, 9, 8] with a parameterized cross section which is independent of temperature. Constant values of transport coefficients have been used in [11] and in [13] these quantities have been evaluated using conformal quantum field theory for a strongly coupled system.

In the kinetic theory approach the dynamics of interaction resides in the differential cross-section which goes as an input in the expressions of all these transport coefficients. In almost all estimations of the transport coefficients a vacuum cross-section was employed. In this work we consider a medium dependent interaction cross section evaluated at finite temperature to estimate first the viscosities and thermal conductivity and use them to study the temperature dependence of the relaxation times of the dissipative flows.

2. First order transport coefficients in Chapman-Enskog method

The evolution of the phase space distribution of the pions is governed by the equation

$$p^\mu \partial_\mu f(x,p) = C[f] \tag{2.1}$$

where $C[f]$ is the collision integral. For binary elastic collisions $p+k \to p'+k'$ which we consider, this is given by [9]

$$C[f] = \int d\Gamma_k \, d\Gamma_{p'} \, d\Gamma_{k'} [f(x,p')f(x,k')\{1+f(x,p)\}\{1+f(x,k)\} \\ -f(x,p)f(x,k)\{1+f(x,p')\}\{1+f(x,k')\}] \, W \quad , \tag{2.2}$$

where the interaction rate, $W = \frac{s}{2} \frac{d\sigma}{d\Omega}(2\pi)^6 \delta^4(p+k-p'-k')$ and $d\Gamma_q = \frac{d^3q}{(2\pi)^3 q_0}$. The $1/2$ factor comes from the indistinguishability of the initial state pions. For small deviation from local equilibrium we write, in the first Chapman-Enskog approximation

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p), \quad \delta f(x,p) = f^{(0)}(x,p)[1+f^{(0)}(x,p)]\phi(x,p) \quad , \tag{2.3}$$

where the equilibrium distribution function is given by $f^{(0)}(x,p) = \left[e^{\frac{p^\mu u_\mu(x)-\mu(x)}{T(x)}} - 1\right]^{-1}$, with $T(x)$, $u_\mu(x)$ and $\mu(x)$ representing the local temperature, flow velocity and chemical potential respectively. Putting (2.3) in (2.1) the deviation function $\phi(x,p)$ is seen to satisfy

$$p^\mu \partial_\mu f^{(0)}(x,p) = -\mathscr{L}[\phi] \, , \tag{2.4}$$

where the linearized collision term

$$\mathscr{L}[\phi] = \quad f^{(0)}(x,p) \int d\Gamma_k \, d\Gamma_{p'} \, d\Gamma_{k'} f^{(0)}(x,k)\{1+f^{(0)}(x,p')\}\{1+f^{(0)}(x,k')\} \\ [\phi(x,p)+\phi(x,k)-\phi(x,p')-\phi(x,k')] \, W \, . \tag{2.5}$$

Using the form of $f^{(0)}(x,p)$ as given above on the left side of (2.4) and eliminating time derivatives with the help of equilibrium thermodynamic laws we arrive at,

$$[Q\partial_\nu u^\nu + p_\mu \nabla^{\mu\nu}(p_\sigma u^\sigma + m_\pi h)(T^{-1}\partial_\nu T + Du_\nu) - \langle p_\mu p_\nu \rangle \langle \partial^\mu u^\nu \rangle] f^{(0)}(1+f^{(0)}) = -T\mathscr{L}[\phi] \, . \tag{2.6}$$

In this equation $Q = -\frac{1}{3}m^2 + (p_\mu u^\mu)^2\{\frac{4}{3} - \gamma^{'}\} + \{(\gamma^{''}-1)mh - \gamma^{'''}T\}p_\mu u^\mu$, and $\langle \nabla^\mu U^\nu \rangle = \frac{1}{2}[\nabla^\mu U^\nu + \nabla^\nu U^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\sigma U^\sigma]$. To be a solution, ϕ must be a linear combination of the thermodynamic forces appearing on the left hand side of the transport equation as the following

$$\phi = A\partial_\nu u_\nu + B_\mu \nabla^{\mu\nu}(T^{-1}\partial_\nu T - Du_\nu) - C_{\mu\nu}\langle \partial^\mu u^\nu \rangle \, , \tag{2.7}$$

which on substituting on the right hand side of (2.6) we obtain a set of three integral equation satisfied by the coefficients, $A, B_\mu, C_{\mu\nu}$

$$\mathscr{L}[A] = -Qf^{(0)}(p)\{1+f^{(0)}(p)\}/T \, , \\ \mathscr{L}[B_\mu] = -\Delta_{\mu\sigma}p^\sigma(p.u-h)f^{0)}(p)\{1+f^{(0)}(p)\}/T \, , \\ \mathscr{L}[C_{\mu\nu}] = -\langle p_\mu p_\nu \rangle f^{(0)}(p)\{1+f^{(0)}(p)\}/T \, . \tag{2.8}$$

Here, $C_{\mu\nu} = C\langle p_\mu p_\nu \rangle$ and $B_\mu = B\Delta_{\mu\nu}p^\mu$. The other details are discussed in [14, 15, 16].

In an imperfect fluid, the dissipative part of the energy momentum stress tensor is [17],

$$\Delta T^{\mu\nu} = 2\eta\langle\partial^\mu u^\nu\rangle + \zeta\Delta^{\mu\nu}\partial_\sigma u^\sigma + \lambda\{\Delta^{\mu\alpha}U^\nu + \Delta^{\nu\alpha}U^\nu\}(\partial_\alpha T - Tu.\partial u_\alpha) \tag{2.9}$$

The first two terms corresponds to the viscous effects while the last term indicates thermal dissipation. The dissipative part of heat flow or the energy 4-flow is related to thermal conductivity by the following equation, [18],

$$\Delta I^\mu = \lambda\Delta^{\mu\alpha}(\partial_\alpha T - Tu.\partial u_\alpha) \tag{2.10}$$

Again these quantities can be expressed in integral forms over the particle distribution function as,

$$\Delta T^{\mu\nu} = \int d\Gamma_p f^{(0)}(1+f^{(0)})C_{\mu\nu}\langle p^\mu p^\nu\rangle\langle\partial^\mu u^\nu\rangle + \int d\Gamma_p f^{(0)}(1+f^{(0)})QA\Delta^{\mu\nu}\partial_\sigma u^\sigma \tag{2.11}$$

$$\Delta I^\mu = \int \frac{d^3p}{(2\pi)^3 p^0}(p.U - h)p^\sigma\Delta^\mu_\sigma f_0\{1+f_0\} \tag{2.12}$$

Comparing, we obtain the expressions of transport coefficients,

$$\begin{aligned}\zeta &= -\int \frac{d^3p}{(2\pi)^3p^0}QAf_0(1+f_0)\\ \lambda &= \frac{1}{3T}\int \frac{d^3p}{(2\pi)^3p^0}B_\nu p^\nu(p.u-h)f_0(1+f_0)\\ \eta &= -\frac{1}{10}\int \frac{d^3p}{(2\pi)^3p^0}f_0(1+f_0)C\langle p^\alpha p^\beta\rangle\langle p_\alpha p_\beta\rangle\end{aligned} \tag{2.13}$$

Here we follow the procedure outlined in [9] in which A,B_μ,$C_{\mu\nu}$ is expanded in terms of orthogonal Laguerre polynomials of half integral order. After some simplifications (discussed in detail in Refs. [15]) the first approximation to transport coefficients comes out to be,

$$\zeta = T\frac{\alpha_2^2}{a_{22}},\ \ \lambda = -\frac{T}{3m}\frac{\beta_1^2}{b_{11}},\ \ \eta = \frac{T}{10}\ \frac{\gamma_0^2}{c_{00}}\ \ . \tag{2.14}$$

where a_{22}, b_{11} and c_{00} can be expressed in terms of the integral $X_\alpha(z)$ as

$$a_{22} = z^2X_3(z) \tag{2.15}$$

$$b_{11} = -z[X_2(z) + X_3(z)] \tag{2.16}$$

and

$$c_{00} = 2[X_1(z) + X_2(z) + \frac{1}{3}X_3(z)] \tag{2.17}$$

where

$$\begin{aligned}X_\alpha(z) = {} & \frac{8z^4}{[S_2^1(z)]^2}\,e^{(-2\mu/T)}\int_0^\infty d\psi\ \cosh^3\psi\sinh^7\psi\int_0^\pi d\Theta\sin\Theta\frac{1}{2}\frac{d\sigma}{d\Omega}(\psi,\Theta)\int_0^{2\pi}d\phi\\ & \int_0^\infty d\chi\sinh^{2\alpha}\chi\int_0^\pi d\theta\sin\theta\frac{e^{2z\cosh\psi\cosh\chi}}{(e^E-1)(e^F-1)(e^G-1)(e^H-1)}\,M_\alpha(\theta,\Theta)\end{aligned} \tag{2.18}$$

with

$$
\begin{aligned}
E &= z(\cosh\psi\cosh\chi - \sinh\psi\sinh\chi\cos\theta) - \mu/T \\
F &= z(\cosh\psi\cosh\chi - \sinh\psi\sinh\chi\cos\theta') - \mu/T \\
G &= E + 2z\sinh\psi\sinh\chi\cos\theta \\
H &= F + 2z\sinh\psi\sinh\chi\cos\theta' \; .
\end{aligned}
\tag{2.19}
$$

The functions M_α stand for

$$
\begin{aligned}
M_1(\theta,\Theta) &= 1 - \cos^2\Theta \; , \\
M_2(\theta,\Theta) &= \cos^2\theta + \cos^2\theta' - 2\cos\theta\cos\theta'\cos\Theta \; , \\
M_3(\theta,\Theta) &= [\cos^2\theta - \cos^2\theta']^2
\end{aligned}
\tag{2.20}
$$

and

$$
\cos\theta' = \cos\theta\cos\Theta - \sin\theta\sin\Theta\cos\phi \; . \tag{2.21}
$$

The quantities α_2, β_1 and γ_0 are discussed in next section.

3. Second order transport coefficients in Grad's 14 moment method

The basic idea of the moment method is to obtain an approximate solution of the transport equation (2.1) by expanding the distribution function $f(x,p)$ in momentum space around its local equilibrium value when the deviation from it is small. We write

$$
f(x,p) = f^{(0)}(x,p) + \delta f(x,p), \quad \delta f(x,p) = f^{(0)}(x,p)[1+f^{(0)}(x,p)]\phi(x,p) \tag{3.1}
$$

where the equilibrium distribution function is as before for a bosonic system with ϕ is a quantity which amounts the deviation.

Putting (3.1) in (2.1) the left hand side of the later splits into a term containing derivative over the equilibrium distribution and another containing derivative over ϕ,

$$
p_\mu\partial^\mu f_0 + f_0(1+f_0)p_\mu\partial^\mu\phi = -\mathscr{L}[\phi], \tag{3.2}
$$

which after some simplification reduces to

$$
\begin{aligned}
\Pi^\mu\partial_\mu f_0 = \; & f_0(1+f_0) \\
& \times \Bigg[(\tau-\hat{h})\Pi_\alpha\frac{\nabla^\alpha T}{T} + \frac{1}{Tn}\Pi_\alpha\nabla^\alpha P - \langle\Pi_\mu\Pi_\nu\rangle\langle\nabla^\mu u^\nu\rangle + \hat{Q}\nabla^\mu u_\mu - \tau\Pi_\mu Du^\mu \\
& + \tau[\{\tau(1-\gamma')+(\gamma''-1)\hat{h}-\gamma'''\}\frac{\delta}{P}\nabla_\alpha I_q^\alpha - \frac{\delta'}{nT}\nabla_\alpha I_q^\alpha]\Bigg] \quad ,
\end{aligned}
\tag{3.3}
$$

with $\Pi^\mu = p^\mu/T$, $\tau = p\cdot u/T$ and $\hat{Q} = Q/T^2$, where, $Q = -\frac{1}{3}m_\pi^2 + (p\cdot u)^2\{\frac{4}{3}-\gamma'\} + p\cdot u\{(\gamma''-1)h - \gamma'''T\}$. The reduced enthalpy per particle is defined as, $\hat{h} = h/T$ and P and I_q^μ stand for the pressure and heat flow vector respectively. The γ's and δ's are mentioned in details in [19].

For the remaining two terms in (3.2) we need to define the deviation function ϕ and its derivative. Since the distribution function is a scalar depending on the particle momentum p^μ and the space-time coordinate x^μ, the deviation function is expressed as a sum of scalar products of tensors formed from p^μ and tensor functions of x^μ. In terms of irreducible tensors ϕ is written as

$$\phi(x,p) = A(x,\tau) - B_\mu(x,\tau)\langle\Pi^\mu\rangle + C_{\mu\nu}(x,\tau)\langle\Pi^\mu\Pi^\nu\rangle. \tag{3.4}$$

The notation $\langle\rangle$ denotes the irreducible tensors defined as $\langle\Pi^\mu\rangle = \Delta^{\mu\nu}\Pi_\nu$ and $\langle\Pi^\mu\Pi^\nu\rangle \equiv [\frac{1}{2}(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\nu\alpha}\Delta^{\mu\beta}) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}]\Pi_\alpha\Pi_\beta$.

Now the x and τ-dependent coefficient functions A, B_μ and $C_{\mu\nu}$ are further expanded in a power series in τ such that the last power is the one which gives a non-zero contribution to the collision term,

$$\begin{aligned} A(x,\tau) =& \ A_0 + A_1(x)\tau + A_2(x)\tau^2 = \sum_{s=0}^{2} A_s(x)\tau^s, && (3.5)\\ B_\mu(x,\tau) =& \ B_{0\mu}(x) + B_{1\tau}(x)\tau = \sum_{s=0}^{1}(B_s)_\mu(x)\tau^s, && (3.6)\\ C_{\mu\nu}(x,\tau) =& \ (C_0)_{\mu\nu}(x). && (3.7) \end{aligned}$$

This leaves us with six x-dependent coefficients A_0, A_1, A_2, $B_{0\mu}$, $B_{1\mu}$ and $C_{0\mu\nu}$. It is convenient to express them in terms of the thermodynamic fluxes (irreducible flows) in the following way,

$$\begin{aligned} A_2 =& \ \frac{\Pi}{nT\alpha_2} && (3.8)\\ A_1 =& \ \frac{(a_1a_4 - a_2a_3)}{(a_2^2 - a_1a_3)}\frac{\Pi}{nT\alpha_2} && (3.9)\\ A_0 =& \ \frac{(a_2a_4 - a_3^2)}{(a_1a_3 - a_2^2)}\frac{\Pi}{nT\alpha_2} && (3.10)\\ B_{1\nu} =& \ \frac{I_q^\mu\Delta^{\mu\nu}}{nT\beta_1} && (3.11)\\ B_{0\nu} =& \ \frac{I_q^\mu\Delta^{\mu\nu}}{nT\beta_1}(-\frac{b_1}{b_0}) && (3.12)\\ \langle(C_0)^{\mu\nu}\rangle =& \ -\frac{5}{\rho\gamma_0}\langle\Pi^{\mu\nu}\rangle \ , && (3.13) \end{aligned}$$

where Π and $\langle\Pi^{\mu\nu}\rangle$ are bulk and shear viscous fluxes respectively. The details can be found in [19]. Defining all the space-time dependent coefficients of equation (3.4) in terms of the known functions it is now possible to specify the deviation function ϕ completely. Knowing ϕ, we now go back and use it in the Boltzmann equation (3.2) to evaluate the equations of motion for the dissipative fluxes.

3.1 Bulk viscous pressure equation

Taking inner product of both sides of equation (3.2) with τ^2 and applying the (inner product) properties of irreducible tensors [18] we obtain the equation of motion for bulk viscous pressure equation,

$$\begin{aligned}\Pi = \zeta[\nabla_\mu u^\mu - & \frac{1}{n^2\alpha_2^2}\{\frac{a_3^2 - 2a_2a_3a_4 + a_1a_4^2}{a_2^2 - a_1a_3} + a_5\}D\Pi \\ - & \frac{1}{n^2\alpha_2}\{\frac{3}{\beta_1}(\frac{b_1b_2}{b_0} - b_3) + (1-\gamma')\delta(\frac{S_2^1}{S_2^2})a_4 \\ + & \{(\hat{h}(\gamma''-1) - \gamma''')\delta(\frac{S_2^1}{S_2^2}) - \delta'\}a_3\}\nabla_\mu I_q^\mu] \,. \end{aligned} \tag{3.14}$$

Retaining only the first term on the right hand side of (3.14) the equation for the bulk viscous pressure reduces to the same in the first order theory of dissipative fluids with the coefficient of this term as the bulk viscous coefficient ζ. Equation (3.14) is indeed hyperbolic and contains a time derivative of the bulk viscous pressure. This yields a relaxation time for bulk viscous pressure given by,

$$\tau_\zeta = \zeta\frac{1}{n^2\alpha_2^2}[\frac{a_3^2 - 2a_2a_3a_4 + a_1a_4^2}{a_2^2 - a_1a_3} + a_5], \tag{3.15}$$

with

$$\begin{aligned} a_1 = & \frac{n}{T}\{\frac{S_2^0}{S_2^1}\}, \\ a_2 = & \frac{n}{T}\{z\frac{S_3^0}{S_2^1} - 1\}, \\ a_3 = & \frac{n}{T}z^2\{\frac{S_2^0}{S_2^1} + 3z^{-1}\frac{S_3^1}{S_2^1}\}, \\ a_4 = & \frac{n}{T}z^3\{15z^{-2}\frac{S_3^2}{S_2^1} + 2z^{-1} + \frac{S_3^0}{S_2^1}\}, \\ a_5 = & \frac{n}{T}z^4[6z^{-1}\{\frac{S_3^1}{S_2^1} + 15z^{-2}\frac{S_3^3}{S_2^1}\} + \{\frac{S_2^0}{S_2^1} + 15z^{-2}\frac{s_2^2}{S_2^1}\}]. \end{aligned} \tag{3.16}$$

$$\alpha_2 = z^3[\frac{1}{3}(\frac{S_3^0}{S_2^1} - z^{-1}) + (\frac{S_2^0}{S_2^1} + \frac{3}{z}\frac{S_3^1}{S_2^1})\{(1-\gamma'')\frac{S_3^1}{S_2^1} + \gamma'''z^{-1})\} - (\frac{4}{3} - \gamma')\{\frac{S_3^0}{S_2^1} + 15z^{-2}\frac{S_3^2}{S_2^1} + 2z^{-1}\}] \tag{3.17}$$

The terms S_n^α are defined as $S_n^\alpha(z) = \sum_{k=1}^{\infty} e^{k\mu/T}k^{-\alpha}K_n(kz)$, $K_n(x)$ denoting the modified Bessel function of order n with $z = m_\pi/T$.

3.2 Heat flow equation

In this case we take the inner product of both sides of equation (3.2) with $\langle\Pi^\mu\rangle\tau$. Following similar techniques as above we get the equation for heat flow,

$$I_q^\mu = T\lambda[\{\frac{\nabla^\mu T}{T} - \frac{\nabla^\mu P}{nh}\} - \frac{1}{nT}\{\beta'' DI_q^\mu + \gamma''\nabla_\nu\langle\Pi^{\mu\nu}\rangle + \alpha''\nabla^\mu\Pi\}], \tag{3.18}$$

with

$$\beta'' = -\frac{1}{\beta_1}\{\frac{9T}{n\beta_1}(b_3 - \frac{b_1 b_2}{b_0}) - \frac{3T}{n}\frac{b_2}{\hat{h}}\}, \quad (3.19)$$

$$\gamma'' = \frac{1}{\beta_1}\{\frac{\gamma_1}{\gamma_0)} + \frac{3T}{n}\frac{b_2}{\hat{h}}\}, \quad (3.20)$$

$$\alpha'' = \frac{3T}{n}\frac{1}{\beta_1}[\frac{1}{\alpha_2}\{b_1\frac{a_2a_4 - a_3^2}{a_1a_3 - a_2^2} + b_2\frac{a_1a_4 - a_2a_3}{a_2^2 - a_1a_3} + b_3\} + \frac{b_2}{\hat{h}}]. \quad (3.21)$$

So from the above equation the relaxation time for heat flow is given by,

$$\tau_\lambda = \lambda T\frac{1}{nT}\beta'', \quad (3.22)$$

with

$$b_0 = -\frac{n}{T},$$
$$b_1 = -\frac{n}{T}z\frac{S_3^1}{S_2^1},$$
$$b_2 = -\frac{n}{T}\{5z\frac{S_3^2}{s_2^1} + z^2\},$$
$$b_3 = -\frac{n}{T}\{30z\frac{S_3^3}{S_2^1} + 5z^2\frac{S_2^2}{S_2^1} + z^3\frac{S_3^1}{S_2^1}\}. \quad (3.23)$$

$$\beta_1 = 3z^2[1 + 5z^{-1}\frac{s_3^2}{S_2^1} - (\frac{S_3^1}{S_2^1})^2]. \quad (3.24)$$

3.3 Shear viscous pressure equation

Multiplying both sides of equation (3.2) with $\langle\Pi^{\mu\nu}\rangle$ we applying the inner product properties of irreducible tensors as before. This produces the equation of motion for shear viscous pressure given by,

$$\langle\Pi^{\mu\nu}\rangle = \eta[2\langle\nabla^\mu u^\nu\rangle - \frac{1}{nT}\{\gamma'''D\langle\Pi^{\mu\nu}\rangle - \beta'''\nabla^\mu I_q^\nu\}], \quad (3.25)$$

with

$$\gamma''' = \frac{z^2[\frac{S_2^2}{S_2^1} + 6z^{-1}\frac{S_3^3}{S_2^1}]}{[z\frac{S_3^2}{S_2^1}]^2}, \quad (3.26)$$

$$\beta''' = \frac{6}{\beta_1}[\hat{h} - (6\frac{S_3^3}{S_3^2} + s\frac{S_2^2}{S_3^2})]. \quad (3.27)$$

The coefficient of shear viscosity can be followed from the first term of the right hand side of eqn. (3.25) with,

$$\gamma_0 = -10\frac{S_3^2}{S_2^1}. \quad (3.28)$$

From (3.25) the relaxation time for shear viscous pressure is obtained as,

$$\tau_\eta = \eta \frac{1}{nT} \gamma'''. \tag{3.29}$$

4. The in-medium $\pi\pi$ cross-section

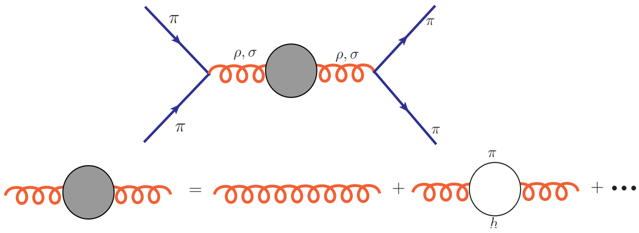

Figure 1: $\pi - \pi$ scattering with self-energy corrections

The $\pi\pi$ cross-section is the key dynamical input for the evaluation of transport coefficients mentioned in earlier sections. Here the scattering is assumed to proceed via σ and ρ meson exchange within the thermal medium. From the effective interaction [20]

$$\mathscr{L} = g_\rho \vec{\rho}^\mu \cdot \vec{\pi} \times \partial_\mu \vec{\pi} + \frac{1}{2} g_\sigma m_\sigma \vec{\pi} \cdot \vec{\pi} \sigma \tag{4.1}$$

the matrix elements for $\pi\pi$ scattering are given by the following expressions where the widths of the σ and ρ mesons have been introduced in the propagators involved in the corresponding s-channel processes. We thus have

$$\begin{aligned}
\mathscr{M}_{I=0} &= 2g_\rho^2 \left[\frac{s-u}{t-m_\rho^2} + \frac{s-t}{u-m_\rho^2} \right] \\
&+ g_\sigma^2 m_\sigma^2 \left[\frac{3}{s-m_\sigma^2 + i m_\sigma \Gamma_\sigma} + \frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2} \right] \\
\mathscr{M}_{I=1} &= g_\rho^2 \left[\frac{2(t-u)}{s-m_\rho^2 + i m_\rho \Gamma_\rho} + \frac{t-s}{u-m_\rho^2} - \frac{u-s}{t-m_\rho^2} \right] \\
&+ g_\sigma^2 m_\sigma^2 \left[\frac{1}{t-m_\sigma^2} - \frac{1}{u-m_\sigma^2} \right] .
\end{aligned} \tag{4.2}$$

Defining the isospin averaged amplitude as $|\mathscr{M}|^2 = \frac{1}{9} \sum_I |\mathscr{M}_I|^2$ and ignoring the non-resonant $I=2$ contribution, the cross-section is found to agree very well with the estimate based on measured phase-shifts given in [4]. In this way it is ensured that the dynamical model is normalized against experimental data.

To obtain the in-medium cross-section we replace the vacuum width in the above expressions by the ones in the medium as indicated in fig.1. The width is related to the imaginary part of the self-energy through the relation [21]

$$\Gamma(T,M) = -M \mathrm{Im}\Pi(T,M) \tag{4.3}$$

where Π denotes the one-loop self energy diagrams shown in fig. 1 and are evaluated using the real-time formalism of thermal field theory. The σ meson self-energy is obtained from the $\pi\pi$ loop diagram whereas in case of the ρ meson the $\pi\pi$, $\pi\omega$, πh_1, πa_1 graphs are evaluated using interactions from chiral perturbation theory [22]. The longitudinal and transverse parts of the ρ self-energy are defined in terms of Π^μ_μ as [23]

$$\Pi^T = -\frac{1}{2}(\Pi^\mu_\mu + \frac{q^2}{\vec{q}^2}\Pi_{00}), \quad \Pi^L = \frac{1}{\vec{q}^2}\Pi_{00}, \quad \Pi_{00} \equiv u^\mu u^\nu \Pi_{\mu\nu} \; . \tag{4.4}$$

The momentum dependence being weak [23] we take an average over the polarizations,

$$\Pi = \frac{1}{3}[2\Pi^T + \Pi^L] \; . \tag{4.5}$$

The imaginary part of the self-energy obtained by evaluating the loop diagrams is given by [24]

$$\begin{aligned}
&\mathrm{Im}\Pi(q_0,\vec{q}) = -\pi \int \frac{d^3k}{(2\pi)^3 4\omega_\pi \omega_h} \times \\
&\Big[N_1\{(1 - f^{(0)}(\omega_\pi) - f^{(0)}(\omega_h))\delta(q_0 - \omega_\pi - \omega_h) \\
&+(f^{(0)}(\omega_\pi) - f^{(0)}(\omega_h))\delta(q_0 - \omega_\pi + \omega_h)\} + \\
&N_2\{(f^{(0)}(\omega_h) - f^{(0)}(\omega_\pi))\delta(q_0 + \omega_\pi - \omega_h) \\
&-(1 - f^{(0)}(\omega_\pi) - f^{(0)}(\omega_h))\delta(q_0 + \omega_\pi + \omega_h)\}\Big]
\end{aligned} \tag{4.6}$$

where $f^{(0)}(\omega) = \frac{1}{e^{(\omega-\mu_\pi)/T}-1}$ is the Bose distribution function with arguments $\omega_\pi = \sqrt{\vec{k}^2 + m_\pi^2}$ and $\omega_h = \sqrt{(\vec{q}-\vec{k})^2 + m_h^2}$. The terms N_1 and N_2 stem from the vertex factors and the numerators of vector propagators, details of which can be found in [24]. The angular integration is done using the δ-functions which define the kinematic domains for occurrence of scattering and decay processes which lead to loss or gain of ρ (or σ) mesons in the medium. To account for the substantial 3π and $\rho\pi$ branching ratios of the heavy particles in the loop the self-energy function is convoluted with their widths,

$$\begin{aligned}
\Pi(q,m_h) \;=\; & \frac{1}{N_h}\int_{(m_h-2\Gamma_h)^2}^{(m_h+2\Gamma_h)^2} dM^2 \times \\
& \frac{1}{\pi}\mathrm{Im}\left[\frac{1}{M^2 - m_h^2 + iM\Gamma_h(M)}\right]\Pi(q,M)
\end{aligned} \tag{4.7}$$

with

$$\begin{aligned}
N_h \;=\; & \int_{(m_h-2\Gamma_h)^2}^{(m_h+2\Gamma_h)^2} dM^2 \times \\
& \frac{1}{\pi}\mathrm{Im}\left[\frac{1}{M^2 - m_h^2 + iM\Gamma_h(M)}\right] \; .
\end{aligned} \tag{4.8}$$

The contribution from the loops with these unstable particles can thus be looked upon as multi-pion effects in $\pi\pi$ scattering.

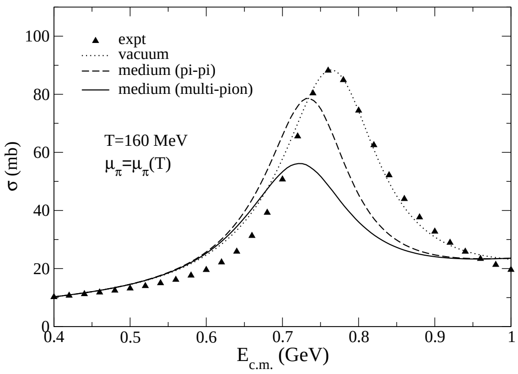

Figure 2: The $\pi\pi$ cross-section as a function of centre of mass energy. The dashed and solid lines respectively indicate the cross-section obtained using the vacuum and in-medium widths of the ρ and σ mesons.

It is well known that hadrons undergo chemical freeze-out quite early at a temperature close to the crossover temperature $T\sim 170$ MeV [25]. The number-changing inelastic collisions cease at this point and a chemical potential gradually builds up with decreasing temperature until kinetic freeze-out [26] which we take to be ~ 100 MeV. Here the temperature-dependent pion chemical potential is taken from Ref. [27] which is parameterized as

$$\mu_\pi(T) = a + bT + cT^2 + dT^3 \tag{4.9}$$

with $a = 0.824$, $b = 3.04$, $c = -0.028$, $d = 6.05\times 10^{-5}$ and T, μ_π in MeV.

We now plot in fig. 2 the total $\pi\pi$ cross-section defined by $\sigma(s) = \frac{1}{2}\int d\Omega \frac{d\sigma}{d\Omega}$ with $\frac{d\sigma}{d\Omega} = \frac{|\mathscr{M}|^2}{64\pi^2 s}$. The increase in the widths of the exchanged ρ and σ on account of thermal emission and absorption is reflected in a significant change in both the magnitude and shape of the cross-section as a function of the c.m. energy.

5. Results

In result section let us start with the results of shear viscosity to entropy density ratio η/s. In Fig. (3) for $\mu_\pi = 0$ the upper set of curves with filled circles show the usual decreasing trend as seen, for example in [7]. This trend is reversed when $\mu_\pi(T)$ is used and η/s increases with T. The values in all cases remain well above $1/4\pi$. In the two set of curves the distinctly separated three curves with vacuum and in medium cross sections respectively exhibit the effect of the thermal medium on the shear viscosity discussed so far. The curves with thermal ρ propagator including heavy meson loops show a larger enhancement indicating a greater effect of medium on the shear viscosity at finite temperature.

Then we have the results for bulk viscosity ζ as a function of temperature T. In Fig. (4) the three sets of curves correspond to different values of the pion chemical potential. The clear separation between the curves in each set displays a significant effect brought about by the medium dependence of the cross-section. A large dependence on the pion chemical potential is also inferred since the three sets of curves appear nicely separated.

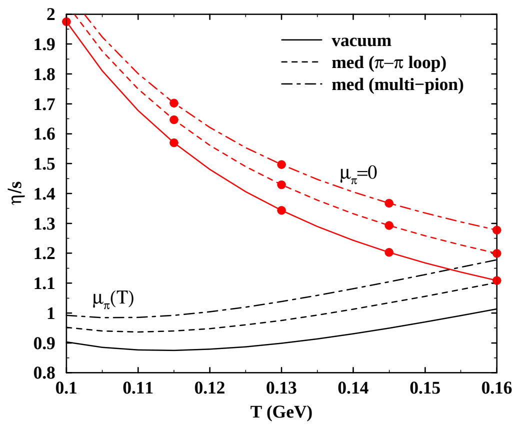

Figure 3: η/s as a function of T.

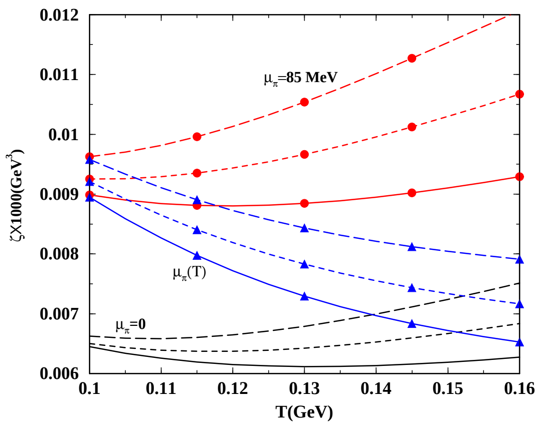

Figure 4: ζ as a function of T. In each set the solid line indicates vacuum cross-section, the dotted line for in-medium modification due to pion loop and the dashed line for loops with heavy mesons.

We next turn to the results of thermal conductivity. In fig. (5) we plot λT as a function of T evaluated in the Chapman-Enskog approach. The effect of a hot medium as well as temperature dependent chemical potential is clearly visible for those plots.

We now present the results of numerical evaluation of the relaxation times. We start with τ_ζ, as a function of temperature. In Fig. (6) the upper set of curves merges the lower one at $100 MeV$ representing the point of kinetic freeze-out indicating that $\mu_\pi(T)$ interpolates between the points representing chemical and kinetic freezeouts. In each set the τ_ζ shows a decreasing trend with temperature which is in accordance with [4]. The three different curves in each set show the effect of the medium on account of the $\pi\pi$ cross section. These curves with medium cross sections appear to be enhanced with respect to the vacuum ones indicating the effect of a thermal medium on τ_ζ.

Next we plot the τ_λ against temperature for the same two different values of pion chemical potentials mentioned above. We notice that the medium modified cross sections evaluated at finite

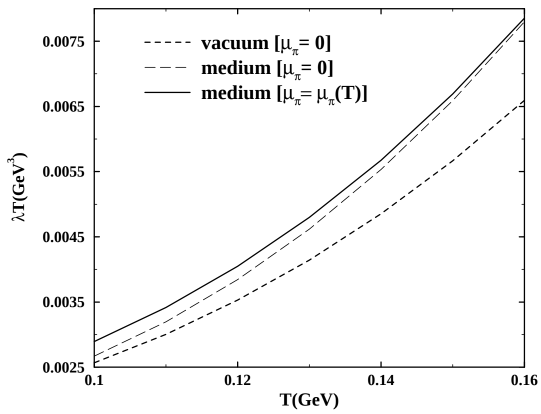

Figure 5: λT as a function of T for $\pi\pi$ cross-section in vacuum and in medium.

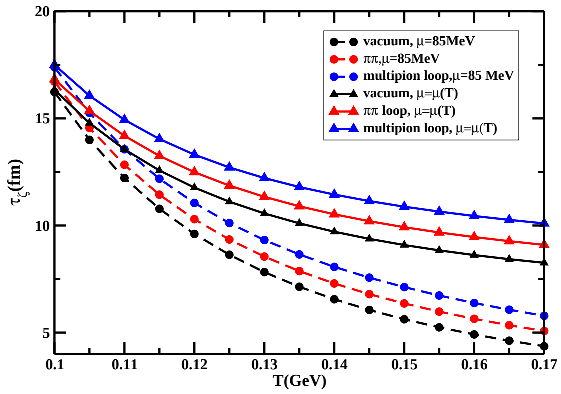

Figure 6: Relaxation time of bulk viscous pressure as a function of T for different $\pi\pi$ cross-section with temperature dependent pion chemical potential.

temperature influence the temperature dependence of τ_λ which appear to be more enhanced for heavier mesons in the ρ propagator than the $\pi\pi$ loop only. In Fig. (7) the nicely separated three curves in each set reveal the effects of medium on the temperature dependence of τ_λ.

Finally we present our result of τ_η, i.e, the relaxation time of the shear viscous flow for a medium induced $\pi\pi$ cross section. In each set two different values of chemical potential demonstrates the effect of μ_π on the values of τ_η. Moreover the effect of medium is shown by the enhancement of the curves which appears to be more significant for multipion case than $\pi\pi$ loop. In all the three cases (τ_ζ, τ_λ and τ_η) the effect of medium on relaxation times increases with increasing temperature.

6. Discussions

In this work the main focus was to emphasize the role of medium modifications of the cross-section in the evaluation of the transport coefficients. The transport coefficients and their temper-

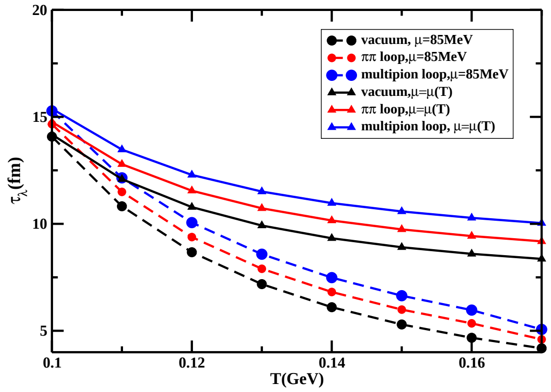

Figure 7: Relaxation time of heat flow as a function of T for different $\pi\pi$ cross-section with temperature dependent pion chemical potential.

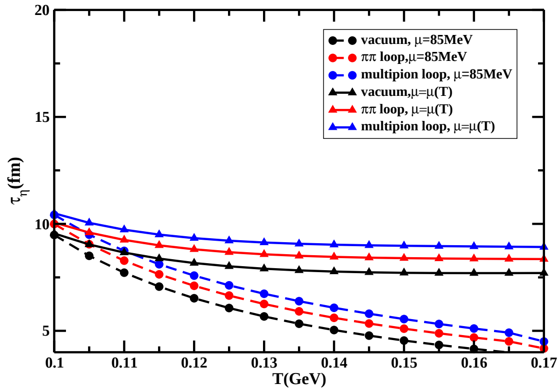

Figure 8: Relaxation time of shear viscous pressure as a function of T for different $\pi\pi$ cross-section with temperature dependent pion chemical potential.

ature dependence could affect the quantitative estimates of signals of heavy ion collisions particularly where hydrodynamic simulations are involved. For example, it has been argued in [28] that corrections to the freeze-out distribution due to bulk viscosity can be significant. As a result the hydrodynamic description of the p_T spectra and elliptic flow of hadrons could be improved by including a realistic temperature dependence of the transport coefficients. So a realistic evaluation of these quantities is essential to obtain the proper temperature profile and consequently the cooling laws of the evolving system. In addition it is found that the relaxation times of the bulk viscous flow and the heat flow to be of similar magnitude to that of the shear viscous flow which suggests that they should all be taken into consideration in dissipative hydrodynamic simulations.

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